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On the choice of mathematical functions to model damage in anisotropic soft tissues

Arthesh Basak [a,*], Rajagopal Amirtham [b]

[a] Department of Civil Engineering, GITAM Visakhapatnam, Andhra Pradesh, India
[b] Department of Civil Engineering, Indian Institute of Technology Hyderabad, Telangana, India

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ABSTRACT

Soft tissues display highly non linear behaviour under various mechanical loads. Due to various diseases, injury or when exposed to supra physiological loads, soft tissues demonstrate a softening behaviour or damage. Continuum damage models coupled with continuum material models have been successful in mathematically predicting the damage in tissues. Continuum models also incorporate anisotropy to give better predictions of the response of tissues at higher stretch values. In this work, three different mathematical decaying functions (exponential, logarithmic and stretched exponential) have been adopted to propose continuum damage models for anisotropic soft tissues. The model predictions were compared with the experimental data of uniaxial extension from literature. The stretched exponential function with the least computed error demonstrated the most accurate performance for predicting the damage in soft tissues. A general algorithm for the procedure was also remarked upon.

1. Introduction

Soft tissues can be mechanically simplified as being constituted of a ground matrix with embedded fibres [1]. It has been shown that the ground matrix behaves isotropically at lower stretches and takes up the majority of the loads [2]. The fibres get activated at higher stretches and participate in the load bearing process, meanwhile inducing non linearity and anisotropy to the mechanical repsonse of the tissue [3]. A tissue, under various circumstances may experience damage, both *in vivo* as well as *ex vivo*. Factors influencing *in vivo* damage may be attributed to diseases or certain medical interventions [4–6] or even injury from strenuous activities [7]. Damage, *ex vivo*, is usually manifested during experiments carried out to determine the material properties of the tissues like uniaxial extension, biaxial extension and shear. During such tests either under fatigue loading or supra physiological stretching, does the tissue exhibit damage.

Damage simply is the manifestation of a softening behaviour or degradation of the mechanical response in a material. A mathematical approach to model damage maybe credited to the works of [8]. Prior to **circa** 1970, damage models were predominantly phenomenological and it was only later that semi analytical and analytical models were proposed [9]. Continuum mechanics based damage models, also called continuum damage models (CDMs), have gained popularity in the last couple of decades to model the softening response of biological tissues. It is in the works of [10] that we can probably see the advent of CDMs.

To model the softening response, decaying functions are amalgamated to the strain energy density functions. In [11] an exponential function was used to model the non local damage in soft tissues. Various methods to determine the material model parameters as well as exponential damage model parameters were discussed in [12]. An exponential damage model coupled with a visco hyperelastic material model was proposed in [13]. To model the degradation of the collagen fibres as a parameter for damage, an exponential model was also proposed in [14]. To incorporate the squares of stretches in the fibre directions and also cross linking of collagen fibres, the exponential damage model of [15] was demonstrated to be highly efficient. Sigmoidal damage functions have also proven to be efficient in predicting the damage in soft tissues [16–18]. A comparative study of various exponential, sigmoidal and polynomial damage functions can be found in [19].

* Corresponding author.
E-mail address: abasak@gitam.edu (A. Basak).

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In this paper, the material model of [20] has been adopted to define the constitutive behaviour of soft tissues. The damage parameters from [21] have been adopted and implemented in exponential, stretched exponential and logarithmic decay functions to give an insight on the effect of the choice of mathematical function to model the softening behaviour of tissues at high loads.

2. Constitutive laws

2.1. Material model

Cauchy-elastic materials exhibit path independent material responses. Hyperelastic or Green-elastic materials are a special class of Cauchy-elastic materials whose constitutive laws are governed by a strain energy density function [22]. For soft tissues, the strain energy density function, Ψ, is usually considered to be composed of an isotropic function that incorporates the mechanical response of the ground matrix and an anisotropic function that incorporates the response of the collagen fibres at higher stretches [1]. An additive split can be carried out in strain energy density function as $\Psi = \left[\Psi^{vol} + \Psi^{iso} + \Psi^{aniso}\right]$ into its volumetric, isotropic and anisotropic components respectively. The strain energy function is thus expressed in [20] as

$$\Psi = \frac{\kappa}{2}[\mathscr{J} - 1]^2 + \frac{\mu}{2}\left[\bar{I}_1 - 3\right] + \frac{\omega_1}{\omega_2}\left[e^{\omega_2\left[\bar{I}_1^*/\gamma - 1\right]^2} - 1\right] \tag{1}$$

