

Perturbative Bottom-up Approach for Neutrino Mass Matrix in Light of Large θ_{13} and Role of Lightest Neutrino Mass

Rupak Dutta,^{*} Upender Ch,[†] Anjan K. Giri,[‡] and Narendra Sahu[§]

Indian Institute of Technology Hyderabad, Yeddumailaram 502205, AP, India.

Abstract

We discuss the role of lightest neutrino mass (m_0) in the neutrino mass matrix, defined in a flavor basis, through a *bottom-up* approach using the current neutrino oscillation data. We find that if $m_0 < 10^{-3}\text{eV}$, then the deviation δM_ν in the neutrino mass matrix from a tree-level, say tribimaximal neutrino mass matrix, does not depend on m_0 . As a result δM_ν 's are exactly predicted in terms of the experimentally determined quantities such as solar and atmospheric mass squared differences and the mixing angles. On the other hand for $m_0 \gtrsim 10^{-3}\text{eV}$, δM_ν strongly depends on m_0 and hence can not be determined within the knowledge of oscillation parameters alone. In this limit, we provide an exponential parameterization for δM_ν for all values of m_0 such that it can factorize the m_0 dependency of δM_ν from rest of the oscillation parameters. This helps us in finding δM_ν as a function of the solar and atmospheric mass squared differences and the mixing angles for all values of m_0 . We use this information to build up a model of neutrino masses and mixings in a top-down scenario which can predict large θ_{13} perturbatively.

PACS numbers: 14.60.Pq

^{*}Electronic address: rupak@iith.ac.in

[†]Electronic address: ph11p1003@iith.ac.in

[‡]Electronic address: giria@iith.ac.in

[§]Electronic address: nsahu@iith.ac.in

I. INTRODUCTION

Compelling evidences of neutrino oscillations observed in solar, atmospheric and reactor experiments indicate that neutrinos are massive and hence they mix with each other [1]. In the basis where the charged lepton masses are real and diagonal, the mixing matrix is given by [2]:

$$U_{\text{PMNS}} = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta_{\text{CP}}} \\ -c_{23} s_{12} - s_{23} c_{12} s_{13} e^{i\delta_{\text{CP}}} & c_{23} c_{12} - s_{23} s_{12} s_{13} e^{i\delta_{\text{CP}}} & s_{23} c_{13} \\ s_{23} s_{12} - c_{23} c_{12} s_{13} e^{i\delta_{\text{CP}}} & -s_{23} c_{12} - c_{23} s_{12} s_{13} e^{i\delta_{\text{CP}}} & c_{23} c_{13} \end{pmatrix} P, \quad (1)$$

where $c_{ij} = \cos \theta_{ij}$, $s_{ij} = \sin \theta_{ij}$ and δ_{CP} is the Dirac CP violating phase on which the oscillation probability depends. The diagonal matrix $P = \text{diag}(e^{i\phi_1}, e^{i\phi_2}, 1)$, consists of two Majorana phases ϕ_1 and ϕ_2 which are not relevant in neutrino oscillation experiments [3, 4]. However, they affect lepton number violating amplitudes such as neutrinoless double beta decay. Thus in a flavor basis, where charged leptons are real and diagonal, the neutrino mass matrix is given by:

$$M_\nu = U_{\text{PMNS}}^* \text{diag}(m_1, m_2, m_3) U_{\text{PMNS}}^\dagger, \quad (2)$$

where m_1, m_2, m_3 are the mass eigenvalues.

In the last decade, data from solar and reactor neutrino experiments have provided information on the sign and magnitude of Δm_{\odot}^2 and a precise value of θ_{12} [5, 6]. The atmospheric parameters $|\Delta m_{\text{atm}}^2|$ and θ_{23} have been measured and their precision will be increased by T2K [7] and NO ν A [8]. The sign of solar mass splitting Δm_{\odot}^2 is precisely known, while the sign of atmospheric mass splitting Δm_{atm}^2 is still unknown. This opens up a possibility of whether neutrino masses follow normal ordering, i.e., $m_1 < m_2 < m_3$ or inverted ordering, i.e., $m_3 < m_1 < m_2$. In other words, the lightest mass, either m_1 (normal ordering) or m_3 (inverted ordering) is yet to be determined. Recent measurement from T2K [9], MINOS [10], Double Chooz [11], Daya Bay [12], and RENO [13] confirms a non-zero value of θ_{13} at 5σ confidence level. This opens up a range of possibilities to measure the sign of the atmospheric mass splitting and the unknown CP phase δ_{CP} .

The absolute mass scale of neutrino is hitherto not known and can only be measured in a tritium beta decay experiment. The KATRIN experiment, which will investigate the kinematics of tritium beta decay, aims to measure the neutrino mass with a sensitivity of 0.2 eV [14]. At present the best upper limit at 95% confidence level on the sum of the neutrino masses comes from the cosmic

microwave background data and is given by [15]:

$$\sum_i m_i < 0.44eV. \quad (3)$$

Once the absolute mass scale of the lightest neutrino, either m_1 in normal ordering or m_3 in inverted ordering, is determined one can reconstruct the neutrino mass matrix in the flavor basis using the experimental values of the elements of U_{PMNS} matrix. This may unravel exact flavor structure in the neutrino mass matrix.

In light of recent non-zero θ_{13} , a large number of flavor models have been proposed by using the top-down approach, where one assumes a specific symmetry [16–21] to explain the observed masses and mixings of the light neutrinos. However, it is worth exploring the symmetries of neutrino mass matrix through a data driven approach [22]. But as we discussed above the mass of lightest neutrino is yet to be known and therefore, the low energy neutrino data may not unravel the full flavour structure.

In this paper we develop a perturbative bottom-up approach to unravel the flavor structure of neutrino mass matrix and discuss the role of lightest neutrino mass. We set the full neutrino mass matrix: $M_\nu = (M_\nu)_0 + \delta M_\nu$, where δM_ν is the perturbation around the tree-level mass matrix $(M_\nu)_0$, which is determined using some of the well known mixing scenarios such as tribimaximal (TBM) mixing [23], bimaximal (BM) mixing [24] and/or democratic (DC) mixing [25]. Among all these, the TBM is closer to the experimentally observed mixing pattern and hence mostly studied. All these mixing scenarios, however, predict θ_{13} to be zero and hence ruled out by the recent measurement on the reactor neutrino angle. However, a perturbative approach to realize a large value of θ_{13} is still a viable option. So, we assume that the non-zero value of θ_{13} is generated perturbatively. Using the 3σ range of values of the elements of U_{PMNS} matrix we determine δM_ν as a function of the lightest neutrino mass, say m_0 , where $m_0 = m_1$ in the normal ordering and $m_0 = m_3$ in the inverted ordering. In this way we find that for $m_0 < 10^{-3}$ eV, all the δM_ν are independent of m_0 and hence the lightest neutrino mass does not play any role in the perturbative determination of neutrino mass matrix. Such models, for example, can be generated in two right-handed neutrino extensions of the standard model (SM). However, for $m_0 \gtrsim 10^{-3}$ eV the lightest neutrino mass plays an important role in the perturbative determination of neutrino mass matrix. We factorize the m_0 dependency of δM_ν using an exponential parameterization: $\delta M_\nu(m_0) \propto \delta M_\nu|_{m_0=0} \text{Exp}[-m_0/0.1 \text{ eV}]$. This helps us in finding the required perturbations of δM_ν as a function of experimentally determined quantities

such as solar and atmospheric mass squared differences and the mixing angles. To this end we apply the result of bottom-up approach to develop a model which predict large θ_{13} perturbatively.

The paper is organized as follows. In section II A, we start with a brief description of the bottom-up approach that is developed in this paper to find the desired perturbation of the mass matrix in the flavor basis. Then, in section II B, we use the knowledge of our current experimental results to find the deviation in the tree-level mass matrix of the neutrinos, obtained by using TBM mixing ansatz. In section III, we use the result of bottom-up approach to develop a model in which the perturbation to the TBM mixing is obtained. We then fit the model parameters using the data from the bottom-up approach of section II B and conclude in section IV.

II. PHENOMENOLOGY

A. Bottom up approach

The recent global fit to the neutrino oscillation data has ruled out the possibility of a zero reactor angle at 10.2σ confidence level [26, 27]. Evidence of non zero θ_{13} at 3σ level was first established by data from T2K [7], MINOS [10] and Double Chooz [11]. More recently, Daya Bay [12] and RENO [13] experiments confirm large θ_{13} at more than 5σ confidence level (CL) from the reactor $\bar{\nu}_e \rightarrow \bar{\nu}_e$ oscillations. The current best fit values and the 3σ allowed values of all the mixing parameters are summarized in Table. I. The values written within brackets are for inverted ordering

Oscillation parameters	Best fit value	3σ range
$\Delta m_{\odot}^2 [10^{-5} \text{ eV}^2]$	7.62	[7.12 – 8.20]
$ \Delta m_{\text{atm}}^2 [10^{-3} \text{ eV}^2]$	2.55 (2.43)	[2.31 – 2.74] ([2.21 – 2.64])
$\sin^2 \theta_{12}$	0.320	[0.27 – 0.37]
$\sin^2 \theta_{23}$	0.613 (0.600)	[0.36 – 0.68] ([0.37 – 0.67])
$\sin^2 \theta_{13}$	0.0246 (0.0250)	[0.017 – 0.033]

TABLE I: Current status of oscillation parameters taken from Ref. [26].

of the neutrino mass spectrum. At this juncture we note that the only unknown quantity in the neutrino mass matrix is the lightest neutrino mass apart from the CP violating phases (one Dirac and two Majorana phases).

The neutrino mixing matrix derived from the current best fit parameters is

$$U_{\text{EXPT}}^c = \begin{pmatrix} 0.81442 & 0.55868 & 0.15684 \\ -0.45317 & 0.44353 & 0.77325 \\ 0.36244 & -0.70083 & 0.61439 \end{pmatrix}, \quad (4)$$

and the corresponding 3σ allowed values of the mixing matrix is

$$U_{\text{EXPT}} = \begin{pmatrix} 0.78 - 0.85 & 0.51 - 0.60 & 0.13 - 0.18 \\ -(0.39 - 0.57) & 0.36 - 0.64 & 0.59 - 0.82 \\ 0.19 - 0.44 & -(0.54 - 0.76) & 0.56 - 0.79 \end{pmatrix}, \quad (5)$$

where we have assumed δ_{CP} to be zero for simplicity. The corresponding neutrino mass matrix in the flavor basis can be written as

$$(M_\nu)_{\text{EXPT}} = U_{\text{EXPT}} \text{diag}(m_1, m_2, m_3) U_{\text{EXPT}}^T \quad (6)$$

where $(M_\nu)_{\text{EXPT}}$ is a real symmetric matrix. Therefore, it is described by three masses: m_1, m_2, m_3 and three mixing angles: $\theta_{12}, \theta_{23}, \theta_{13}$. The mass eigenvalues m_1, m_2, m_3 in the basis where the charged leptons are real and diagonal can be expressed as: $m_2 = \sqrt{\Delta m_\odot^2 + m_1^2}$ and $m_3 = \sqrt{\Delta m_{\text{atm}}^2 + m_1^2}$ for normal hierarchy and $m_1 = \sqrt{m_3^2 + \Delta m_{\text{atm}}^2}$ and $m_2 = \sqrt{m_3^2 + \Delta m_{\text{atm}}^2 + \Delta m_\odot^2}$ for inverted hierarchy.

