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Soret and Dufour Effects on Free Convection of Non-Newtonian Power Law Fluids with Yield Stress from a Vertical Flat Plate in Saturated Porous Media

P. A. Lakshmi Narayana,[1*] P. V. S. N. Murthy,[1] and A. Postelnicu[2]

[1]Department of Mathematics, Indian Institute of Technology, Kharagpur, KHARAGPUR, West Bengal, 721 302, India

[2]Department of Fluid Mechanics and Thermal Engineering, Transilvania University, Brasov, 500036, Romania

*E-mail: puranam.ananth@gmail.com

ABSTRACT

In this paper we deal with the problem of free convection heat and mass transfer of a non-Newtonian power law fluid with yield stress from a vertical flat plate embedded in a fluid-saturated Darcy porous medium, considering Soret and Dufour effects. Here, we consider two different types of boundary conditions. In the first case, we assume the vertical wall is maintained at uniform wall temperature and concentration, and in the second case, we assume the wall is maintained at uniform wall heat and mass flux conditions. In both these cases, similarity solutions are possible. The results are analyzed with reference to the Soret and Dufour parameters along with other parameters that arise due to the non-Newtonian character of the fluid. Temperature and concentration profiles in the boundary layer are presented, together with the Nusselt and Sherwood numbers, for various combinations of problem parameters.

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NOMENCLATURE

C	concentration	x, y	Cartesian coordinates along and normal to the surface, respectively
c_p	specific heat at constant pressure	**Greek Symbols**	
c_s	concentration susceptibility	α	thermal diffusivity
Df	Dufour number	α_0	threshold pressure
D	mass diffusivity	β_T	coefficient of thermal expansion
f	dimensionless stream function	β_C	coefficient of concentration expansion
K	Darcy permeability	ϕ	dimensionless concentration
k_T	thermal diffusion ratio	η	similarity variable
Le	Lewis number, α/D	θ	dimensionless temperature
N	buoyancy ratio parameter	ρ	density
n	viscosity index	ψ	stream function
Nu_x	local Nusselt number	Ω	rheological parameter
$\mathrm{Ra}_{n,x}$	local Rayleigh number	**Subscripts**	
u, v	Darcian velocities in the x and y directions, respectively	w	condition at wall
Sr	Soret number	∞	condition at infinity
Sh_x	local Sherwood number	**Superscript**	
T	temperature	$'$	differentiation with respect to η

1. INTRODUCTION

Coupled heat and mass transfer by natural convection in a fluid-saturated porous medium has attracted considerable attention due to many important engineering applications relevant to this problem. A substantial amount of work has been reported in the literature on Newtonian fluid flow in a porous medium due to thermal buoyancy alone as well as due to combined buoyancy forces resulting from temperature and concentration variations (Nield and Bejan, 2006; Vafai, 2005). Some of the applications where the combined heat and mass transfer in porous media are often encountered in the chemical industry, or in reservoir engineering in connection with the thermal recovery process, in the study of dynamics of hot and salty springs of a sea. Underground spreading of chemical waste and other pollutants, grain storage, evaporation cooling, and solidification are few other application areas where combined thermosolutal convection in porous media are observed. Most of the theories were proposed to analyze the flow through a porous medium and to predict the heat transfer rates are based on the assumption that the fluid is Newtonian and Darcy's law holds.

Many engineering applications involve the study of non-Newtonian fluids. An illustrative example is found in oil reservoir engineering in connection with the production of heavy crude oils that are power law fluids with yield stress. This process involves the cyclic injection of steam into the well for the purpose of increasing the temperature of the oil reservoir, a procedure referred to as "steam soak" or "huff and puff" in the industry. The increase in the temperature of the reservoir decreases the fluid viscosity, resulting in a substantial increase in the mobility of the heavy crude oil, thus improving the production flow rate by gravity drainage. It is obvious that the efficiency of this process can be increased by obtaining insight into the combined effects of convective heat and mass transfer and convective flow in a power law fluid–filled porous medium. On the other hand, a number of industrially important fluids including fossil fuels that may saturate underground beds exhibit non-Newtonian fluid behavior. Non-Newtonian shear flows are so widespread in industrial processes and the environment that it would be no exaggeration to affirm that Newtonian shear flows are the exception rather than the rule.

Chen and Chen (1988) have studied the problems of free convection flow of non-Newtonian fluids past an isothermal vertical flat plate embedded in a porous medium. Mehta and Narasimha Rao (1994) analyzed the buoyancy-induced flow of non-Newtonian fluids in a porous medium past a vertical flat plate with nonuniform surface heat flux conditions. Combined free and forced convection heat transfer in power law fluid–saturated porous media was analyzed by Nakayama and Shenoy (1993). Also, Nakayama and Shenoy (1992) presented a unified similarity transformation for Darcy and non-Darcy forced, free, and mixed convection heat transfer in non-Newtonian inelastic fluid-saturated porous media. Mansour and Gorla (2000) studied the combined convection in non-Newtonian fluids along a nonisothermal vertical plate in a porous medium. Rastogi and Poulikakos (1995) studied the problem of double diffusion from a vertical surface in a porous region saturated with a non-Newtonian fluid. Cheng (2007) analyzed the double diffusion from a vertical wavy surface in a porous medium saturated with a non-Newtonian power law fluid subjected to constant wall temperature and concentration.

Turning now to the class of non-Newtonian power law fluids with yield stress in a saturated porous medium, we can cite a couple of papers dealing with the heat and mass transfer. Chaoyang and Chuanjing (1989) studied the problem of boundary-layer flow and heat transfer and Jumah and Mazumdar (2000) dealt with free convection heat and mass transfer, while Cheng (2006) analyzed the natural convection heat and mass transfer from a vertical plate with variable wall heat and mass fluxes.

The Soret effect corresponds to species differentiation developing in an initially homogeneous mixture subjected to a thermal gradient, and the Dufour effect corresponds to diffusion of heat caused by concentration gradients. In several earlier studies, Soret and Dufour effects are neglected on the basis that they are of smaller-order magnitude than the effects described by Fourier's and Fick's laws. These effects are considered as second-order phenomena, but they may become significant in areas such as hydrology, petrology, or geosciences.

The importance of the Soret effect at low Rayleigh number has been analyzed by Bergman and Srinivasan (1989). However, Eckert and Drake (1972) indicated the processes where the Dufour effect may become significant. Kafoussias and Williams (1995) studied the thermal diffusion and diffusion thermo effects on forced, free, and mixed convection with temperature-dependent viscosity. Postelnicu (2004) studied the influence of a magnetic field, considering Soret and Dufour effects, from a vertical surface in porous medium. Partha et al. (2006) studied the effect of a magnetic field and double dispersion on free convective heat and mass transport considering the Soret and Dufour effects in a non-Darcy porous medium. The influence of a chemical reaction on flow field considering Soret and Dufour effects from a vertical surface in porous medium was analyzed by Postelnicu (2007). The effect of Soret and Dufour parameters on free convection heat and mass transfer from a vertical surface in a doubly stratified Darcian porous medium has been reported by Lakshmi Narayana and Murthy (2007).