where $\mathscr{J} = \det(F_{iJ})$ is the Jacobian of the deformation gradient (F_{iJ}). The scalar parameter $\bar{I}_1 = \bar{C}_{IJ}\delta_{IJ}$ is the first invariant of the isochoric right Cauchy-Green tensor $\bar{C}_{IJ}$. The anisotropic pseudo invariant $\bar{I}_1^* = \bar{C}^{*IJ}\delta_{IJ}$ is a measure of the stretching along fibre directions, and its mathematical expression is given in (9). The isochoric right Cauchy Green tensor $\bar{C}_{IJ}$ defined in the Cartesian space of orthogonal basis $\boldsymbol{X}_I$ can be transformed to its corresponding definition $(\bar{C}^{*IJ})$ in the space defined by the oblique basis $\boldsymbol{e}^J$ as illustrated in Fig. 1 and the transformation is carried out using the fourth order fibre orientation transformation tensor β^{IJ}_{KL} such that [20]

$$\bar{C}^{*IJ} = \beta^{IJ}_{KL}\bar{C}_{KL} \tag{2}$$

The angles θ_1 and θ_2 in Fig. 1 shows the angles made by the two family of fibres with respect to the longitudinal axis of the tissue. The term γ is a parameter associated with the undeformed configuration. The parameters $\kappa, \mu, \omega_1 > 0$ are material parameters possessing the dimensions of stress and $\omega_2 > 0$ is a dimensionless constant.

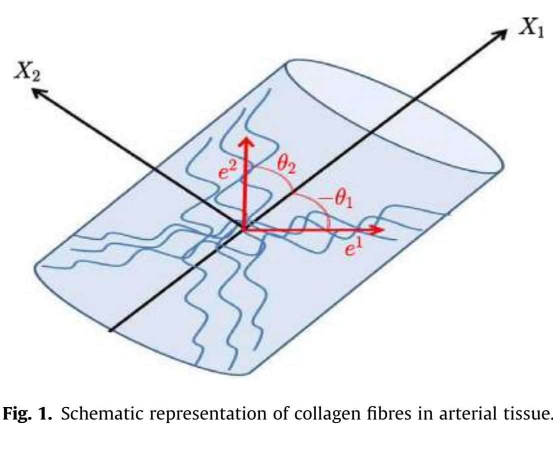

Fig. 1. Schematic representation of collagen fibres in arterial tissue.

2.2. Damage model

Damage can be parametrized using a scalar parameter $D_\imath \in [0, 1]$ as when

$$D_\imath = \begin{bmatrix} 0 & no & damage & \\ 1 & complete & mechanical & failure \end{bmatrix} \tag{3}$$

The subscript '$\imath$' can either be isotropic or anisotropic depending on the whether the damage is defined for the isotropic matrix or the anisotropic fibres. The undamaged second Piola–Kirchhoff stress $(\mathbb{S}_{IJ})$ can be determined as

$$\mathbb{S}_{IJ} = 2\frac{\partial \Psi}{\partial C_{IJ}} \tag{4}$$

Using Clausius–Duhem inequality $(-\dot{\Psi} + 0.5S_{IJ}\dot{C}_{IJ} \geqslant 0)$ the damaged second Piola–Kirchhoff stress (S_{IJ}) can be derived as

$$S_{IJ} = \left[\frac{\partial \Psi^u_{vol}}{\partial C_{IJ}} + (1 - D_{iso}) \cdot \mathbb{S}^{iso}_{IJ} + (1 - D_{aniso}) \cdot \mathbb{S}^{aniso}_{IJ}\right] \tag{5}$$

Such that

$$\mathbb{S}^{iso}_{IJ} = 2\frac{\partial \Psi^{iso}}{\partial C_{IJ}}; \mathbb{S}^{aniso}_{IJ} = 2\frac{\partial \Psi^{aniso}}{\partial C_{IJ}} \tag{6}$$

To define D_{iso} the scalar invariant called equivalent stretch (λ_{eq}) can be adopted from [17] such that

$$\lambda^4_{eq} = \bar{C}_{IJ}\bar{C}_{IJ} \tag{7}$$

An exponential decay function of the equivalent stretch can be implemented [21] such that

$$D_{iso} = 1 - e^{\left[a_{iso}\left(\lambda^d_{eq} - \lambda_{eq}\right)\right]} \tag{8}$$

where a_{iso} is a parameter that dictates the slope of the decaying function and λ^d_{eq} defines the onset of damage in the tissue matrix.