We define the perturbed neutrino mass matrix δM_ν as

$$\delta M_\nu = (M_\nu)_{\text{EXPT}} - (M_\nu)_0, \quad (7)$$

where $(M_\nu)_{\text{EXPT}}$ is the neutrino mass matrix given by Eq. 6 and $(M_\nu)_0$ is the neutrino mass matrix corresponding to a mixing matrix U_0 , that is obtained in a specific model such as TBM mixing, BM mixing and/or DC mixing *etc.* This δM_ν is a symmetric matrix whose elements denote the amount of perturbation that is needed to add to the $(M_\nu)_0$ so that it is consistent within the 3σ of the experimental results. In order to gauge the size of the δM_ν matrix, we do a random scan of all the parameters: $\theta_{13}, \theta_{23}, \theta_{12}, \Delta m_\odot^2$ and $|\Delta m_{\text{atm}}^2|$ within their 3σ allowed range of values. Note that $(M_\nu)_0$ depends only on the value of the lightest neutrino mass m_0 as the mixing angles are expected to be fixed via certain symmetries as done in TBM, BM and/or DC mixing scenarios. Hence, to see the effect of m_0 on the elements of δM_ν we vary m_0 in the range $[10^{-9}, 10]$ eV. We then define the predicted neutrino mass matrix $(M_\nu)_M$ of any model as:

$$(M_\nu)_M = (M_\nu)_0 + \delta M_\nu. \quad (8)$$

Note that the matrix elements of $(M_\nu)_M$ and δM_ν are function of the oscillation parameters θ_{12} , θ_{23} , and θ_{13} as well as of the lightest neutrino mass m_0 . We then perform a naive statistical analysis and compare the elements of the $(M_\nu)_M$ to the experimental data of Eq. 6 by a χ^2 function which is defined by:

$$\chi^2 = [(M_\nu^c)_{\text{EXPT}} - (M_\nu)_M]^2 / m_0 \sigma^2 = [\delta M_\nu^c - \delta M_\nu]^2 / m_0 \sigma^2, \quad (9)$$

where $\sigma = \sqrt{(M_\nu^c)_{\text{EXPT}}}$ and $\delta M_\nu^c = (M_\nu^c)_{\text{EXPT}} - (M_\nu)_0$. Note that $(M_\nu^c)_{\text{EXPT}}$ is the experimental mass matrix corresponding to the best fit values of the oscillation parameters and is diagonalized by the mixing matrix of Eq. 4. It is worth noting that as $\delta M_\nu \rightarrow \delta M_\nu^c$, we get the minimum χ^2 and hence δM_ν^c is the required amount of perturbation that should be added to $(M_\nu)_0$. We will discuss the properties of the δM_ν matrix using the TBM mixing ansatz. Although we focus mainly on the TBM mixing, some of the features presented here are applicable to other tree-level mass matrix.

B. Perturbation in Tribimaximal mixing Scenario

We now proceed to discuss the tribimaximal (TBM) mixing scenario. For the TBM mixing, we have $s_{12} = 1/\sqrt{3}$, $s_{23} = 1/\sqrt{2}$, and $s_{13} = 0$. We know that apart from the recently measured value of θ_{13} , the solar and atmospheric mixing angles are in a very good agreement with the TBM ansatz. Modifications to this TBM scenario have been studied by many authors [28].

We begin by writing the TBM mixing matrix in the standard parameterization of Eq. 1, that is

$$U_{\text{TBM}} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \\ \sqrt{1/6} & -\sqrt{1/3} & \sqrt{1/2} \end{pmatrix} = \begin{pmatrix} 0.81650 & 0.57735 & 0 \\ -0.40825 & 0.57735 & 0.70711 \\ 0.40825 & -0.57735 & 0.70711 \end{pmatrix}. \quad (10)$$

The corresponding neutrino mass matrix in the flavor basis and the perturbed matrix δM_ν are

$$(M_\nu)_{\text{TBM}} = U_{\text{TBM}} \text{diag}(m_1, m_2, m_3) U_{\text{TBM}}^T, \quad \delta M_\nu = (M_\nu)_{\text{EXPT}} - (M_\nu)_{\text{TBM}}. \quad (11)$$

We wish to determine the elements of this δM_ν matrix in a bottom-up approach. We follow the procedure given in section II A and perform a random scan over all the oscillation parameters within their 3σ range of values. In Fig. 1, we show $\delta M_\nu(i, j)$ versus χ^2 for $m_0 = 10^{-9}$ eV. The minimum of each curve corresponds to the value of $\delta M_\nu^c(i, j)$. A similar plot is shown in Fig. 2 for inverted hierarchy as well. We know that the neutrino mass matrix in the flavor basis does depend on the

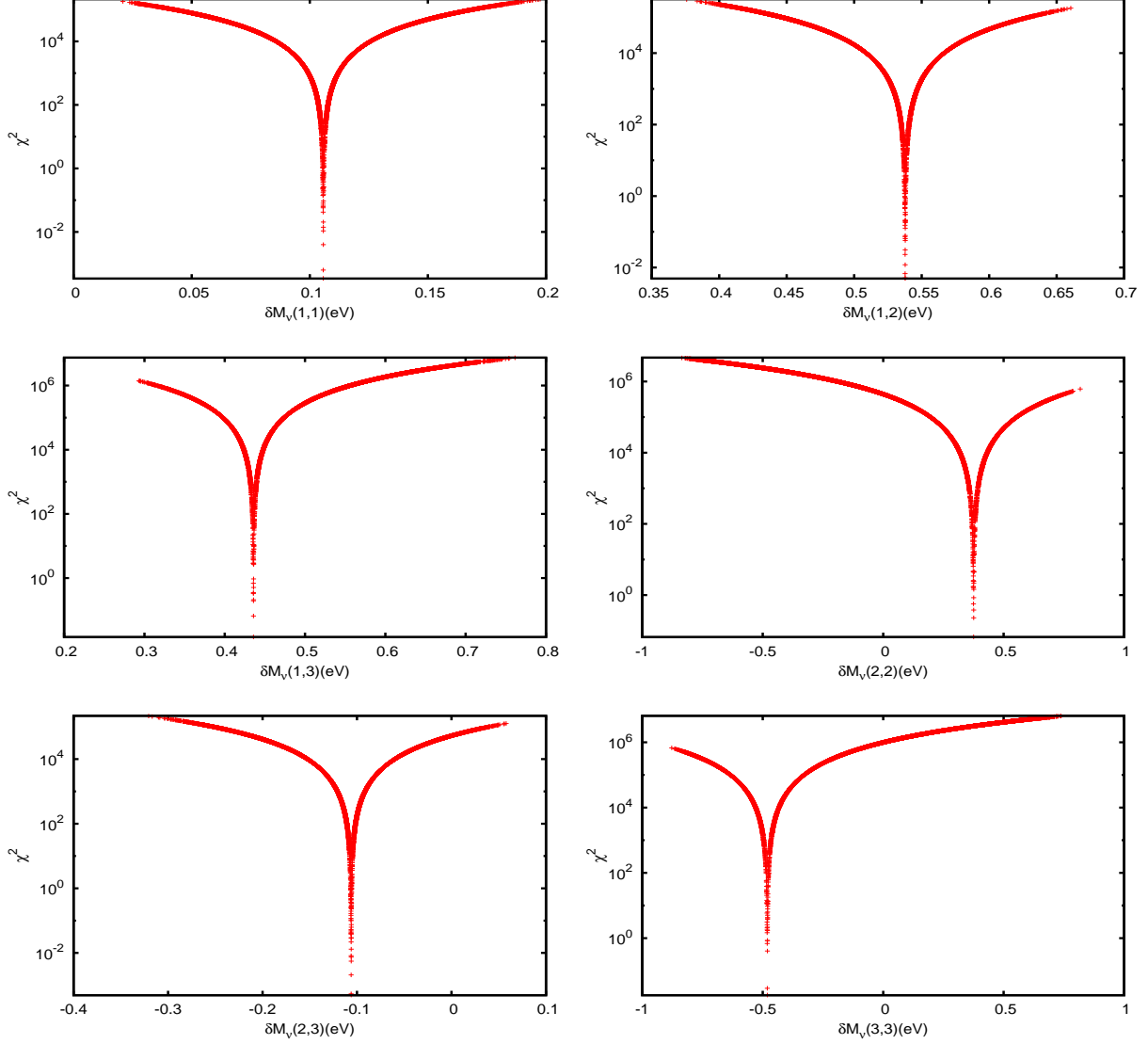


FIG. 1: χ^2 of $(\delta M_\nu(i, j) \times 10^2)$ for normal ordering of neutrino masses with TBM mixing in tree-level.

value of the lightest mass eigenvalue m_0 . In order to see the effect of m_0 on the value of $\delta M_\nu^c(i, j)$ qualitatively, we vary m_0 in the range $(10^{-10}, 10)$ eV while keeping other parameters such as θ_{12} , θ_{13} , θ_{23} , $|\Delta m_{\text{atm}}^2|$, and Δm_\odot^2 fixed at their best fit values. In Fig. 3, we plot $\delta M_\nu^c(i, j)$ versus m_0 and it is clear that for $m_0 < 10^{-3}$ eV, the value of $\delta M_\nu^c(i, j)$ remains constant. In other words, $\delta M_\nu(i, j)$ are independent of the value of m_0 . However, for $m_0 \gtrsim 10^{-3}$ eV, the value of $\delta M_\nu(i, j)$ keeps changing. In the limit, $m_0 \gg \sqrt{|\Delta m_{\text{atm}}^2|} = 4.9 \times 10^{-2}$ eV, we have $m_3 = m_2 = m_1 = m_0$ and hence $\delta M_\nu(i, j)$ become zero. This particular feature is common to both normal and inverted