In this paper, we deal with the problem of combined free convection heat and mass transfer of a non-Newtonian power law fluid with yield stress from a vertical flat plate in a fluid-saturated Darcy porous medium considering Soret and Dufour effects. Two different types of boundary conditions are considered in this study, namely, *(i)* the vertical wall is maintained at uniform wall temperature and concentration *(ii)* in the second case we assume the wall is maintained at uniform wall heat and mass flux conditions. It is shown that in both these cases a similarity solution is possible. The results are analyzed with reference to the Soret and Dufour parameters along with the parameters pertaining to the non-Newtonian character of the fluid.

2. MATHEMATICAL FORMULATION

Consider the free convection boundary layer flow along a vertical impermeable surface embedded in a porous medium saturated with a non-Newtonian fluid as shown in Fig. 1. The x coordinate is taken along the plate, in the ascending direction and the y coordinate is measured normal to the plate, while the origin of the reference system is considered at the leading edge of the vertical plate.

The wall is *(i)* maintained at constant temperature and concentration, T_{w} and C_{w} (UWT/UWC), respectively, which are higher than the temperature and concentration in the ambient medium given by T_∞ and C_∞, respectively, which will be called hereinafter case I and *(ii)* subjected to constant heat and mass fluxes Q_T and Q_C (UWHF/UWCF), which will be termed hereinafter case II.

We assume the flow is governed by Darcy's law, which is valid only when the order of the pore-dependant Reynolds number is very small. The thermophysical properties of the fluid are assumed to be constant except for the density dependency of the buoyancy term in the momentum equation. The Boussinesq approximation is valid and the fluid and the porous medium are in local thermodynamic equilibrium. Under these assumptions, boundary layer equations may be written as in Chaoyang and Chuanjing (1989) as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

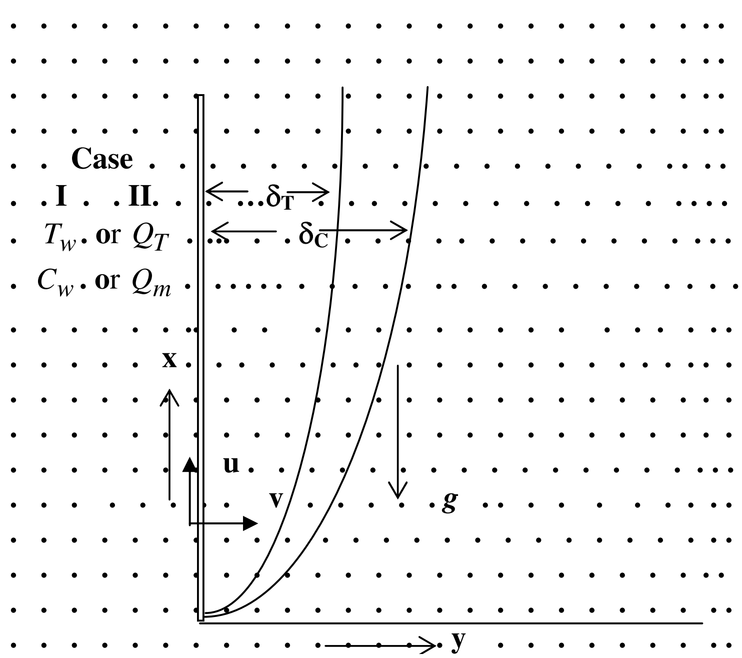


Figure 1. Free convective heat and mass transfer from a semi-infinite vertical wall in a non-Newtonian fluid-saturated Darcy porous medium

$$u^n = \frac{K}{\mu}\left\{-\frac{dP}{dx} - \rho g_x - \alpha_0\right\}$$
$$\text{if} \quad \left|-\frac{dP}{dx} - \rho g_x\right| > \alpha_0 \tag{2a}$$

$$u = 0 \quad \text{if} \quad \left|-\frac{dP}{dx} - \rho g_x\right| \le \alpha_0 \tag{2b}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{D\,k_T}{C_s\,C_p}\frac{\partial^2 C}{\partial y^2} \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} + \frac{D\,k_T}{T_m}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

In the above equations α_0 and n are the threshold pressure gradient and the viscosity index, respectively. The rheological fluids with $n < 1$ are called pseudo plastic, while those with $n > 1$ are termed as dilatants. Here T_m is the reference temperature. Other notations are usual and are given in the list of symbols. The appropriate boundary conditions are

$$v = 0, \qquad T = T_{\mathrm{w}}, \qquad C = C_{\mathrm{w}}$$
$$\text{at} \quad y = 0, \quad \text{in case I} \tag{6a}$$

$$v = 0, \quad \frac{\partial T}{\partial y} = -\frac{Q_T}{k}, \quad \frac{\partial C}{\partial y} = -\frac{Q_C}{D}$$
$$\text{at} \quad y = 0, \quad \text{in case II} \tag{6b}$$

$$u \to 0, \quad T \to T_\infty, \quad C \to C_\infty \quad \text{as} \quad y \to \infty \tag{7}$$

For the freestream, Eq. (2) gives

$$-\frac{dP}{dx} - \rho_\infty g_x = 0 \tag{8}$$

Eliminating dP/dx between Eqs. (2) and (8), we have

$$u^n = \frac{K}{\mu}\,[(\rho_\infty - \rho)g - \alpha_0]$$
$$\text{if} \quad |(\rho_\infty - \rho)g_x| > \alpha_0 \tag{9a}$$

$$u = 0, \quad \text{if} \quad |(\rho_\infty - \rho)g_x| \le \alpha_0 \tag{9b}$$

Equations (9) express the fact that the flow through the porous medium stops when the externally controlled pressure gradient matches the hydrostatic pressure gradient. Taking into account the linear variation of temperature and concentration in the density,

$$\rho = \rho_\infty\,[1 - \beta_T(T - T_\infty) - \beta_C(C - C_\infty)] \tag{10}$$

the Boussinesq-approximated momentum equation is given by

$$u^n = \frac{\rho_\infty g K}{\mu}\left[\beta_T(T - T_\infty) + \beta_C(C - C_\infty) - \frac{\alpha_0}{g\rho_\infty}\right]$$
$$\text{if} \quad \beta_T\,|T - T_\infty| - \beta_C\,|C - C_\infty| > \frac{\alpha_0}{g\rho_\infty} \tag{11a}$$

$$u = 0$$
$$\text{if} \quad \beta_T\,|T - T_\infty| - \beta_C\,|C - C_\infty| \le \frac{\alpha_0}{g\rho_\infty} \tag{11b}$$

