Low complexity VLSI Architecture Design methodology for Wigner Ville Distribution

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Abstract—In this paper, we propose a low complexity VLSI architecture design Methodology for Wigner Ville Distribution (WVD) computation. The proposed methodology performs both auto and cross WVD computations using only the half number of Fast Fourier transform (FFT) computations as opposed to the state of the art methodologies. The FPGA implementation for proposed methodology was performed for 16 bit fixed point and 32 bit single precision floating point numbers on the Xilinx Virtex-7 FPGA (XC7vx485tffg). The proposed methodology saves 49% energy consumption when compared with the state of the art methodology. However it can be noted that the proposed methodology is independent of the VLSI implementation platform and technology node.

Index Terms—Wigner Ville Distribution (WVD), VLSI Architecture, Fast Fourier transform (FFT), Auto WVD, Cross WVD. I. INTRODUCTION

 $\sum_{\text{used in different practical signal processing applications}} \text{MME FREQUENCY signal analysis tools have been used in different practical signal processing applications.}$ IME FREQUENCY signal analysis tools have been where the signals are non-stationary [1]-[4]. Wigner Ville Distribution (WVD) is one of the most fundamental tools in quadratic Time Frequency distributions which offers high Time-Frequency (TF) resolution among TF signal analysis tools [5]-[17]. WVD suffers with cross frequency terms which can be removed by applying the approaches proposed in [18]- [22]. However, WVD has been used in the computationally intensive source separation algorithms as for instance Underdetermined Blind Source Separation (UBSS)[5]-[17] where Spatial Time Frequency Distribution matrix generator is one of the most important modules comprising of both Auto WVD (AWVD) and cross WVD (XWVD) of input mixture signals [23]-[24]. Such systems have been found to be useful in the resource constrained applications including remote health monitoring where the device has to be portable and backed up by the limited battery power. In such applications, energy efficiency and low-complexity holistic algorithm-architecture design methodology would play a significant role. Hence, a low-complexity architecture design of AWVD and XWVD would help accomplishing such efficient system designs which is the objective of this paper.

The WVD is non-casual and computationally intensive in nature. The computation of the WVD requires the analytical signal computation [25]-[26], kernel computation and Fast Fourier Transform (FFT) computation [27]. In AWVD computation, the kernel is computed using instantaneous auto correlation of the input signal whereas in XWVD computation, the kernel is computed using instantaneous cross correlation of the input signals. An efficient methodology was reported in [27] for AWVD computation by exploiting the conjugate symmetry property possessed by AWVD kernel resulting reduction of number of FFT computations by half. Another methodology

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was reported in [28] for AWVD computation which was faster than [27] for signal length shorter than 1024 samples and FFT length less than 256 points. On the other hand, the XWVD is implemented in [29], [30] using kernel computation followed by FFT computation. However, the XWVD kernel does not have such conjugate symmetry property as that of the AWVD kernel. Hence methodologies reported in [27]-[28] cannot be applicable here for the implementation of the XWVD. The existing XWVD design methodologies as reported in [29], [30] require more number of FFT computations than that is required in AWVD computation as reported in [27], [28]. The state of the art methodologies for WVD computation can be implemented using kernel computation followed by either single FFT or parallel FFTs [27]-[30]. For example, the single FFT implementation for N point AWVD computation [27], [28], the FFT should be reused $\frac{N}{2}$ number of times whereas in parallel FFT implementation requires $\frac{N}{2}$ parallel FFT computations. Similarly for N point XWVD computation [29], [30], the single FFT implementation requires to reuse the FFT N number of times whereas in parallel FFT implementation requires N parallel FFT computations.

Motivated by the fore mentioned arguments and applications, in this paper, we introduce a low complexity design methodology of WVD to minimize the need of significant number of FFT computations resulting reduction of hardware complexity, energy consumption and also resulting the computational speed up of the WVD computation. The highlights of this paper are:

- Exploration of partial conjugate symmetry in the XWVD kernel. The partial conjugate symmetry in XWVD computation saves half the number of FFT computations.
- The proposed methodology works for both AWVD and XWVD computations unlike the state of the art methodologies [28], [30].
- An extensive vector matching is performed to validate the proposed methodology using MATLAB simulation.
- The architecture has been coded in VHDL and FPGA prototyped on the Xilinx Virtex-7 FPGA (XC7vx485tffg). The proposed methodology saves 49% energy consumption when compared with the state of the art methodologies [28], [30].