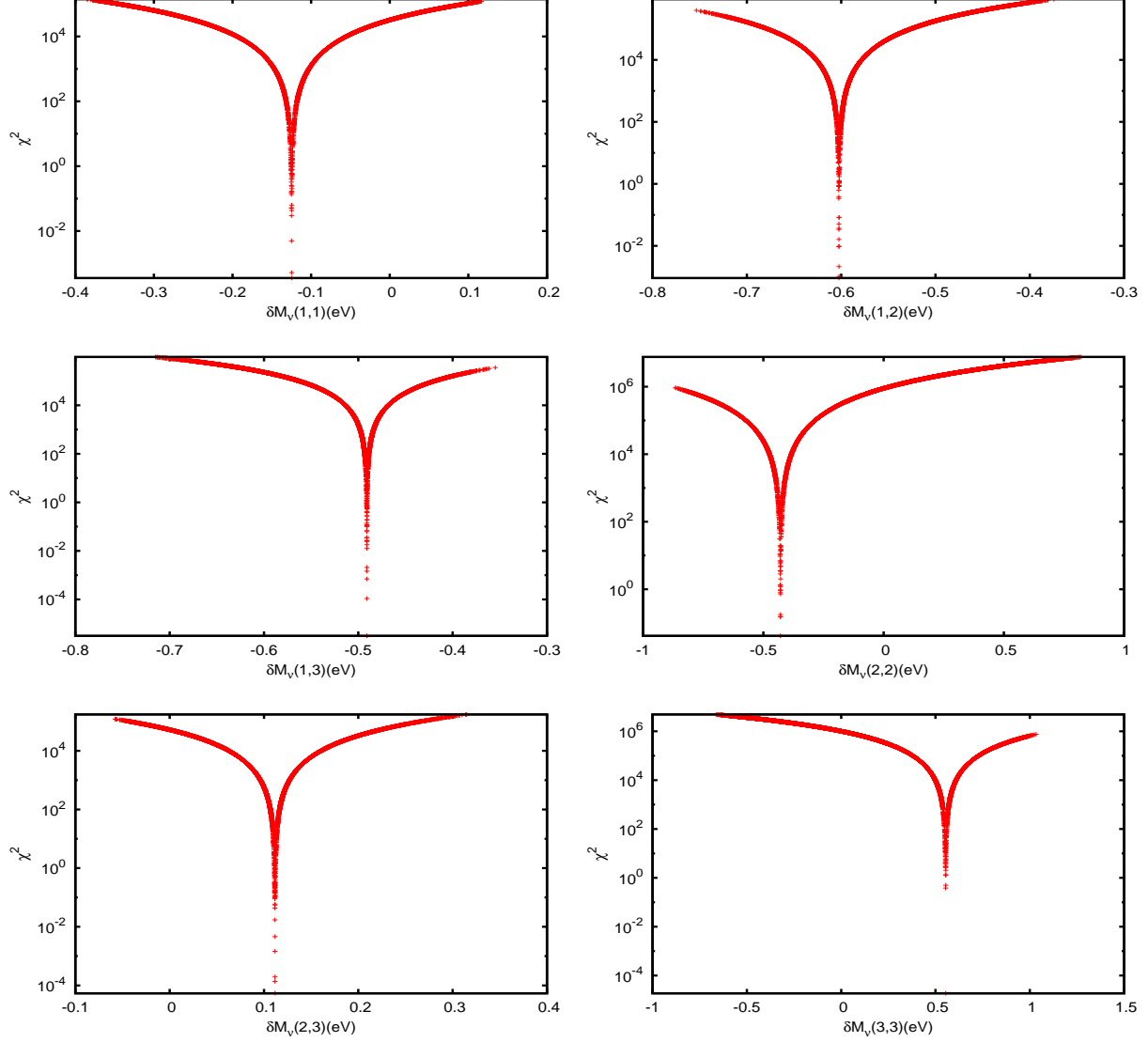


FIG. 2: χ^2 of $(\delta M_\nu(i, j) \times 10^2)$ for inverted ordering of neutrino masses with TBM mixing in tree-level.

hierarchies as can be seen from Fig. 3. To appreciate the plots in Fig. 3, we write down $\delta M_\nu(i, j)$ as a function of neutrino masses and mixing angles:

$$\delta M_\nu(1, 1) = ((c_{12}c_{13})^2 - 2/3)m_1 + ((s_{12}c_{13})^2 - 1/3)m_2 + s_{13}^2 m_3$$

$$\begin{aligned} \delta M_\nu(1, 2) = & (c_{12}c_{13}(-s_{12}c_{23} - c_{12}s_{23}s_{13}) + 1/3)m_1 \\ & + (s_{12}c_{13}(c_{12}c_{23} - s_{12}s_{23}s_{13}) - 1/3)m_2 + s_{13}c_{13}s_{23}m_3 \end{aligned}$$

$$\delta M_\nu(1, 3) = (c_{12}c_{13}(s_{12}s_{23} - c_{12}c_{23}s_{13}) - 1/3)m_1$$

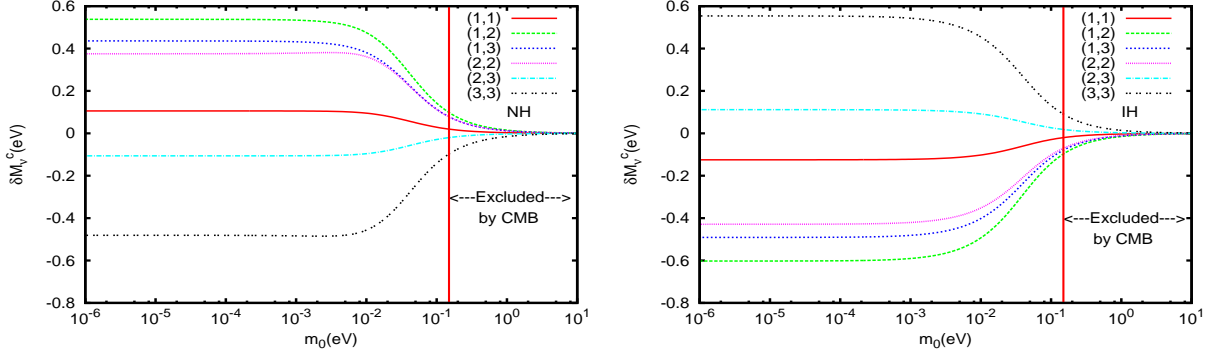


FIG. 3: Variation of $(\delta M_\nu^c \times 10^2)$ with respect to the lightest neutrino mass for NH (left) and IH (right) of neutrino masses with TBM mixing. An upper cut-off at $m_0 \approx 0.15\text{eV}$ [15] is put to ensure that $\delta M_\nu(i, j) \neq 0$ for all $m_0 < 0.15\text{eV}$.

$$\begin{aligned}
& + (s_{12}c_{13}(-c_{12}s_{23} - s_{12}c_{23}s_{13}) + 1/3)m_2 + s_{13}c_{13}c_{23}m_3 \\
\delta M_\nu(2, 2) & = ((-s_{12}c_{23} - c_{12}s_{23}s_{13})^2 - 1/6)m_1 \\
& + ((c_{12}c_{23} - s_{12}s_{23}s_{13})^2 - 1/3)m_2 + ((c_{13}s_{23})^2 - 1/2)m_3 \\
\delta M_\nu(2, 3) & = ((-s_{12}c_{23} - c_{12}s_{23}s_{13})(s_{12}s_{23} - c_{12}c_{23}s_{13}) + 1/6)m_1 \\
& + ((c_{12}c_{23} - s_{12}s_{23}s_{13})(-c_{12}s_{23} - s_{12}c_{23}s_{13}) + 1/3)m_2 + (s_{23}c_{13}c_{23}c_{13} - 1/2)m_3 \\
\delta M_\nu(3, 3) & = ((s_{12}s_{23} - c_{12}c_{23}s_{13})^2 - 1/6)m_1 \\
& + ((-c_{12}s_{23} - s_{12}c_{23}s_{13})^2 - 1/3)m_2 + ((c_{13}c_{23})^2 - 1/2)m_3.
\end{aligned} \tag{12}$$

Note that the higher eigenvalues m_2 and $m_3(m_1)$ can be re-expressed as a function of lightest neutrino mass $m_0 \equiv m_1(m_3)$. Therefore, it is obvious that all the elements: $\delta M_\nu(i, j)$ are function of the only unknown quantity m_0 . From Fig. 3 we see that $\delta M_\nu(1, 1)$, $\delta M_\nu(1, 2)$, $\delta M_\nu(1, 3)$, $\delta M_\nu(2, 2)$ start with a positive value, while $\delta M_\nu(2, 3)$ and $\delta M_\nu(3, 3)$ start with a negative value. An exactly opposite spectrum is observed in case of inverted hierarchy as expected. In either case we observe that for $m_0 \ll 10^{-3}\text{eV}$, $\delta M_\nu(i, j)$ are independent of m_0 . Therefore, in this limit it is reasonable to set $m_0 \approx 0$.

In the opposite limit when $m_0 \gg \sqrt{|\Delta m_{\text{atm}}^2|} = 4.9 \times 10^{-2}\text{eV}$, we have $m_3 = m_2 = m_1 = m_0$. In this limit we get $M_\nu(i, j) = m_0(I)$, where I is the identity matrix. Hence all the $\delta M_\nu(i, j)$ are

zero as expected ¹. This feature can be easily read from Fig. 3. However, the observation tells us that this should not be the case and we need small mass splittings between the mass eigenvalues in order to satisfy the solar and atmospheric mass differences. Therefore, we put an upper cut-off at $m_0 \approx 0.15\text{eV}$ [15], as required by CMB data, to ensure that $\delta M_\nu(i, j) \neq 0$ for all $m_0 < 0.15\text{eV}$. In this region $m_1 \neq m_2 \neq m_3$.

For $10^{-3}\text{eV} < m_0 < 0.15\text{eV}$, which is comparable to solar and atmospheric values, we observe appreciable effect of m_0 on various $\delta M_\nu(i, j)$ as shown in Fig. 3. We have factored out this dependency of all $\delta M_\nu(i, j)$ on m_0 using an exponential parameterization:

$$\left| \delta M_\nu(m_0, \sqrt{\Delta m_\odot^2}, \sqrt{|\Delta m_{\text{atm}}^2|}, \theta_{12}, \theta_{13}, \theta_{23}) \right| = \left| \delta M_\nu(m_0 = 0, \sqrt{\Delta m_\odot^2}, \sqrt{|\Delta m_{\text{atm}}^2|}, \theta_{12}, \theta_{13}, \theta_{23}) \right| \times \text{Exp}(-m_0/a). \quad (13)$$

where a is determined from a fit to the recent experimental data and found to be $a \approx 0.1\text{eV}$. As a result we could do all our analysis for $m_0 \leq 10^{-3}\text{eV}$ which is equivalent to setting $m_0 = 0$. Then we generalize it to any value of m_0 using Eq. 13.