Introducing the following similarity transformation:

$$\eta = \frac{y}{x}\,\mathrm{Ra}_x^{1/2}, \qquad \psi(\eta) = \alpha\mathrm{Ra}_{n,x}^{1/2} f(\eta) \tag{12}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_{\mathrm{w}} - T_\infty}, \qquad \phi(\eta) = \frac{C - C_\infty}{C_{\mathrm{w}} - C_\infty}$$
$$\text{in case I} \tag{13a}$$

$$T - T_\infty = \frac{Q_T}{k}\,x\,\theta(\eta)$$
$$C - C_\infty = \frac{Q_C}{D}\,x\,\phi(\eta), \quad \text{in case II} \tag{13b}$$

where

$$\mathrm{Ra}_{n,x} = \frac{x}{\alpha}\left[\frac{\rho K g \beta_T (T_{\mathrm{w}} - T_\infty)}{\mu}\right]^{1/n}, \quad \text{in case I}$$

and

$$\mathrm{Ra}_{n,x} = \frac{x}{\alpha}\left[\frac{\rho K g \beta_T Q_T x}{k\mu}\right]^{1/n}, \quad \text{in case II}$$

Equations (3), (4), and (11) reduce to

- Case I

$$f' = (\theta + N\phi - \Omega)^{1/n} \quad \text{if} \quad (\theta + N\phi) > \Omega \tag{14a}$$

$$f' = 0, \quad \text{if} \quad (\theta + N\,\phi) \leq \Omega \tag{14b}$$

$$\theta'' + \mathrm{Df}\,\phi'' + \frac{1}{2} f\theta' = 0 \tag{15}$$

$$\frac{1}{\mathrm{Le}}\phi'' + \mathrm{Sr}\,\theta'' + \frac{1}{2} f\phi' = 0 \tag{16}$$

- Case II

$$f' = (\theta + N\,\phi - \Omega)^{1/n} \quad \text{if} \quad (\theta + N\,\phi) > \Omega \tag{17a}$$

$$f' = 0, \quad \text{if} \quad (\theta + N\phi) \leq \Omega \tag{17b}$$

$$\theta'' + \mathrm{Df}\,\phi'' - f'\theta + \frac{n+1}{2n} f\theta' = 0 \tag{18}$$

$$\frac{1}{\mathrm{Le}}\phi'' + \mathrm{Sr}\,\theta'' - f'\phi + \frac{n+1}{2n} f\phi' = 0 \tag{19}$$

Here, Ω is the rheological parameter, N is the buoyancy ratio parameter, Df and Sr are the Dufour and Soret numbers, and these are given by

$$\Omega = \frac{\alpha_0}{\rho_\infty g \beta_T \Delta T}, \quad N = \frac{\beta_C \Delta C}{\beta_T \Delta T}$$
$$\mathrm{Df} = \frac{D k_T \Delta C}{c_s c_p \alpha \Delta T}, \quad \mathrm{Sr} = \frac{D k_T \Delta T}{T_m \alpha \Delta C}, \quad \text{in case I}$$

$$\Omega = \frac{\alpha_0 k}{\rho_\infty g \beta_T Q_T x}, \quad N = \frac{\beta_C Q_C k}{\beta_T Q_T D}$$
$$\mathrm{Df} = \frac{k Q_C k_T}{c_s c_p \alpha Q_T}, \quad \mathrm{Sr} = \frac{D k_T Q_T}{T_m \alpha Q_C k}, \quad \text{in case II}$$

and the diffusivity ratio parameter is given by $\mathrm{Le} = \alpha/D$.

In order to obtain similarity transformation for the UWHF/UWCF case, we assume α_0 is linearly proportional to x so that the rheological parameter Ω will be independent of x. It is worth mentioning that this restriction is not required for the UWT/UWC case.

The boundary conditions (6) and (7) become

$$f = 0, \quad \theta = 1, \quad \phi = 1 \quad \text{at} \quad \eta = 0, \quad \text{in case I} \tag{20a}$$

$$f = 0, \quad \theta' = -1, \quad \phi' = -1 \quad \text{at} \quad \eta = 0, \quad \text{in case II} \tag{20b}$$

$$f' \to 0, \quad \theta \to 0, \quad \phi \to 0 \quad \text{as} \quad \eta \to \infty \tag{21}$$

The nondimensional heat and mass transfer coefficients are given by

$$\frac{\mathrm{Nu}_x}{\mathrm{Ra}_{n,x}^{1/2}} = -\theta'(0), \quad \frac{\mathrm{Sh}_x}{\mathrm{Ra}_{n,x}^{1/2}} = -\phi'(0), \quad \text{for case I}$$

where Nu_x and Sh_x are the local Nusselt and Sherwood numbers.

3. RESULTS AND DISCUSSION

The sets of ordinary differential equations for case I and case II are solved using shooting and matching techniques. For case I, Eqs. (14)–(16) along with the boundary conditions (20a) and (21) are integrated by giving appropriate initial guess values for $f'(0)$, $\theta'(0)$, and $\phi'(0)$ to match the solutions with the corresponding boundary conditions at $f'(\infty)$, $\theta(\infty)$, and $\phi(\infty)$. Similarly for case II, Eqs. (17)–(19) are solved subject to the boundary conditions (20b) and (21) by giving appropriate guess values for $f'(0)$, $\theta(0)$, and $\phi(0)$ to match the solutions with the corresponding boundary conditions at $f'(\infty)$, $\theta(\infty)$, and $\phi(\infty)$. The solution procedure is explained in Partha et al. (2006). The following range of parameters have been used for the numerical computation: $-0.2 \leq N \leq 1$, $0.2 \leq n \leq 1.5$, $0 \leq \Omega \leq 0.4$, $0 \leq \mathrm{Df} \leq 0.1$, $0 \leq \mathrm{Sr} \leq 1$, and $0.5 \leq \mathrm{Le} \leq 30$ for case I; $-0.2 \leq N \leq 1$, $0.5 \leq n \leq 1.5$, $0 \leq \Omega \leq 0.4$, $0 \leq \mathrm{Df} \leq 0.1$, $0 \leq \mathrm{Sr} \leq 1$, and $0.5 \leq \mathrm{Le} \leq 5$ for case II. The results obtained here are accurate up to the fourth decimal place. With $n = 1$ and $\Omega = 0$, the present problem reduces to the one with the Soret and Dufour effects on free convection of Newtonian flow from a vertical flat plate in a porous medium in the absence of an inertia effect that was analyzed by Partha et al (2006).

3.1. Case I: UWT/UWC

Aiding buoyancy: In Fig. 2, variation of the nondimensional heat transfer coefficient is plotted against the Dufour parameter Df for varying Soret parameter Sr by considering pseudoplastic fluids with $n = 0.5$ and dilatants with $n = 1.5$. For both values of n, it is seen that increase in the Soret parameter enhances the heat transfer coefficient in the medium. Also, it is observed that the heat transfer coefficient is more for pseudoplastics when compared to dilatants.

In Fig. 3, variation of the nondimensional mass transfer coefficient is plotted against the Soret parameter Sr for varying Dufour parameter Df in pseudoplastics ($n = 0.5$) and dilatants ($n = 1.5$). For both types of fluids, it is noticed that an increase in the Dufour parameter reduced the mass transfer coefficient in the medium. Also, it is observed that the mass transfer coefficient is more for pseudoplastics when compared to dilatants.