To quantify the anisotropic damage D_{aniso} the pseudo invariants of [21] can be adopted using (2) as

$$\bar{I}_1^* = \bar{C}^{*IJ}\delta_{IJ}; \; \bar{I}_2^* = 0.5\left[\left(\bar{I}_1^*\right)^2 - \bar{C}^{*IJ}\bar{C}^{*IJ}\right] \tag{9}$$

quantify the stretch along fibre directions and the shear developed between the matrix and fibres respectively, are conducive to model the anisotropic damage. These parameters are implemented in three different decaying functions to predict the softening behaviour of tissues when strained above the physiological loads.

The spatial Cauchy stress (σ_{ij}) can be determined as [23]

$$\sigma_{ij} = \mathscr{J}^{-1}F_{iI}S_{IJ}F_{jJ} \tag{10}$$

3. Decaying functions for anisotropic damage

A general form of the anisotropic damage function can be written as

$$D_{aniso} = 1 - \prod_{i=1}^{n} f^i_d(X_i) \tag{11}$$

The function $f^i_d(X_i)$ represents the damage function of the i^{th} internal variable (X_i). Since $\bar{I}_1^*$ and $\bar{I}_2^*$ are the chosen parameters to define the anisotropic damage, the function can now be written as

$$D_{aniso} = 1 - \left[f^1_d\left(\bar{I}_1^{*d} - \bar{I}_1^*\right) \cdot f^2_d\left(\bar{I}_2^{*d} - \bar{I}_2^*\right)\right] \tag{12}$$

The damage initiation parameter $\bar{I}_1^{*d}$ quantifies the onset of the softening behaviour in tissues due to loss of stiffness in the fibres. The parameter $\bar{I}_2^{*d}$, on the other hand, quantifies the initiation of damage in the fibre–matrix interface due to shear. The parameters $\bar{I}_1^{*d}, \bar{I}_2^{*d}$ needs to be judiciously extracted from the experimental data.

The damage functions $f_d^i(X_i)$ are implemented as exponential, logarithmic and stretched exponential decaying functions and the model predictions are plotted against the experimental data of muscular vaginal tissue from [24].

3.1. Exponential decaying function

Exponentially decaying functions, of the chosen internal variables, maybe adopted of the form

$$D_{aniso} = 1 - \left[e^{a_1\left(\bar{I}_1^{*d} - \bar{I}_1^*\right)} \cdot e^{a_2\left(\bar{I}_2^{*d} - \bar{I}_2^*\right)} \right] \tag{13}$$

The parameters $a_1, a_2 \geqslant 0$ can be obtained by calibrating the model to the experimental data through a curve fitting procedure. The values obtained after fitting the model to the experimental data of uniaxial extension [24] are mentioned in Table 1.

3.2. Logarithmic decaying function

Product of logarithmic decaying functions can be adopted for the anisotropic damage such that

$$D_{aniso} = 1 - \left[1 + a_1 \log_e\left(\bar{I}_1^*/\bar{I}_1^{*d}\right)\right]^{-1} \cdot \left[1 + a_2 \log_e\left(\bar{I}_2^*/\bar{I}_2^{*d}\right)\right]^{-1} \tag{14}$$

The parameters $a_1, a_2 \geqslant 0$, obtained after curve fit with the experimental data [24] are tabulated in Table 1.

3.3. Stretched exponential decaying function

A stretched exponential function is an exponential function of the n^{th} power of the dependent variable. Assuming a cubic function of the internal parameters, the anisotropic damage parameter can be expressed as

$$D_{aniso} = 1 - \left[e^{a_1\left(\bar{I}_1^{*d} - \bar{I}_1^*\right)^3} \cdot e^{a_2\left(\bar{I}_2^{*d} - \bar{I}_2^*\right)^3} \right] \tag{15}$$

The parameters $a_1, a_2 \geqslant 0$ too are tabulated in Table 1 after curve fit with the experimental data [24].

4. Goodness of fit and error analysis

The goodness of fit was measured using the R^2 value which is a measure of the correlation between the experimental stress and the stress predicted by the model. The $R^2 \in [0, 1]$ implies a good fit if the value lies closer to unity.

Error in the model prediction was quantified using χ^2 value such that

$$\chi^2 = \sum_{j=1}^{p} \frac{\left(\sigma_{expt} - \sigma_{model}\right)^2}{\sigma_{expt}} \tag{16}$$

where p is the number of data points. σ_{expt} is the value of the Cauchy stress obtained experimentally and σ_{model} is the Cauchy stress predicted by the model.

The values of R^2 and χ^2 obtained against each of the exponential, logarithmic and stretched exponential damage functions have been mentioned in Table 1.

Table 1
Damage model parameters with associated R^2 and χ^2 values.