We first compare the perturbed matrix elements $\delta M_\nu^c(i, j)$ with the elements of tree level mass matrix $(M_\nu)_{\text{TBM}}(i, j)$. In case of normal hierarchy, the experimental mass matrix, the tree-level neutrino mass matrix, and its deviations from the central values are given by:

$$\frac{(M_\nu^c)_{\text{EXPT}}}{10^{-2}\text{eV}} = \begin{pmatrix} 0.397 & 0.829 & 0.145 \\ 0.829 & 3.191 & 2.128 \\ 0.145 & 2.128 & 2.335 \end{pmatrix}, \quad \frac{(M_\nu)_{\text{TBM}}}{10^{-2}\text{eV}} = \begin{pmatrix} 0.291 & 0.291 & -0.291 \\ 0.291 & 2.816 & 2.234 \\ -0.291 & 2.234 & 2.816 \end{pmatrix}, \quad (14)$$

and

$$\frac{\delta M_\nu^c}{10^{-2}\text{eV}} = \begin{pmatrix} 0.106 & 0.538 & 0.436 \\ 0.538 & 0.375 & -0.106 \\ 0.436 & -0.106 & -0.481 \end{pmatrix}. \quad (15)$$

Similarly, for inverted hierarchy, the mass matrix $(M_\nu^c)_{\text{EXPT}}$ and $(M_\nu)_{\text{TBM}}$ are,

$$\frac{(M_\nu^c)_{\text{EXPT}}}{10^{-2}\text{eV}} = \begin{pmatrix} 4.830 & -0.577 & -0.517 \\ -0.577 & 2.061 & -2.379 \\ -0.517 & -2.379 & 3.044 \end{pmatrix}, \quad \frac{(M_\nu)_{\text{TBM}}}{10^{-2}\text{eV}} = \begin{pmatrix} 4.956 & 0.026 & -0.026 \\ 0.026 & 2.490 & -2.490 \\ -0.026 & -2.490 & 2.490 \end{pmatrix} \quad (16)$$

¹ In our case we set all the CP-violating phases to be zero. As a result, the unitary PMNS matrix is simply an orthogonal matrix and hence we get $M_\nu(i, j) = \mathcal{O}m_0(I)\mathcal{O}^T = m_0(I)$. This is not true if one assumes the CP-violating phases to be non-zero.

and the corresponding perturbed matrix is

$$\frac{\delta M_\nu^c}{10^{-2} \text{ eV}} = \begin{pmatrix} -0.125 & -0.602 & -0.491 \\ -0.602 & -0.429 & 0.112 \\ -0.491 & 0.112 & 0.554 \end{pmatrix}. \quad (17)$$

We introduce a new parameter ϵ as:

$$\epsilon^c(i, j) = \frac{\delta M_\nu^c(i, j)}{(M_\nu)_{\text{TBM}}(i, j)} \quad (18)$$

such that it will give a measure of required perturbation with respect to the corresponding tree-level value. Thus for normal and inverted hierarchies of neutrino mass spectrum, we get:

$$(\epsilon^c)_{\text{NH}} = \begin{pmatrix} 0.364 & 1.848 & -1.848 \\ 1.848 & 0.133 & -0.047 \\ -1.848 & -0.047 & 0.171 \end{pmatrix} \quad (\epsilon^c)_{\text{IH}} = \begin{pmatrix} 0.02 & 23.154 & 18.885 \\ 23.154 & 0.172 & 0.045 \\ 18.885 & 0.045 & 0.222 \end{pmatrix}. \quad (19)$$

From Eq. (19), it is clear that (1, 2), and (1, 3) elements of the TBM mass matrix needs to be modified largely to be consistent with the experiment. The perturbations of rest of the elements are small in comparison to their tree-level masses. In case of NH the modification is mild, while it is significant in case of IH case.

We now try to explore the θ_{13} , θ_{23} , and θ_{12} dependency of all the elements of the neutrino mass matrix in the flavor basis where the charged lepton is assumed to be diagonal. In order to see the effect of a particular oscillation parameter on the value of δM_ν , we vary that parameter in the 3σ allowed range and do a marginalization over the other oscillation parameters. From Eq. (12) we see that in either case of NH and IH the dependency of $\delta M_\nu(1, 2)$ and $\delta M_\nu(1, 3)$ on θ_{12} is almost negligible because of the cancellation in the second term of each expression. However, both $\delta M_\nu(1, 2)$ and $\delta M_\nu(1, 3)$ do depend on θ_{23} and θ_{13} in either case of NH and IH as can be seen from Fig. 4. The dependency of $\delta M_\nu(1, 2)$ and $\delta M_\nu(1, 3)$ elements on θ_{13} and θ_{23} can be understood from the following analytical approximation. In case of NH, $m_0 \equiv m_1 \rightarrow 0$. Hence we have $m_3 = \sqrt{\Delta m_{\text{atm}}^2}$ and $m_2 = \sqrt{\Delta m_\odot^2}$. As a result from Eq. (12) we get

$$\begin{aligned} |\delta M_\nu(1, 2)| &= \left[(s_{12}c_{13}(c_{12}c_{23} - s_{12}s_{23}s_{13}) - 1/3)\sqrt{\Delta m_\odot^2} + s_{13}c_{13}s_{23}\sqrt{\Delta m_{\text{atm}}^2} \right] \text{Exp}[-m_0/a] \\ |\delta M_\nu(1, 3)| &= \left[(s_{12}c_{13}(-c_{12}s_{23} - s_{12}c_{23}s_{13}) + 1/3)\sqrt{\Delta m_\odot^2} + s_{13}c_{13}c_{23}\sqrt{\Delta m_{\text{atm}}^2} \right] \text{Exp}[-m_0/a] \end{aligned} \quad (20)$$

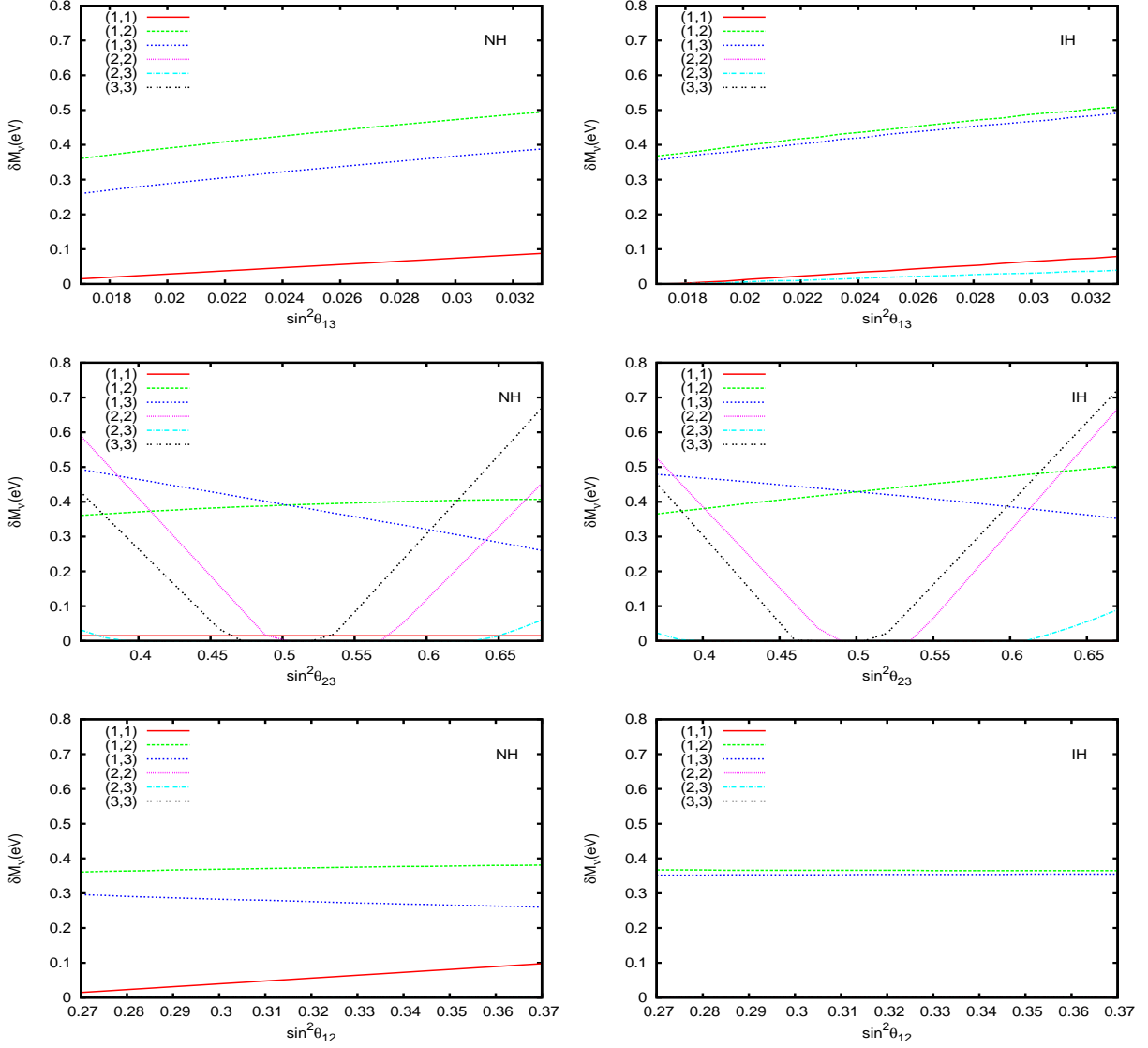


FIG. 4: $(\delta M_\nu(i, j) \times 10^2)$ as function of mixing angles for $m_0 = 0$ in case of NH (left) and IH (right) of neutrino mass matrix.

where the exponential factor in $|\delta M_\nu(1, 2)|$ and $|\delta M_\nu(1, 3)|$ gives the lightest neutrino mass dependency. Because of the cancellation, the 1st term in the square bracket is always suppressed in comparison to the second term. Therefore, we get:

$$\begin{aligned}
 |\delta M_\nu(1, 2)| &\approx \sqrt{\Delta m_{\text{atm}}^2} \sin \theta_{13} \cos \theta_{13} \sin \theta_{23} \text{Exp}[-m_0/a] \\
 |\delta M_\nu(1, 3)| &\approx \sqrt{\Delta m_{\text{atm}}^2} \sin \theta_{13} \cos \theta_{13} \cos \theta_{23} \text{Exp}[-m_0/a].
 \end{aligned} \tag{21}$$

On the other hand in case of IH, $m_0 \equiv m_3 \rightarrow 0$. Hence we have $m_2 \approx m_1 = \sqrt{\Delta m_{\text{atm}}^2}$. As a result,

in the same analogy of Eq. (21), we get

$$\begin{aligned}
|\delta M_\nu(1, 2)| &\approx \sqrt{\Delta m_{\text{atm}}^2} \sin \theta_{13} \cos \theta_{13} \sin \theta_{23} \text{Exp}[-m_0/a] \\
|\delta M_\nu(1, 3)| &\approx \sqrt{\Delta m_{\text{atm}}^2} \sin \theta_{13} \cos \theta_{13} \cos \theta_{23} \text{Exp}[-m_0/a].
\end{aligned}
\tag{22}$$