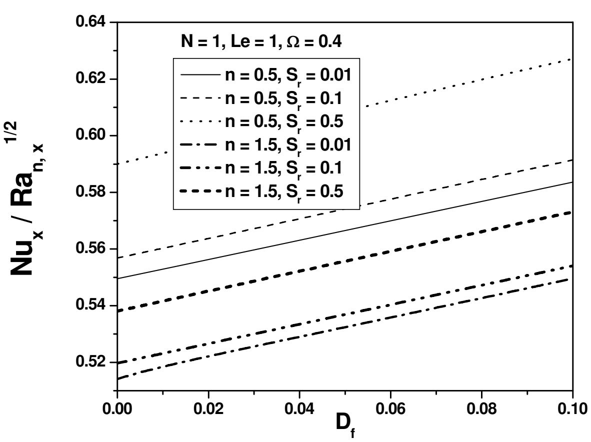


Figure 2. Variation of nondimensional heat transfer coefficient against Df for varying Sr in pseudoplastic and dilatant fluids, other parameters are $N = 1$, Le = 1, and $\Omega = 0.4$

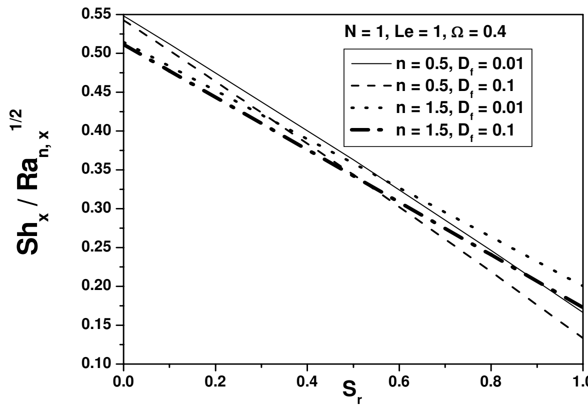
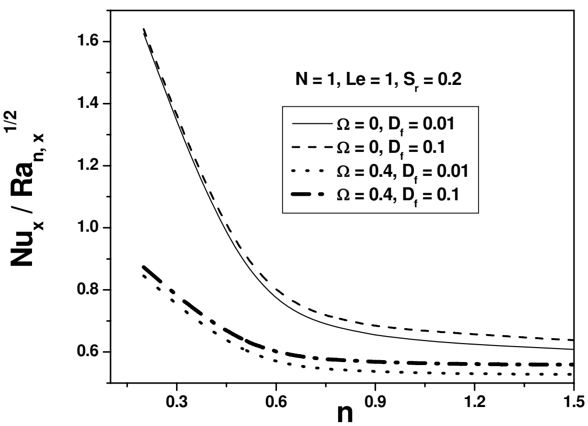


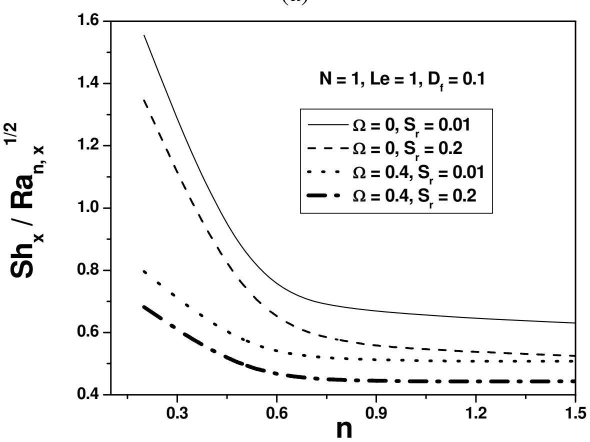
Figure 3. Variation of nondimensional mass transfer coefficient against Sr for varying Df in pseudoplastic and dilatant fluids, other parameters are $N = 1$, Le = 1, and $\Omega = 0.4$

In Fig. 4a, the nondimensional heat transfer coefficient is shown as a function of power law index parameter n for varying Ω and Df and fixed Soret number. From Fig. 4a, it is seen that the nondimensional heat transfer coefficient is increased with increasing Dufour parameter, while a reduction in heat transfer coefficient is obtained for increased Ω. Also, it is observed that the heat transfer coefficient is decreased with the power law index parameter n. Variation of the nondimensional mass transfer coefficient is plotted against power law index parameter n for varying Ω and Sr and fixed Dufour parameter in Fig. 4b. From this figure, it is seen that the mass transfer coefficient is reduced for increasing Ω and Sr. On the other hand, the mass transfer coefficient decreases with the power law index parameter n.

Variation of the nondimensional heat transfer coefficient is shown as a function of the diffusivity ratio parameter Le for varying power law index parameter n and the Soret parameter Sr in Fig. 5a. From

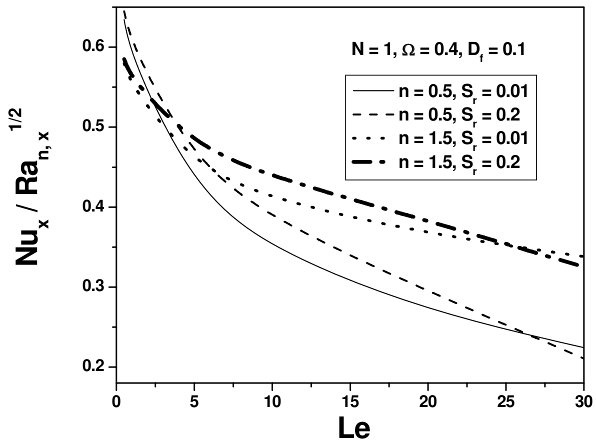


(a)

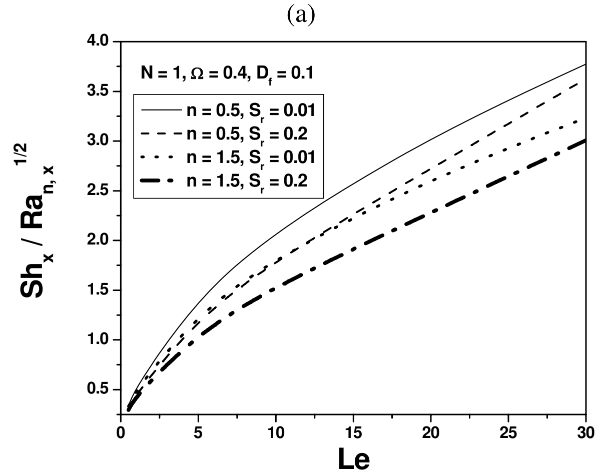


(b)

Figure 4. a) Variation of nondimensional heat transfer coefficient against n for varying Ω and Df, other parameters are $N = 1$, Le = 1, and Sr = 0.2; **b)** variation of nondimensional mass transfer coefficient against n for varying Ω and Sr, other parameters are $N = 1$, Le = 1, and Df = 0.1



(a)



(b)

Figure 5. Variation of nondimensional **a)** heat transfer coefficient and **b)** mass transfer coefficient against Le for varying Sr in pseudoplastic and dilatant fluids, other parameters are $N = 1$, $\Omega = 0.4$, and Df = 0.1

Fig. 5a, it is observed that the heat transfer coefficient for pseudoplastics is more for $\mathrm{Le} \leq 1$ and it is less for $\mathrm{Le} > 1$ when compared to dilatants in the medium. Another observation is that as Le is increased, the heat transfer coefficient is reduced in the porous medium. Also, it is clear that as the Soret parameter increased, the nondimensional heat transfer coefficient is increased up to certain Le, but as the Lewis number is increased further a reduction is seen with increasing values of the Soret parameter. Variation of the nondimensional mass transfer coefficient against Lewis number for varying Soret number for both pseudoplastics and dilatants is shown in Fig. 5b. Increasing Le enhanced the mass transfer coefficient in the medium, and the mass transfer coefficient in the medium is more for pseudoplastics and it is less for dilatants. As the Soret parameter is increased, a reduction in the mass transfer coefficient occurs for both pseudoplastics and dilatants.