Mathematical Function	Damage Model Parameters	R^2	χ^2
Exponential	$a_1 = 0.85$ $a_2 = 0.2$	0.953	35.97
Logarithmic	$a_1 = 9.8$ $a_2 = 0.3$	0.881	42.65
Stretched Exponential	$a_1 = 0.102$ $a_2 = 2.29$	0.989	31.75

5. General Algorithm

1. Define material model, damage model parameters and damage onset points $\bar{I}_1^{*d}, \bar{I}_2^{*d}, \lambda_{eq}^d$.
2. Intialize load step.
   (a) Calculate deformation gradient $\boldsymbol{F}$
   (b) Calculate internal variables $\lambda_{eq}, \bar{I}_1^*, \bar{I}_2^*$ from (7) and (9).
   (c) λ_{eq}=max$\left(\lambda_{eq}^d, \lambda_{eq}\right)$ $\bar{I}_1^*$=max$\left(\bar{I}_1^{*^d}, \bar{I}_1^*\right)$ $\bar{I}_2^*$=max$\left(\bar{I}_2^{*^d}, \bar{I}_2^*\right)$
   (d) Calculate exponential damage parameter from (13)
   (e) Calculate logarithmic damage parameter from (14)
   (f) Calculate stretched exponential damage parameter from (15)
   (g) Calculate damaged second Piola–Kirchhoff stress from (5)
   (h) Calculate damaged Cauchy stress from (10)
3. Repeat steps a-h till final load step

6. Results and conclusion

Fig. 2 illustrates the evolution of the anisotropic damage parameter D_{aniso} for each of the mathematical functions assumed i.e. exponential, logarithmic and stretched exponential. A cubic power was taken for the stretched exponential function in (15). Taking higher order function of the internal variable results in large values that gives rise to erroneous output. The order needs to be an odd number for a decaying function. The damage in the fibres were assumed to initiate at an uniaxial stretch $\lambda \approx 1.4$ for degradation of the fibre–matrix interface due to shear and at $\lambda \approx 1.78$ for loss of fibre stiffness. These values were assumed by observing the trend of the experimental data and was considered the same for all the damage functions.

From Fig. 2 it can be conceived that both the exponential and logarithmic damage functions of (13) and (14) respectively, behave very similar under lower strains as well as higher strains and display a concave behaviour. The stretched exponential function of (15) on the other hand displays a convex behaviour. It is also to be noted from Fig. 2 that the logarithmic function evolves with almost a constant slope at higher strain values.

In Fig. 3 the prediction of the various damage models when coupled with the material model of (1) has been illustrated. It can be seen that all the damage models demonstrate good performance in the lower stretch regime. At higher stretches the performance of the logarithmic model, post fibre damage, was the most inaccurate. The exponential function on the other hand demonstrated a consistent performance but was erroneous in the post failure softening regime. As can be seen from the experimental data that the post failure response is generally concave in nature and the function that accurately fits the contour is the stretched exponential function. The R^2 values in Table 1 also show the highest value for the stretched exponential function whereas the logarithmic function had the least value. A similar trend was also observed for the χ^2 error. With $R^2 \approx 1$ and the lowest χ^2 error, the stretched exponential function thus demonstrated the best performance in predicting damage of anisotropic soft tissues.