Thus in either cases the dependency of $\delta M_\nu(1, 2)$ and $\delta M_\nu(1, 3)$ elements on θ_{13} and θ_{23} are very similar. This can be checked from Fig. 4. It is evident from Fig. 4 that $\delta M_\nu(2, 2)$, $\delta M_\nu(2, 3)$, and $\delta M_\nu(3, 3)$ elements don't depend on θ_{12} . Those elements depend mostly on the value of θ_{23} . To understand this feature quantitatively, we write down the matrix elements $\delta M_\nu(2, 2)$, $\delta M_\nu(2, 3)$, and $\delta M_\nu(3, 3)$ in the following. For normal ordering of the neutrino mass spectrum, we have $m_0 \equiv m_1 \rightarrow 0$. As a result, in the same analogy of Eq. (21), we have:

$$\begin{aligned}
\delta M_\nu(2, 2) &\approx (\cos^2 \theta_{13} \sin^2 \theta_{23} - \frac{1}{2}) \sqrt{\Delta m_{\text{atm}}^2} \text{Exp}[-m_0/a], \\
\delta M_\nu(2, 3) &\approx (\sin \theta_{23} \cos \theta_{23} \cos^2 \theta_{13} - \frac{1}{2}) \sqrt{\Delta m_{\text{atm}}^2} \text{Exp}[-m_0/a], \\
\delta M_\nu(3, 3) &\approx (\cos^2 \theta_{13} \cos^2 \theta_{23} - \frac{1}{2}) \sqrt{\Delta m_{\text{atm}}^2} \text{Exp}[-m_0/a]
\end{aligned}
\tag{23}$$

Clearly, $\delta M_\nu(2, 2)$, $\delta M_\nu(2, 3)$, and $\delta M_\nu(3, 3)$ don't depend on θ_{12} at all. There is a θ_{13} dependency and is of similar order if θ_{23} is near to its TBM value. However, once the value of θ_{23} deviates away from its TBM value, all these matrix elements mainly depend on the value of θ_{23} . Moreover, all these $\delta M_\nu(i, j)$ increases as we move away from the TBM value. In case of IH, $m_0 \equiv m_3 \rightarrow 0$ and a similar pattern for all the $\delta M_\nu(i, j)$ is expected. This can be easily read from the right panel of Fig. 4.

III. APPLICATION TO TOP-DOWN APPROACH

Now that we know the perturbed matrix from the bottom-up approach, we wish to construct models of neutrino masses and mixings in a top-down approach and elucidate the role of lightest neutrino mass. Our main objective here is that we will use the predictions of bottom-up approach as guide lines for building neutrino mass matrix whose tree-level mixing is governed by a symmetry. We then modify this using some perturbation so that the resulting values are consistent with the results that are obtained using the bottom up approach of section. IIB. We begin by proposing a model where the neutrino mixing is described by $U_M = U_0 V$, where V is the perturbed mixing

matrix around the tree-level value and in general is given by:

$$V = V(1, 2) V(1, 3) V(2, 3), \quad (24)$$

with $V(i, j)$ being given by an orthogonal rotation matrix in the (i, j) plane of neutrino mass matrix. Let us write down $V(i, j)$ explicitly such that the perturbed mixing matrix is

$$\begin{aligned} V = V(1, 2) V(1, 3) V(2, 3) &= \begin{pmatrix} 1 & \beta & 0 \\ -\beta & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & \alpha \\ 0 & 1 & 0 \\ -\alpha & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \gamma \\ 0 & -\gamma & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 & \beta - \alpha\gamma & \alpha + \beta\gamma \\ -\beta & 1 & \gamma - \beta\alpha \\ -\alpha & -\gamma & 1 \end{pmatrix}, \end{aligned} \quad (25)$$

where the determinant of $V(i, j)$ matrix is assumed to be unity. Since U_M is the diagonalising matrix of the proposed neutrino mass matrix, we get

$$\begin{aligned} M_\nu^D &= U_M^T (M_\nu)_M U_M = U_M^T ((M_\nu)_0 + \delta M_\nu) U_M \\ &= V^T U_0^T ((M_\nu)_0 + \delta M_\nu) U_0 V, \end{aligned} \quad (26)$$

where, $(M_\nu)_M$ is the mass matrix in a flavor basis where charged leptons are real and diagonal. We now define a matrix X such that

$$X \equiv V M_\nu^D V^T = U_0^T ((M_\nu)_0 + \delta M_\nu) U_0, \quad (27)$$

where the right-hand-side is determined through the bottom-up approach for a specific value of m_0 , where as the left-hand-side involves six parameters, namely m_0 , Δm_{atm}^2 , Δm_{\odot}^2 , α, β and γ . Note that for $m_0 < 10^{-3}\text{eV}$ we can neglect m_0 dependency as we discussed earlier in Section. II B.

A. Application to Tri-bi-maximal mixing

Let us identify the tree-level mixing matrix as a tri-bi-maximal one, predicted by certain symmetry, then $U_0 = U_{\text{TBM}}$. As a result from Eq. (27) we get

$$X \equiv V M_\nu^D V^T = U_{\text{TBM}}^T ((M_\nu)_{\text{TBM}} + \delta M_\nu) U_{\text{TBM}}. \quad (28)$$

The matrix elements $X(i, j)$ can be expressed in terms of $m_1, m_2, m_3, \alpha, \beta, \gamma$ as

$$\begin{aligned}
X(1, 1) &= m_1 + m_2 \beta^2 + m_3 \alpha^2, \\
X(1, 2) &= (m_2 - m_1) \beta + (m_3 - m_2) \alpha \gamma, \\
X(1, 3) &= (m_3 - m_1) \alpha + (m_3 - m_2) \beta \gamma, \\
X(2, 2) &= m_1 \beta^2 + m_2 + m_3 \gamma^2, \\
X(2, 3) &= -(m_3 - m_1) \alpha \beta + (m_3 - m_2) \gamma, \\
X(3, 3) &= m_1 \alpha^2 + m_2 \gamma^2 + m_3,
\end{aligned} \tag{29}$$

where $m_2 = \sqrt{m_1^2 + \Delta m_{\odot}^2}$ and $m_3 = \sqrt{m_1^2 + \Delta m_{\text{atm}}^2}$, respectively. Here we assume normal ordering of the neutrino masses. We have ignored higher order terms in α, β , and γ . We wish to determine the range of α, β, γ such that $X(i, j)$ computed from Eq. (29) is compatible within the 3σ ranges of $X(i, j)$ derived from the bottom-up approach, i.e, from the right hand side of Eq. (28), for $m_0 \equiv m_1 = 0$ (i.e., $m_0 < 10^{-3}$ eV) as well as for $m_0 \equiv m_1 \neq 0$ (i.e. $m_0 > 10^{-3}$ eV).

1. *Case-I: $m_0 \equiv m_1 \rightarrow 0$*

In Table. II, we report all the elements of the matrix X obtained using the bottom up approach for the TBM mixing scenario. We need to find the range of α, β , and γ so that $X(i, j)$ values

TBM	X(1,1)/eV	X(1,2)/eV	X(1,3)/eV	X(2,2)/eV	X(2,3)/eV	X(3,3)eV
Central values	0.0203	0.0487	0.315	1.01	0.747	4.89
3σ range	$[0.87 \times 10^{-4}, 0.269]$	$[-0.0937, 0.118]$	$[0.0141, 1.15]$	$[0.839, 1.17]$	$[-0.153, 1.05]$	$[4.57, 5.14]$

TABLE II: Values of $X(i, j) \times 10^2$ for TBM mixing from the bottom up approach for $m_0 = 0$.

computed from Eq. (29) are consistent with the values reported in Table. II. We get the range of α, β , and γ by fitting $X(1, 2), X(1, 3)$ and $X(2, 2)$ in Eq. 29 with the corresponding $X(1, 2), X(1, 3)$, and $X(2, 2)$ ranges from Table. II. Once the range of α, β , and γ are known, we then plug those in Eq. 29 to get the range of $X(1, 1), X(2, 3)$, and $X(3, 3)$ and are shown in Fig. 5. Since α, β and γ are non-zero, it is obvious that all the six $X(i, j)$ elements can be fitted within their experimental ranges. However, if more parameters are set to zero then it is not clear if all the six $X(i, j)$ elements can be fitted to their experimental values.

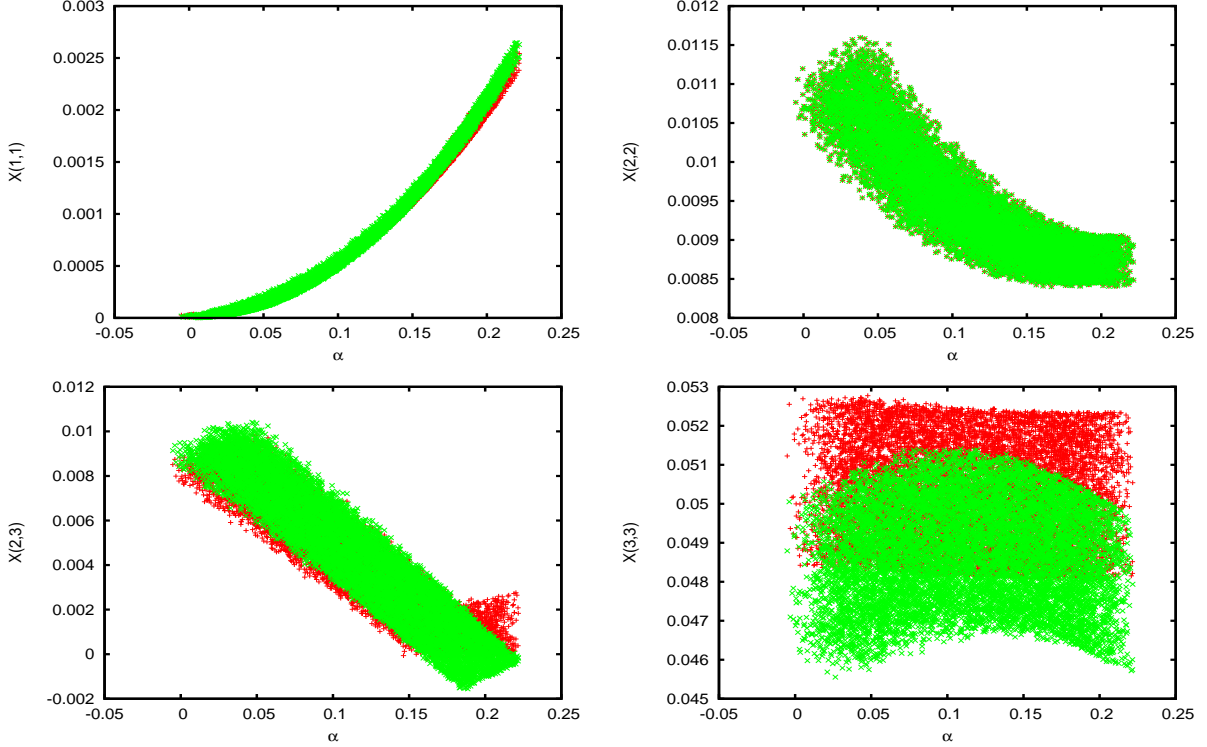


FIG. 5: Range of $X(i, j)$ as a function of α for $\alpha, \beta, \gamma \neq 0$. The theory ranges are denoted by red whereas the experimental ranges from the bottom up approach are denoted by green.