Opposing buoyancy: Variation of the nondimensional heat transfer coefficient against the Dufour parameter for varying power law index parameter n and Soret parameter is shown in Fig. 6. From this figure, it is observed that the heat transfer coefficient is increased

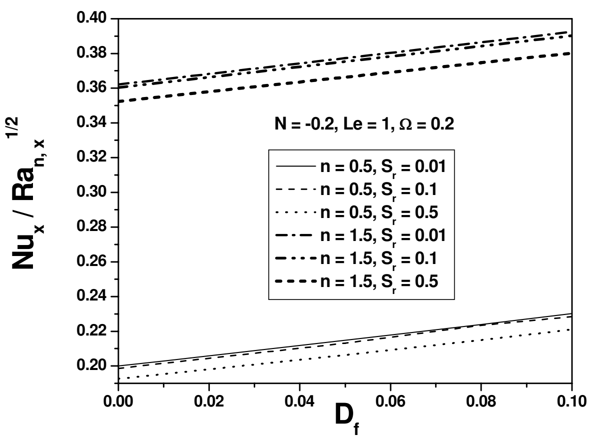


Figure 6. Variation of nondimensional heat transfer coefficient against Df for varying Sr in pseudoplastic and dilatant fluids, other parameters are $N = -0.2$, Le = 1, and $\Omega = 0.2$

with the Dufour parameter, while a reduction in the heat transfer coefficient is observed with the Soret parameter. In the opposing buoyancy case it is evident that the heat transfer coefficient in the medium is more for dilatants while it is less for pseudoplastics.

Variation of the dimensionless mass transfer coefficient against the Soret parameter is plotted in Fig. 7 for various values of the Dufour parameter,

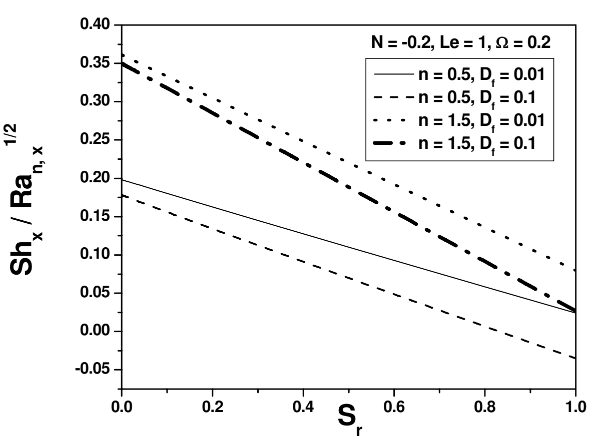
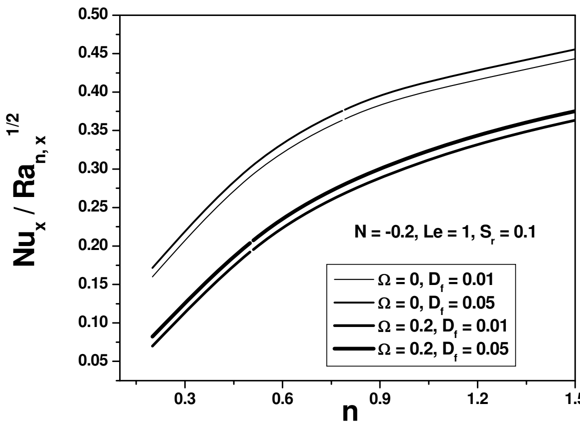


Figure 7. Variation of nondimensional mass transfer coefficient against Sr for varying Df in pseudoplastic and dilatant fluids, other parameters are $N = -0.2$, Le = 1, and $\Omega = 0.2$

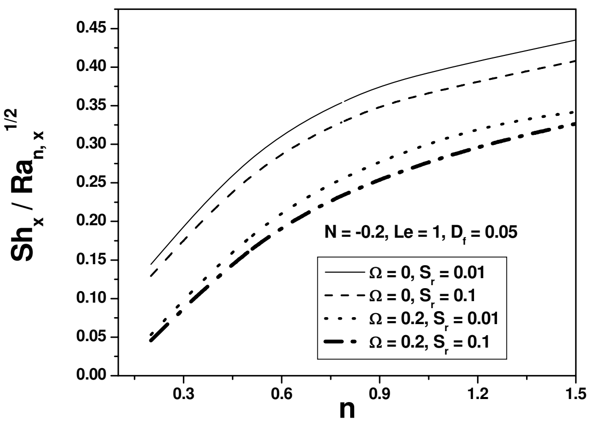
considering both pseudoplastics and dilatants. It is observed that the mass transfer coefficient in the porous medium is decreased with increasing values of Soret parameter for both pseudoplastics and dilatants. A reduction in the mass transfer coefficient is obtained for increasing values of the Dufour parameter, while the mass transfer coefficient is more for dilatants than that of pseudoplastics. Also, it is observed that for large values of Soret parameter, the nondimensional mass transfer coefficient takes negative values in the medium.

In Fig. 8a, variation of the nondimensional heat transfer coefficient is plotted against the power law index parameter n for various values of the rheological parameter Ω and the Dufour parameter. The heat transfer coefficient in the medium is increased with increasing values of n and the Dufour parameter, but a reduction in the heat transfer coefficient is observed with increasing value of rheological parameter Ω. In Fig. 8b, the nondimensional mass transfer coefficient in the medium is plotted against the power law index parameter n for varying rheological parameter Ω and the Soret parameter. As n is increased, the mass transfer coefficient in the medium is increased, but a reduction is found with an increasing value of Ω. Also, it is observed that an increase in the Soret parameter reduced the mass transfer coefficient in the medium.