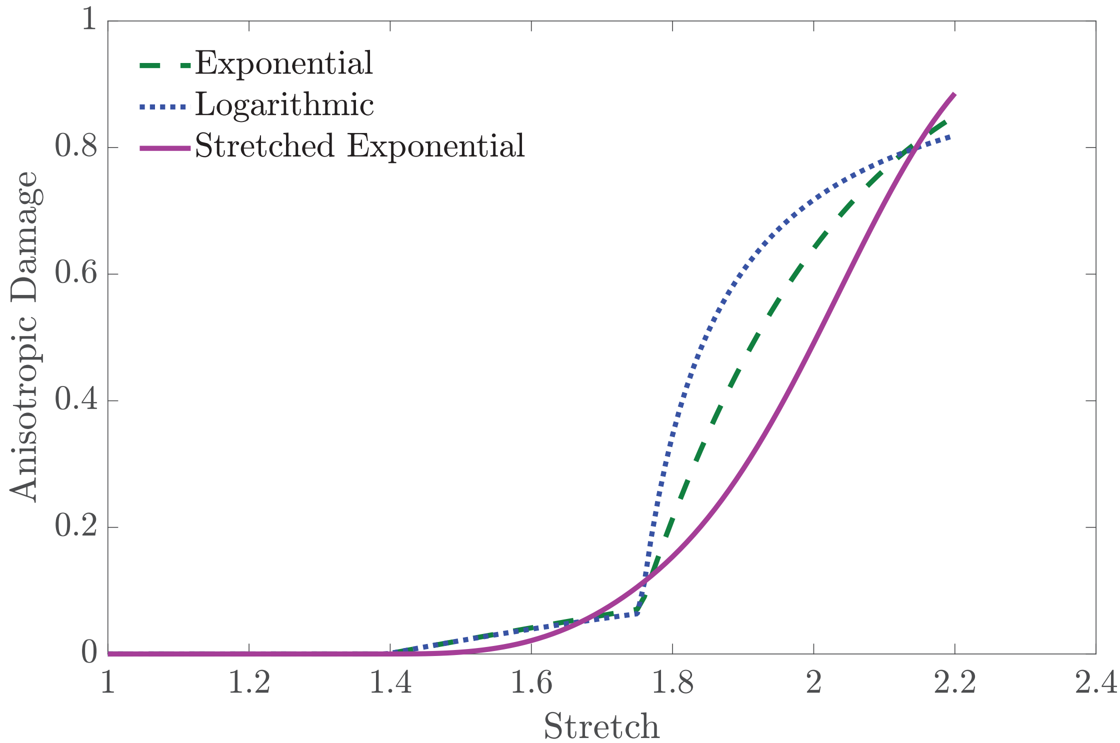

Fig. 2. Evolution of anisotopic damage parameter D_{aniso} for various mathematical functions. The smooth line represents the evolution for a stretched exponential function. The dashed and dotted lines correspond to exponential and logarithmic functions respectively.

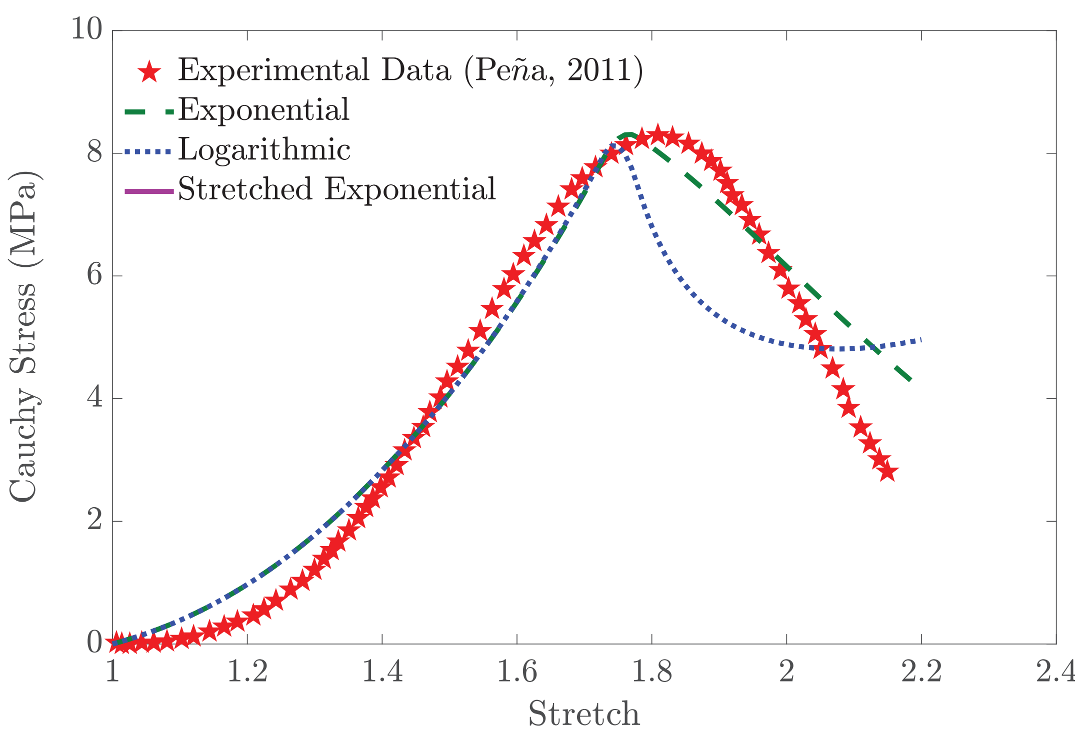

Fig. 3. Prediction of damage for various mathematical decaying functions. The smooth line represents the prediction of the stretched exponential model whereas the dotted and dashed lines represent the predictions of logarithmic and exponential functions respectively. The stars represent the experimental data for uniaxial extension of muscular vaginal tissue from .[24].

CRediT authorship contribution statement

Arthesh Basak: Conceptualization, Methodology, Writing – original draft. **Amirtham Rajagopal:** Conceptualization, Supervision.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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