In order to realize the dependence of Eq. (29) on number of parameters, we analyze six different possibilities: Firstly, we set one of the three parameters to zero while keeping other two non zero (3 ways) and secondly, we set two of the three parameters to zero while keeping the third one non-zero (3 ways).

- First we set $\gamma = 0$ and find the range of α and β by comparing $X(1, 2)$ and $X(1, 3)$ with their experimental values given in Table. II. Then we check the consistency of remaining four elements and shown in Fig. 6. It is clear that, although, the resulting range of $X(i, j)$ s are compatible within their 3σ ranges, we do not get a very good fit for $X(2, 3)$ because γ is a perturbation in the (2, 3) plane. This in turn can be understood from Eq. (29) by setting $m_1 = 0$ and $\gamma = 0$. Since $X(2, 3) \propto m_3\alpha\beta$ and α is required to be small from fitting with $X(1, 3)$, it is clear that theoretical values of $X(2, 3)$ has minimal overlap with its experimental range of values.

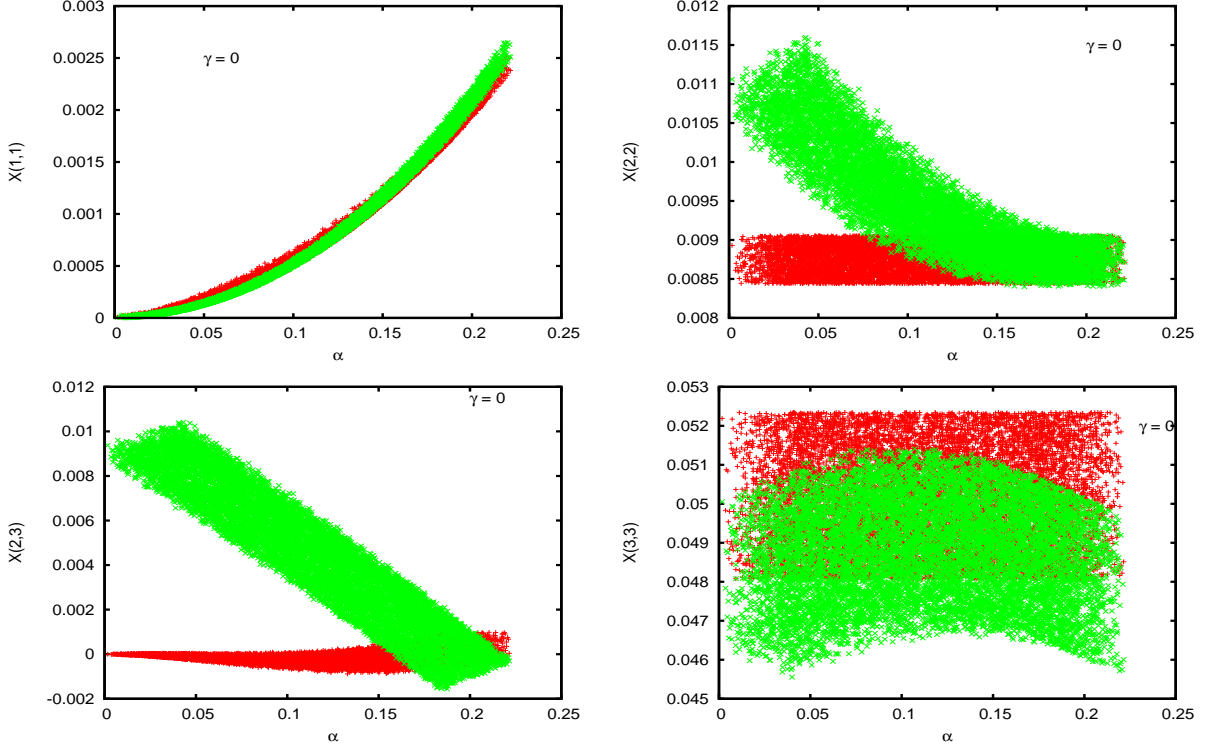


FIG. 6: Range of $X(i, j)$ as a function of α with $\gamma = 0$. The theory ranges are denoted by red whereas the experimental ranges from the bottom up approach are denoted by green.

- We perform the same exercise by setting $\beta = 0$. We fix the range of α and γ from $X(1, 3)$ and $X(2, 2)$. Once the range of α and γ are obtained, we then check the consistency of remaining four elements $X(1, 1)$, $X(1, 2)$, $X(2, 3)$, $X(3, 3)$. This is shown in Fig. 7. Since β gives the perturbation in $(1, 2)$ plane and the perturbations have minimal dependencies on β , we expect a better fit for all the elements. In fact this can be quickly checked from Eq. (29) by setting $m_1 = 0$ and $\beta = 0$.
- Next we set $\alpha = 0$ and fix the range of β and γ from $X(1, 2)$ and $X(2, 2)$. Then we check the consistency of remaining four elements $X(1, 1)$, $X(1, 3)$, $X(2, 3)$, $X(3, 3)$ and are shown in Fig. 8. Since α is the perturbation in $(1, 3)$ plane and it is set to zero, it is obvious that the theoretical values $X(1, 3)$ has a minimal overlap with its 3σ range of experimental values. Note that $X(1, 3)$ is not exactly zero even if $\alpha = 0$. This in fact can be understood from Eq. (29) by setting $m_1 = 0$ and $\alpha = 0$. In this limit $X(1, 3) = (m_3 - m_2)\beta\gamma$ which is not zero. Thus we can get a required θ_{13} by doing perturbation in $(1, 2)$ and $(2, 3)$ plane.

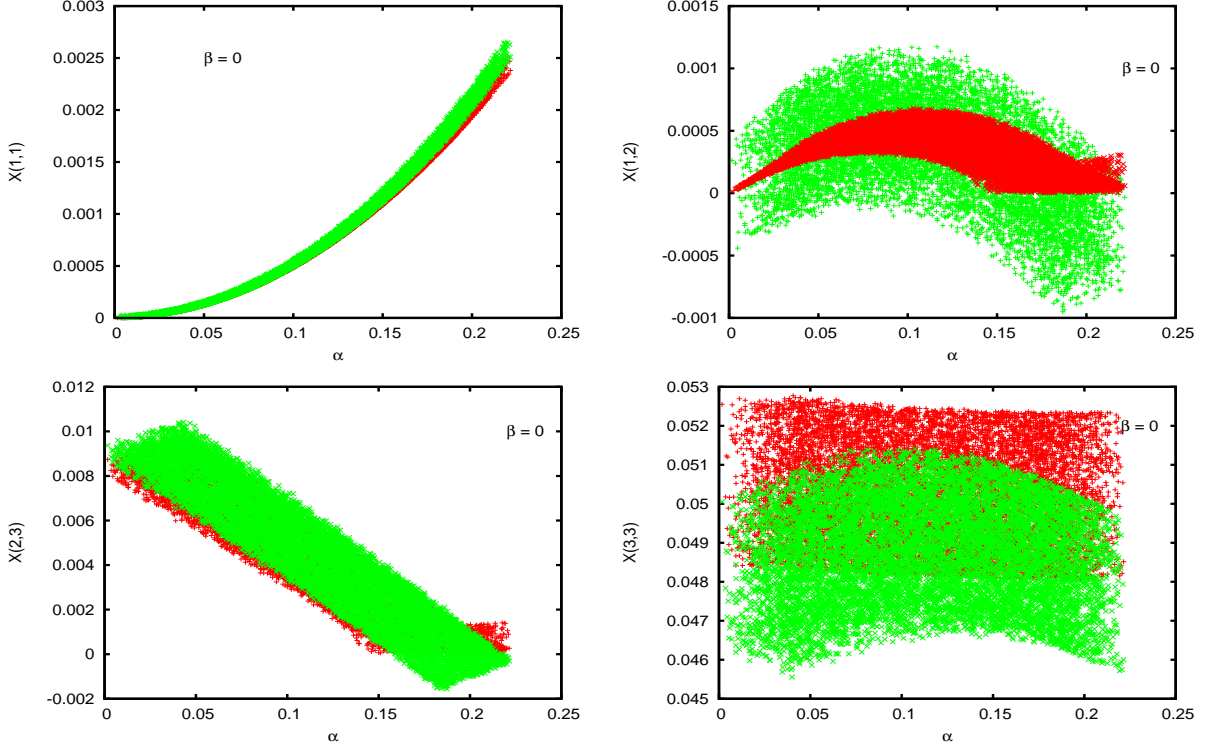


FIG. 7: Range of $X(i, j)$ as a function of α with $\beta = 0$. The theory ranges are denoted by red whereas the experimental ranges from the bottom up approach are denoted by green.

Since we can set α , β , and γ to be zero individually, the order in which we do the rotations to construct the perturbed matrix V is not very important. We further note that if V is product of at least two matrices (that is V is parameterized by at least two variables) then the parameters in V -matrix can not be considered as true-perturbation around their tree-level values. Therefore, the new parameters $\alpha, \beta, \gamma \dots$ are arbitrary.