In Fig. 9a, variation of the nondimensional heat transfer coefficient is shown as a function of Le for pseudoplastics and dilatants for various values of the Soret parameter. From this figure, it is observed that heat transfer coefficient is increased with increasing Le for pseudoplastic fluids, but for dilatants it increased to certain Le and further it is reduced as Le is increased. A reduction in heat transfer coefficient is found with Soret parameter in the medium. In Fig. 9b, variation of the nondimensional mass transfer coefficient is plotted against Le for pseudoplastics and dilatants with varying Soret parameter in the medium. From this figure, it is observed that mass transfer coefficient is increased with the both Le and power law index parameter n. But a reduction in mass trans-

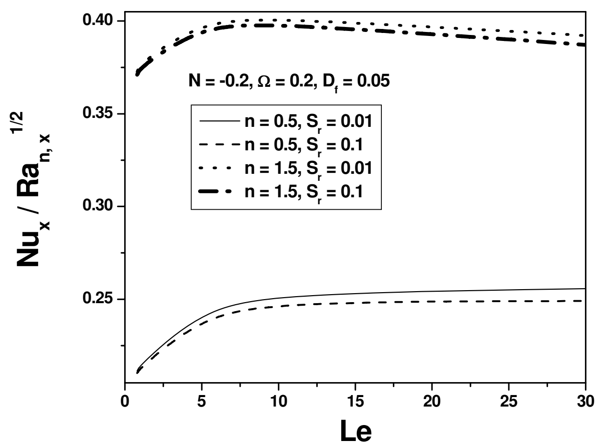


(a)

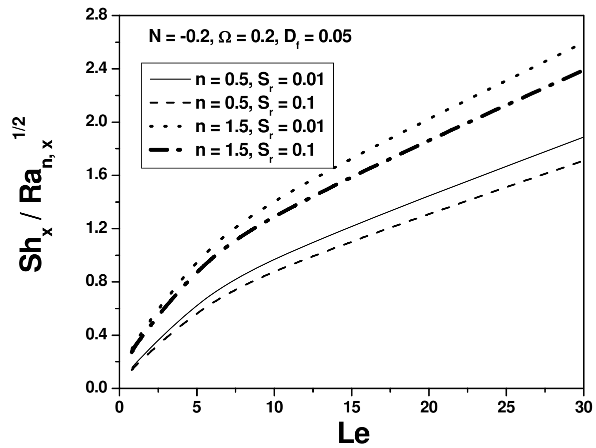


(b)

Figure 8. **a)** Variation of nondimensional heat transfer coefficient against n for varying Ω and Df in pseudoplastic and dilatant fluids, other parameters are $N = -0.2$, Le = 1, and Sr = 0.1; **b)** variation of nondimensional mass transfer coefficient against n for varying Ω and Sr in pseudoplastic and dilatant fluids, other parameters are $N = -0.2$, Le = 1, and Df = 0.05



(a)



(b)

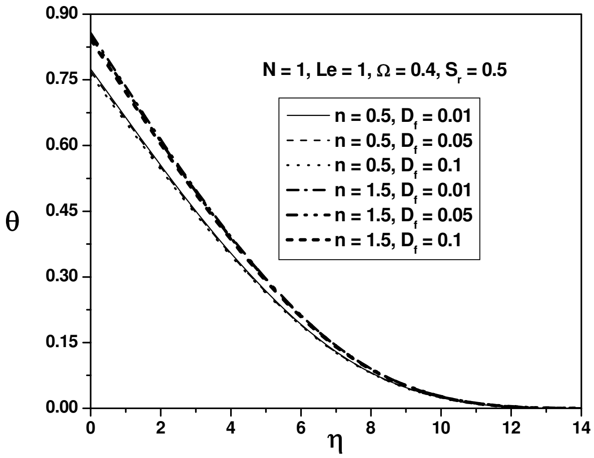
Figure 9. Variation of **a)** nondimensional heat transfer coefficient and **b)** nondimensional mass transfer coefficient against Le for varying Sr in pseudoplastic and dilatant fluids, other parameters are $N = -0.2$, $\Omega = 0$, and Df = 0.05

fer coefficient is found with increased value of Soret parameter.

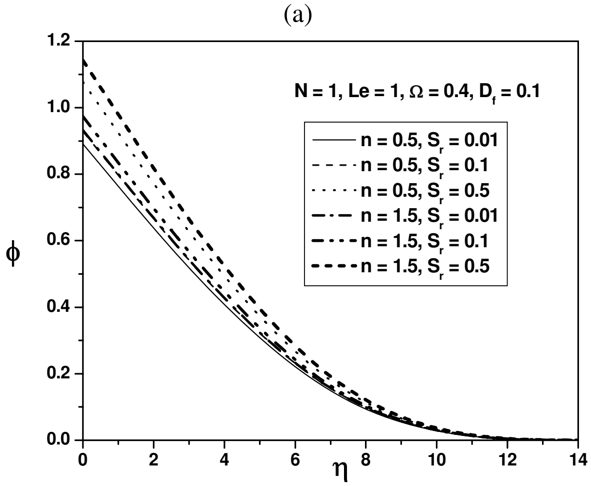
In this case, the behavior of the nondimensional heat and mass transfer coefficients is largely affected by the Soret and Dufour parameters along with the rheological parameter Ω for both pseudoplastics and dilatants. It is worth noting that the nondimensional mass transfer coefficient in the opposing buoyancy case becomes negative for large values of the Soret parameter. This phenomenon can be seen further if the value of the Soret parameter is increased.

3.2. Case II: UWHF/UWCF

Aiding buoyancy: In Fig. 10a, variation of the nondimensional temperature distribution inside the boundary layer is shown for varying power law index parameter n and the Dufour parameter while the Soret parameter is fixed. From this figure, a reduction in temperature distribution is observed for increasing Dufour parameter. Besides, as n increased, temperature distribution is enhanced in the medium. In Fig. 10b, the nondimensional concentration distribu-

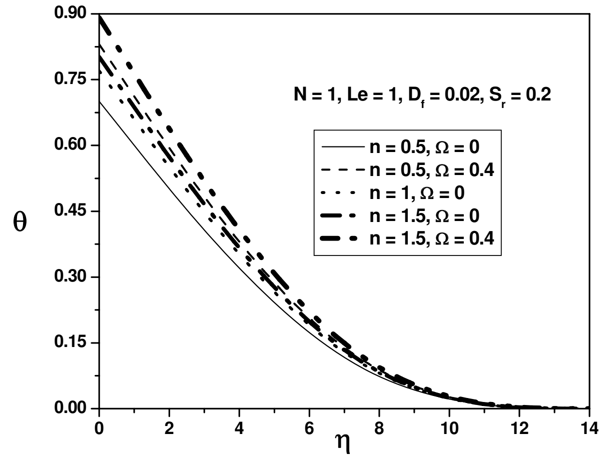


(a)

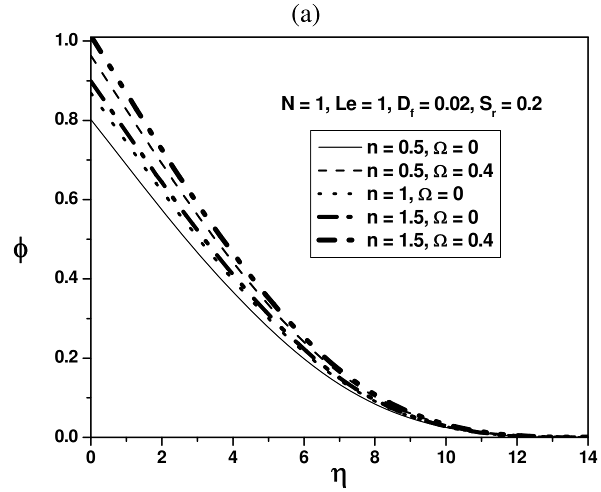


(b)

Figure 10. Variation of **a)** temperature and **b)** concentration inside the boundary layer with varying $n\&D_f$ and $n\&S_r$ respectively for temperature and concentration with fixed values of $N = 1$, Le = 1, $\Omega = 0.4$



(a)



(b)

Figure 11. Variation of **a)** temperature and **b)** concentration inside the boundary layer with varying n and Ω, other parameters are $N = 1$, Le = 1, Df = 0.02, and Sr = 0.2

tion inside the boundary layer is plotted for varying Soret parameter and power law index parameter n with fixed Dufour parameter. It is seen that as the values of n and the Soret parameter are increased as the concentration distribution is enhanced in the medium.