Now let us concentrate on the second case where we put two model parameters to zero while keeping third one to be non-zero. It is clear from Eq. (29) that setting either $(\alpha, \beta = 0)$ or $(\alpha, \gamma = 0)$ will give $X(1, 3) = 0$ which is not compatible with its 3σ range of experimental values reported in Table. II. However, a non zero α with $(\beta, \gamma = 0)$ is still a solution and is compatible with the experimental data set. In this case, we fix the range of α from $X(1, 3)$ and check the consistency of remaining five elements. In the limit $m_1 = 0$ and $\beta = \gamma = 0$, from Eq. (29) we see

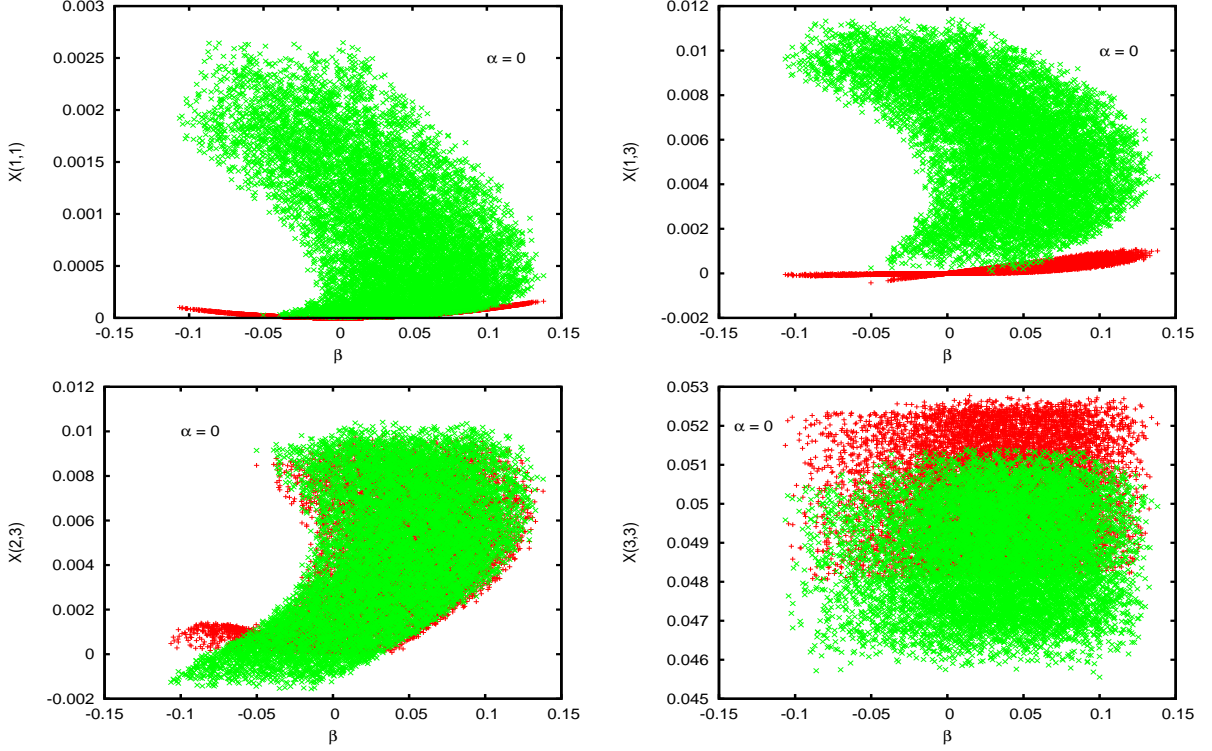


FIG. 8: Range of $X(i, j)$ as a function of β for $\alpha = 0$. The theory ranges are denoted by red whereas the experimental ranges from the bottom up approach are denoted by green.

that $X(1, 2) = X(2, 3) = 0$ and are compatible with their 3σ range of experimental values ². The α dependency of remaining elements $X(1, 1)$, $X(2, 2)$, and $X(3, 3)$ are shown in Fig. 9.

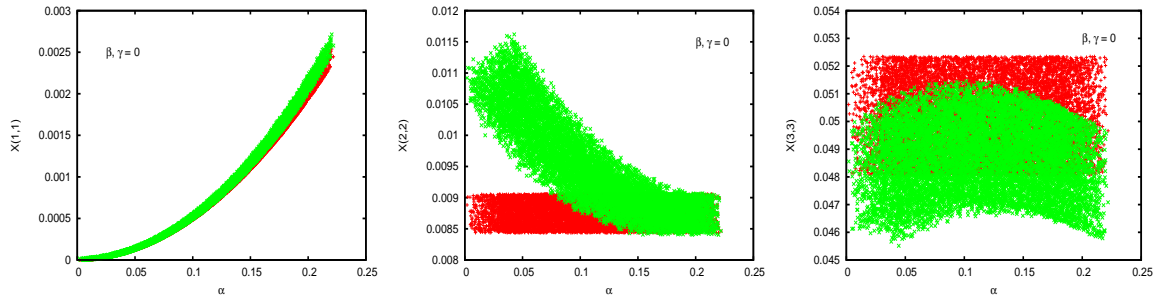


FIG. 9: Range of $X(i, j)$ as a function of α for $(\beta, \gamma = 0)$. The theory ranges are denoted by red whereas the experimental ranges from the bottom up approach are denoted by green.

² Such a case has been obtained using an A_4 symmetry in a type-II seesaw framework [23].

2. *Case-II: $m_0 \equiv m_1 > 10^{-3} \text{eV}$*

Now we discuss the role of the non-zero lightest neutrino mass in determining the form of the perturbed matrix V . In Table. III and Table. IV, we report all the elements of the matrix X for $m_0 = 0.01$ and 0.1eV obtained using the bottom up approach for the TBM mixing scenario.

TBM	X(1,1)/eV	X(1,2)/eV	X(1,3)/eV	X(2,2)/eV	X(2,3)/eV	X(3,3)/eV
Central values	1.02	0.044	0.26	1.45	0.68	5.00
3σ range	[1.0, 1.22]	[-0.054, 0.086]	[0.0079, 0.95]	[1.31, 1.60]	[-0.15, 0.96]	[4.68, 5.26]

TABLE III: Values of $X(i, j) \times 10^2$ for TBM mixing from the bottom up approach for $m_0 = 0.01 \text{eV}$.

TBM	X(1,1)/eV	X(1,2)/eV	X(1,3)/eV	X(2,2)/eV	X(2,3)/eV	X(3,3)/eV
Central values	10.0	0.013	0.075	10.08	0.21	11.16
3σ range	[10.0, 10.06]	[-0.0012, 0.023]	[0.00028, 0.28]	[10.03, 10.11]	[-0.046, 0.30]	[11.02, 11.26]

TABLE IV: Values of $X(i, j) \times 10^2$ for TBM mixing from the bottom up approach for $m_0 = 0.1 \text{eV}$.

To demonstrate the dependency of $X(i, j)$ on m_0 we consider only one out of six set of solutions discussed above. In particular, we choose the most non-trivial one with $\alpha = 0$ and $\beta, \gamma \neq 0$. Because from Fig. 8 we see that, in this case, we can get the required values of θ_{13} even in the absence of perturbation in (1,3) plane. We fix the range of β, γ from the experimental values of $X(1, 2)$ and $X(2, 2)$ given in Table. III and Table. IV for two different values of m_0 . We then check the consistency of remaining four elements. It is evident from Fig. 8 that, in the limit $m_0 = m_1 \rightarrow 0$, the theoretical values of $X(1, 3)$ has minimal overlapping with experimental values. Therefore, we choose this case to further study the dependency of $X(1, 3)$ on non-zero values of m_0 and the result is shown in Fig. 10. It is clear that the fitting with the experimental data improves once we go from $m_0 = 0$ to non zero values of m_0 .

We summarize our findings in Table. V. It is clear from our analysis that any perturbation to the TBM mass matrix where two parameters are non zero can reproduce the experimental data. It is not necessary to have perturbation in (1,3) plane to reproduce the experimental data as frequently discussed in the literature. Perturbations in (1,2) and (2,3) plane are equally good to get a reasonable fit within 3σ of the experimental data. However, once we set two parameters in

(1, 2) and (2, 3) plane to zero, then perturbation along the (1, 3) plane is necessary to get compatible results with the experimental data.

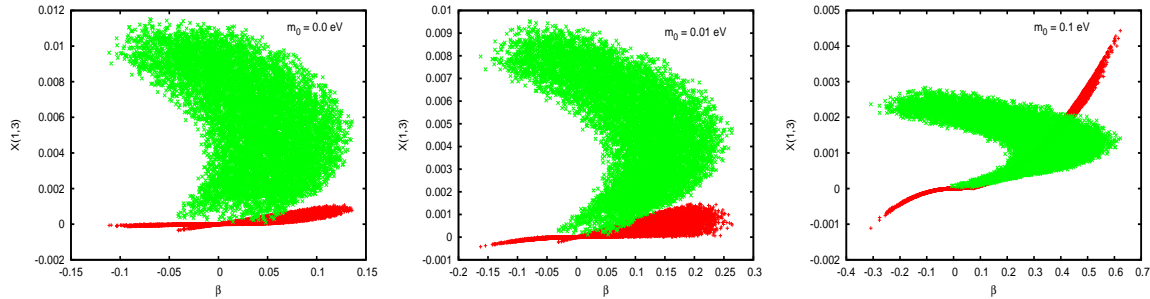


FIG. 10: Range of $X(1,3)$ as a function of β for $\alpha = 0$ at $m_0 = 0.0, 0.01, 0.1$ eV, respectively. The theory ranges are denoted by red whereas the experimental ranges from the bottom up approach are denoted by green.

$\alpha = 0$	$\beta, \gamma \neq 0$	\checkmark
$\beta = 0$	$\alpha, \gamma \neq 0$	\checkmark
$\gamma = 0$	$\alpha, \beta \neq 0$	\checkmark
$\alpha, \beta = 0$	$\gamma \neq 0$	\times
$\alpha, \gamma = 0$	$\beta \neq 0$	\times
$\gamma, \beta = 0$	$\alpha \neq 0$	\checkmark

TABLE V: Six different possibilities out of which four are consistent with data. These are true irrespective of the value of the lightest neutrino mass m_0 .

IV. CONCLUSION

The large value of θ_{13} predicted by Daya Bay and RENO put a stringent constraint on theoretical model buildings for neutrino mixing. All the well known mixing ansatzs such as TBM mixing, BM mixing, and DC mixing predict θ_{13} to be zero and hence not consistent with experiment. However,

a lot of phenomenological models in the top-down scenario have been proposed in order to explain the non zero θ_{13} , where a large value of the 1-3 mixing angle θ_{13} is generated through a perturbation to these mixing scenarios. In this paper we propose a perturbative bottom-up approach to quantify accurately the perturbation required for each element around a tree level mass matrix that is determined using the TBM mixing ansatz. Using the inputs from bottom-up approach we propose a model for neutrino masses and mixings. Though we used TBM mixing as an example, our analysis is more general and can be applied to any other ansatz which predicts $\theta_{13} = 0$ at the tree-level.

We first summarize our results for the bottom-up approach:

- It is known that in a flavor basis the elements of the neutrino mass matrix can be written in terms of the oscillation parameters and the absolute value of lightest neutrino mass (m_0). In this context we point out that for $m_0 \ll 10^{-3}$ eV, the perturbed elements don't depend on the value of m_0 . Therefore, all the perturbed elements can be determined exactly in terms of the oscillation parameters. However, in the opposite limit, where $m_0 \gtrsim 10^{-3}$ eV, we find that the perturbed elements have exponential dependency on the value of m_0 . We factor out the m_0 dependency of all $\delta M_\nu(i, j)$ using an exponential parameterization as given in Eq. (13). This helps us in determining the exact dependency of the perturbed matrix elements on the value of θ_{13} , θ_{23} and θ_{12} for all values of m_0 .
- In order to gauge the size of the perturbation to each element of the mass matrix, we first compare the perturbed matrix elements with the corresponding tree level mass matrix derived from the TBM mixing ansatz and find that only the (1, 2) and (1, 3) elements needs to be significantly modified to be consistent with the experimental data. This is true for normal as well as inverted ordering of the neutrino mass spectrum.
- We show the exact dependency of the perturbed matrix elements on the value of θ_{13} , θ_{23} and θ_{12} . We find that (1, 1), (1, 2) and (1, 3) elements of the perturbed matrix do depend on all the three mixing angles. However, their dependency on θ_{12} is quite small as compared to that of θ_{13} and θ_{23} . This, in act, is true for normal as well as inverted hierarchy of the neutrino mass spectrum. However, the (2, 2), (2, 3), and (3, 3) elements depend only on the value of θ_{23} . Similar conclusions can be drawn for inverted hierarchy as well.

We use the results of bottom-up approach as guide-lines for determining a perturbative model of neutrino masses and mixings. We introduce a typical mixing matrix $U_M = U_{\text{TBM}} V$ for the neutrino

mass matrix, where $V(\alpha, \beta, \gamma)$ is the perturbed mixing matrix. We fix the parameters α , β , and γ from the oscillation data obtained using the bottom up approach, where α , β and γ are the rotation angles in (1,3), (1,2) and (2,3) planes respectively. Here are some important observations from our top down analysis.

- We find that we can set any one parameter to zero, i.e, (α , β , or $\gamma = 0$) and still get a reasonably good fit with the experimental data that are derived using the bottom up approach. Although, keeping $\beta = 0$ and $\alpha, \gamma \neq 0$ gives a much better fit, it is worth mentioning that $\alpha = 0$ and $\beta, \gamma \neq 0$ also gives a reasonably good fit with the experimental data set. Thus, it is not necessary to have a perturbation in the (1, 3) plane in order to be compatible with the experimental data. This is one feature that was not explored in any earlier work.
- Once we set ($\alpha, \beta = 0$) or ($\alpha, \gamma = 0$), our results are not compatible within 3σ range of the experimental data. We, however, can set $\alpha \neq 0$ and ($\beta, \gamma = 0$) and still get a reasonably good fit with the experimental data. This case has been explored extensively in the literature.
- It is worth mentioning that the lightest neutrino mass plays an important role in the parameter fitting. The fitting improves once we go from a zero to non zero values of lightest neutrino mass. In particular for $m_0 \gtrsim 10^{-3}\text{eV}$ we get large overlappings between the theoretical values of $X(i, j)$ with their experimental values.
- Since α , β , and γ can be set to zero separately, their ordering is not important. We further note that, if V is parameterized by more than one variable then the perturbations are arbitrary. In other words, those can not be treated as true perturbations with respect to their tree level mixing angles.

Acknowledgments

RD and AKG would like to thank BRNS, Govt. of India for financial support.

[1] B. Pontecorvo, Sov. Phys. JETP **7**, 172 (1958) [Zh. Eksp. Teor. Fiz. **34**, 247 (1957)]; B. Pontecorvo, Sov. Phys. JETP **26** (1968) 984–988; V. N. Gribov and B. Pontecorvo, Phys. Lett. **B28** (1969) 493.

- [2] Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49** (1973) 652–657.
- [3] S. M. Bilenky, J. Hosek, and S. T. Petcov, Phys. Lett. **B94** (1980) 495.
- [4] P. Langacker, S. T. Petcov, G. Steigman, and S. Toshev, Nucl. Phys. **B282** (1987) 589.
- [5] Q. R. Ahmad *et al* [SNO collaboration], Phys. Rev. Lett. **87**, 071301 (2001) [arXiv:nucl-ex/0106015].
- [6] K. Eguchi *et al* [KamLAND collaboration], Phys. Rev. Lett. **92**, 071301 (2004) [arXiv:hep-ex/0310047].
- [7] Y. Itow *et al* [T2K collaboration], [arXiv:hep-ex/0106019].
- [8] D. S. Ayres *et al* [NO ν A collaboration], [arXiv:hep-ex/0503053].
- [9] K. Abe *et al.* [T2K Collaboration], Phys. Rev. Lett. **107**, 041801 (2011) [arXiv:1106.2822 [hep-ex]].
- [10] P. Adamson *et al.* [MINOS Collaboration], Phys. Rev. Lett. **107**, 181802 (2011) [arXiv:1108.0015 [hep-ex]].
- [11] Y. Abe *et al.* (Double Chooz Collaboration), Phys. Rev. Lett. **108**, 131801 (2012).
- [12] F. P. An *et al.* [DAYA-BAY Collaboration], arXiv:1203.1669 [hep-ex].
- [13] J. K. Ahn *et al.* [RENO Collaboration], arXiv:1204.0626 [hep-ex].
- [14] T. Thummler [KATRIN Collaboration], arXiv:1012.2282 [hep-ex].
- [15] C. L. Bennett, D. Larson, J. L. Weiland, N. Jarosik, G. Hinshaw, N. Odegard, K. M. Smith and R. S. Hill *et al.*, arXiv:1212.5225 [astro-ph.CO].
- [16] R. N. Mohapatra and W. Rodejohann, Phys. Rev. D **72**, 053001 (2005) [hep-ph/0507312]; A. S. Joshipura and A. Y. Smirnov, Nucl. Phys. B **750**, 28 (2006) [hep-ph/0512024]; Z. -z. Xing, H. Zhang and S. Zhou, Phys. Lett. B **641**, 189 (2006) [hep-ph/0607091];
- [17] H. Ishimori, T. Kobayashi, H. Ohki, Y. Omura, R. Takahashi and M. Tanimoto, Phys. Lett. B **662**, 178 (2008) [arXiv:0802.2310 [hep-ph]].
- [18] R. N. Mohapatra, S. Nasri and H. -B. Yu, Phys. Lett. B **639**, 318 (2006) [hep-ph/0605020].
- [19] C. Hagedorn, M. Lindner and R. N. Mohapatra, JHEP **0606**, 042 (2006) [hep-ph/0602244]; C. S. Lam, Phys. Rev. Lett. **101**, 121602 (2008) [arXiv:0804.2622 [hep-ph]]; F. Bazzocchi and S. Morisi, Phys. Rev. D **80**, 096005 (2009) [arXiv:0811.0345 [hep-ph]].
- [20] E. Ma and G. Rajasekaran, Phys. Rev. D **64**, 113012 (2001) [hep-ph/0106291]; S. F. King and M. Malinsky, Phys. Lett. B **645**, 351 (2007) [hep-ph/0610250]; G. Altarelli and F. Feruglio, Nucl. Phys. B **741**, 215 (2006) [hep-ph/0512103]; M. Hirsch, A. S. Joshipura, S. Kaneko and J. W. F. Valle, Phys.

- Rev. Lett. **99**, 151802 (2007) [hep-ph/0703046 [HEP-PH]]. B. Brahmachari, S. Choubey and M. Mitra, Phys. Rev. D **77**, 073008 (2008) [Erratum-ibid. D **77**, 119901 (2008)] [arXiv:0801.3554 [hep-ph]]; F. Feruglio, C. Hagedorn, Y. Lin and L. Merlo, Nucl. Phys. B **832**, 251 (2010) [arXiv:0911.3874 [hep-ph]].
- [21] H. Ishimori, T. Kobayashi, H. Ohki, Y. Shimizu, H. Okada and M. Tanimoto, Prog. Theor. Phys. Suppl. **183**, 1 (2010) [arXiv:1003.3552 [hep-th]]; S. F. King and C. Luhn, Rept. Prog. Phys. **76**, 056201 (2013) [arXiv:1301.1340 [hep-ph]]; G. Altarelli and F. Feruglio, New J. Phys. **6**, 106 (2004) [hep-ph/0405048].
- [22] A. Dighe and N. Sahu, arXiv:0812.0695 [hep-ph]; G. Couture, C. Hamzaoui, S. S. Y. Lu and M. Toharia, Phys. Rev. D **81**, 033010 (2010) [arXiv:0910.3132 [hep-ph]]; E. Bertuzzo, P. A. N. Machado and R. Z. Funchal, JHEP **1306**, 097 (2013) [arXiv:1302.0653 [hep-ph]].
- [23] E. Ma, Phys. Rev. D **70**, 031901 (2004) [hep-ph/0404199]; P. F. Harrison, D. H. Perkins and W. G. Scott, Phys. Lett. B **530**, 167 (2002) [hep-ph/0202074]; Z. -z. Xing, Phys. Lett. B **533**, 85 (2002) [hep-ph/0204049]; X. G. He and A. Zee, Phys. Lett. B **560**, 87 (2003) [hep-ph/0301092].
- [24] V. D. Barger, S. Pakvasa, T. J. Weiler and K. Whisnant, Phys. Lett. B **437**, 107 (1998) [hep-ph/9806387];
G. Altarelli and F. Feruglio, JHEP **9811**, 021 (1998) [hep-ph/9809596];
R. N. Mohapatra and S. Nussinov, Phys. Rev. D **60**, 013002 (1999) [hep-ph/9809415].
- [25] H. Fritzsch and Z. -Z. Xing, Phys. Lett. B **372**, 265 (1996) [hep-ph/9509389]; H. Fritzsch and Z. -z. Xing, Phys. Lett. B **440**, 313 (1998) [hep-ph/9808272].
- [26] D. V. Forero, M. Tortola and J. W. F. Valle, Phys. Rev. D **86**, 073012 (2012) [arXiv:1205.4018 [hep-ph]].
- [27] M. C. Gonzalez-Garcia, M. Maltoni, J. Salvado and T. Schwetz, [arXiv:1209.3023 [hep-ph]].
- [28] B. Brahmachari and A. Raychaudhuri, Phys. Rev. D **86**, 051302 (2012) [arXiv:1204.5619 [hep-ph]]; X. -G. He and A. Zee, Phys. Rev. D **84**, 053004 (2011) [arXiv:1106.4359 [hep-ph]];
B. Grinstein and M. Trott, JHEP **1209**, 005 (2012) [arXiv:1203.4410 [hep-ph]];
S. F. King, Phys. Lett. B **659**, 244 (2008) [arXiv:0710.0530 [hep-ph]];
S. Boudjemaa and S. F. King, Phys. Rev. D **79**, 033001 (2009) [arXiv:0808.2782 [hep-ph]];
D. Meloni, F. Plentinger and W. Winter, Phys. Lett. B **699**, 354 (2011) [arXiv:1012.1618 [hep-ph]];
E. Ma and D. Wegman, Phys. Rev. Lett. **107**, 061803 (2011) [arXiv:1106.4269 [hep-ph]].

S. Gupta, A. S. Joshipura and K. M. Patel, Phys. Rev. D **85**, 031903 (2012) [arXiv:1112.6113 [hep-ph]] ;
E. Ma, Phys. Lett. B **660**, 505 (2008) [arXiv:0709.0507 [hep-ph]] ;
B. Adhikary, B. Brahmachari, A. Ghosal, E. Ma and M. K. Parida, Phys. Lett. B **638**, 345 (2006)
[hep-ph/0603059] ;
E. Bertuzzo, P. A. N. Machado, R. Z. Funchal and , arXiv:1302.0653 [hep-ph] ;
J. A. Acosta, A. Aranda, M. A. Buen-Abad, A. D. Rojas and , Phys. Lett. B **718**, 1413 (2013)
[arXiv:1207.6093 [hep-ph]] .