In Figs. 11a and 11b, variation of the nondimensional temperature and concentration distributions are plotted within the boundary layer for varying rheological parameters n and Ω with fixed Dufour and Soret parameters in the medium. From Figs. 11a and 11b, it is observed that both temperature and concentration distributions are increased with increasing values of rheological parameters n and Ω.

In Figs. 12a and 12b, variation of the nondimensional temperature and concentration distributions are plotted inside the boundary layer for varying Le and n with fixed values of Soret and Dufour parameters. From Fig. 12a, it is seen that the temperature distribution is increased with increasing Le, but it is reduced with n. Contrary to this, the dimensionless concentration distribution is observed to be decreased with increasing Le, while it is enhanced with n, as shown in Fig. 12b.

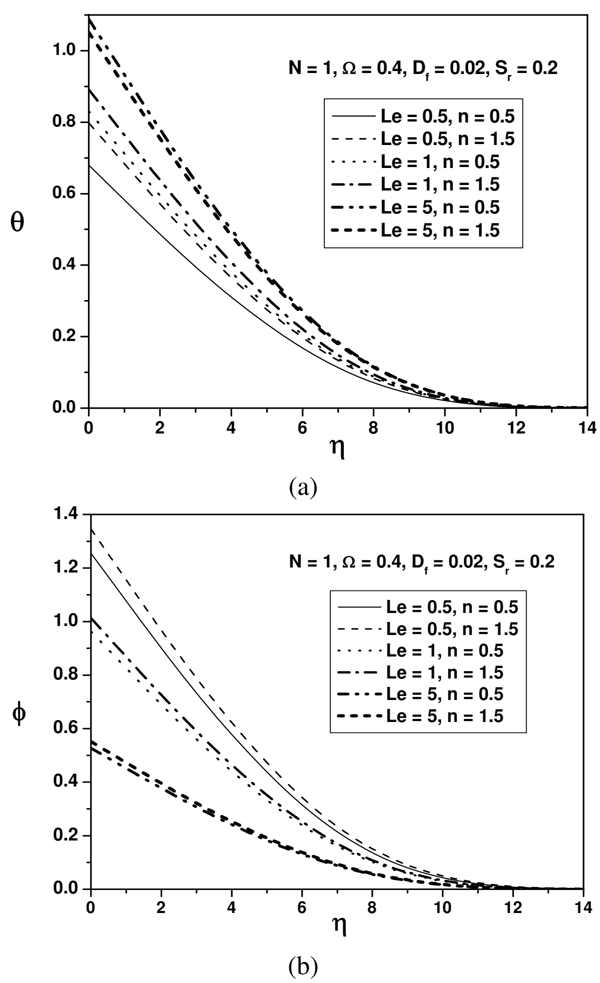


Figure 12. Variation of **a)** temperature and **b)** concentration inside the boundary layer with varying Le and n, other parameters are $N = 1$, $\Omega = 0.04$, Df = 0.02, and Sr = 0.2

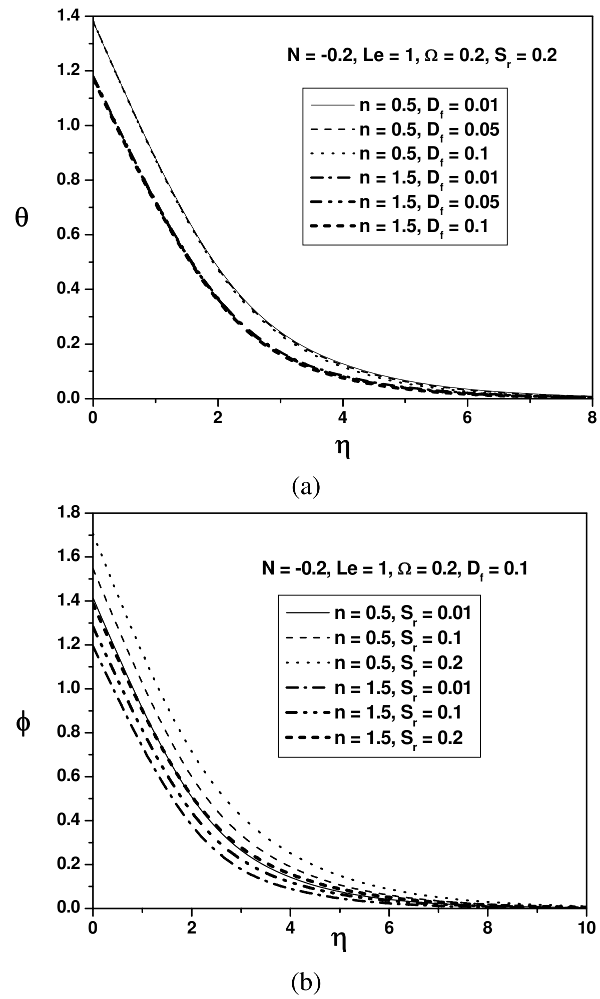


Figure 13. Variation of **a)** temperature and **b)** concentration inside the boundary layer with varying n and Df, other parameters are $N = -0.2$, Le = 1, $\Omega = 0.2$, and Sr = 0.2

Opposing buoyancy: In Fig. 13a, temperature distribution inside the boundary layer is shown when n and Dufour parameters are varied. From Fig. 13a, temperature distribution inside the medium is more for pseudoplastics and it is less for dilatants. Also, it is observed that an increase in the Dufour parameter value reduced the temperature inside the medium. In Fig. 13b, the concentration distribution across the boundary layer is shown for varying n and the Soret parameter. Figure 13b shows that the concentration distribution is more for pseudoplastics and it is less for dilatants. Also, it is observed that an increase in the Soret parameter value enhanced the mass transfer inside the medium.

In Fig. 14a, temperature distribution in the boundary layer is plotted for varying n and Ω, and for fixed nonzero Soret and Dufour parameters. It is seen that as n is increased, temperature distribution is reduced, while it is increased with Ω. A similar behavior is observed for the concentration distribution with varying n and Ω in Fig. 14b.

Temperature distribution across the boundary layer is plotted for varying Le and n for fixed values of Soret and Dufour parameters in Fig. 15a. From this

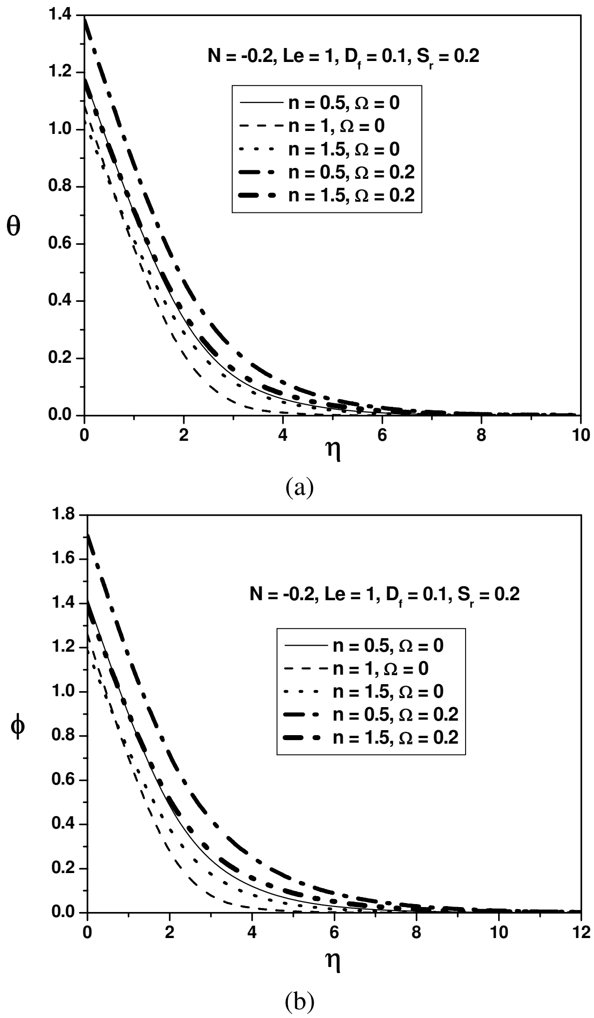


Figure 14. Variation of **a)** temperature and **b)** concentration inside the boundary layer with varying n and Ω, other parameters are $N = -0.2$, Le = 1, Df = 0.1, and Sr = 0.2

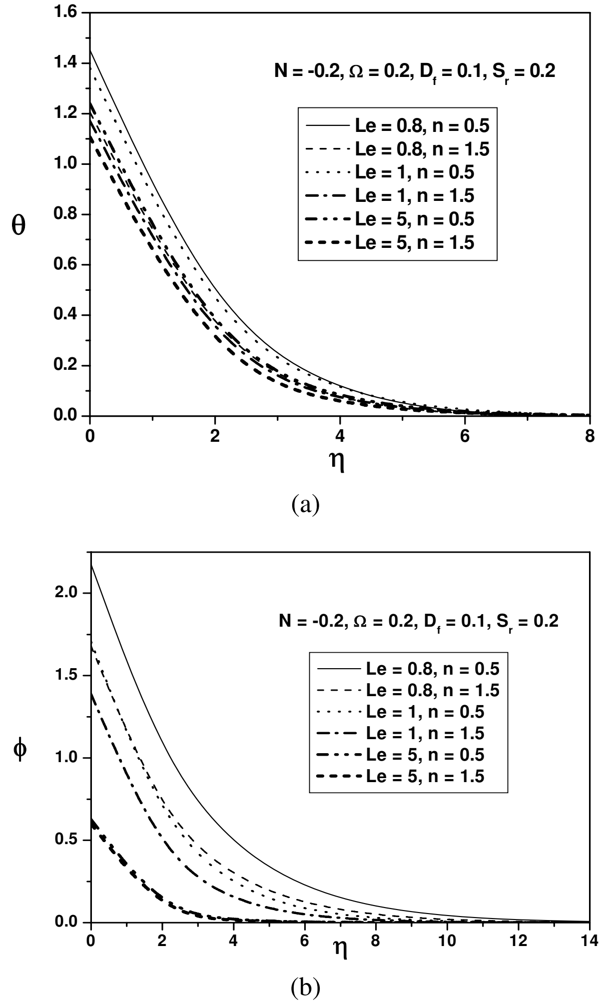


Figure 15. Variation of **a)** temperature and **b)** concentration inside the boundary layer with varying Le and n, other parameters are $N = -0.2$, $\Omega = 0.2$, Df = 0.1, and Sr = 0.2

figure, it is observed that as the value of Le is increased, a reduction in the temperature distribution is found. Similar behavior is found with n. The concentration distribution inside the boundary layer is plotted for varying (Le, n) for fixed $(\mathrm{Sr}, \mathrm{Df})$ in Fig. 15b. From this figure, it is observed that as the value of Le is increased, a reduction in concentration distribution is found. A similar behavior occurs with n.

4. CONCLUSIONS

Natural convection heat and mass transfer, including the Soret and Dufour effects, from a vertical surface embedded in a Darcy porous medium saturated with a non-Newtonian power law fluid with yield stress, has been analyzed. Two different types of boundary conditions have been considered. In case I, the vertical wall is maintained at uniform wall temperature and concentration; while in case II, the wall is maintained at uniform wall heat and mass flux conditions. Some conclusions are summarized below.

4.1. Case I, Aiding Buoyancy

Heat transfer is enhanced when the Soret parameter is increased, for both pseudoplastic (more) and dilatants (less) fluids, and the Dufour parameter is increased. The difference in the heat transfer coefficients is less but enhanced with the Dufour parameter and with the Ω.

The mass transfer is enhanced by increasing the Lewis number. The mass transfer coefficient is reduced when the Dufour parameter is increased, for both pseudoplastic and dilatant fluids, along with the Soret parameter. Also, the mass transfer coefficient is reduced with the power law index parameter n and with the rheological parameter Ω.

4.2. Case I, Opposing Buoyancy

Heat transfer is enhanced when the Dufour parameter increases; this is more for the dilatants compared with the pseudoplastics. The Soret parameter reduced the heat transfer coefficient in the medium. As n increases, the heat transfer coefficient is enhanced but reduced with the rheological parameter Ω.

The mass transfer coefficient in the medium is reduced with increasing values of Soret parameter for both pseudoplastics and dilatants; for increasing values of the Dufour parameter, and with increasing of the rheological parameter Ω. The mass transfer coefficient is enhanced when the viscosity index n is increased.

4.3. Case II, Aiding Buoyancy

A reduction in the temperature distribution is obtained for increased Dufour parameter in the medium, while the temperature distribution is enhanced when n increases. As the values of n and Soret parameters are increased, the concentration distribution is enhanced in the porous medium. Both temperature and concentration distributions are intensified with increasing values of rheological parameters n and Ω.

4.4. Case II, Opposing buoyancy

The temperature distribution inside the porous medium is more for pseudoplastics and it is less for dilatants. On the other hand, an increase in the Dufour value reduces the temperature inside the medium. The concentration distribution in the boundary layer is more for pseudoplastics and it is less for dilatants. On the other hand, an increase in the Soret parameter value enhances the mass transfer inside the medium. For fixed nonzero Soret and Dufour parameters, temperature and concentration profiles are reduced as n is increased.

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