II. PROPOSED METHODOLOGY

The WVD of any two square integral signals $x_i(t)$ and $x_j(t)$ is defined [27] as

$$
\rho_{z_iz_j}(t,f) = \int_{-\infty}^{\infty} z_i^*(t - \frac{\tau}{2}) z_j(t + \frac{\tau}{2}) e^{-j2\pi f\tau} d\tau \tag{1}
$$

where $z_i(t)$ and $z_j(t)$ are the analytic associates of $x_i(t)$ and $x_i(t)$. The analytic associate of $x_i(t)$ is defined as $z_i(t) = x_i(t) + jH\{x_i(t)\}\text{, where } H\{x_i(t)\}\text{ represents }$ Hilbert transform of $x_i(t)$. In (1), if $i = j$ then the WVD is called AWVD, otherwise for $i \neq j$, it is called as XWVD. In practical applications XWVD could be intercepted, if $z_i(t)$ is This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TCSII.2020.2992514, IEEE Transactions on Circuits and Systems II: Express Briefs

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a reference signal and $z_i(t)$ is the signal under analysis. For example, in radar, sonar, seismic and acoustic applications, transmitted signal is the reference signal and received echo is the signal under analysis. In other applications like signal characteristic analysis where reference signal is not available, the AWVD is used for spectral analysis [12]-[14]. The discrete equivalent of (1) is called Discrete time WVD (DWVD) and it can be expressed as

$$
\rho_{z_iz_j}(n,f) = 2 \sum_{m=-\infty}^{m=\infty} z_i^*(n-m)z_j(n+m)e^{-j2\Pi fm}
$$
 (2)

The above DWVD formula is not feasible for real time implementation due to infinite duration in length. To overcome these problems, windowing technique can be applied on the input signals by multiplying with time window function. The windowed DWVD is called as Pseudo DWVD (PDWVD) and expressed as $2\sum_{m=-M}^{m=M} z_i^*(n-m)z_j(n+m)$ $(m)e^{-j2\Pi fm} \cdot w^*(-m)w(m)e^{-j2\Pi fm}$, where $w(m)$ is a window function, of length M , symmetric and real. The DWVD kernel sequence at $n^{\bar{t}h}$ instant is defined as

$$
k_{z_iz_j}^n(m) = z_i^*(n-m)z_j(n+m)w^*(-m)w(m) \tag{3}
$$

Still, the PDWVD formula in (2) is not feasible for implementation because the frequency variable f is a continuous variable. After sampling the frequency variable f for length N , (2) can be rewritten as

$$
\rho_{z_iz_j}(n,l) = 2 \sum_{m=-M}^{m=M} k_{z_iz_j}^n(m) e^{-j2\Pi l \Delta f m}
$$
 (4)

where $\Delta f = \frac{1}{N} = \frac{1}{2M+1}$ [27]. Now the PDWVD formula shown in (4) is feasible for implementation. The PDWVD shown in (4) can be performed using using FFT. The FFT requires the input sequence indexed from 0 to $N - 1$ but (4) has the input sequence from $-M$ to M. The positive indexing can be achieved by making a periodic extension of the kernel sequence considering $N = 2\tilde{M} + 1$ [27]. The extended kernel sequence is expressed in following equation [27]

$$
k_{z_iz_j}^n(m) = \begin{cases} k_{z_iz_j}^n & 0 \le m \le \frac{N}{2} - 1\\ k_{z_iz_j}^{n^*}(m - N) & \frac{N}{2} + 1 \le m \le N - 1\\ 0 & m = \frac{N}{2} \end{cases}
$$
 (5)

The PWVD computation shown in (4) is similar to the FFT computation. In general, the FFT computations are optimized based on symmetry properties of the input signal. Hence, in this proposed methodology, the number of FFT computations can be optimized based on the symmetry properties of the kernel sequence $k_{z_iz_j}^n(m)$. First, we investigate the properties of AWVD kernel sequence when $i = j$. The PDWVD kernel shown in (5) possess conjugate symmetry property when $i = j$

$$
k_{z_iz_i}^n(m) = k_{z_iz_i}^{n^*}(-m)
$$
 (6)

For conjugate symmetry kernel, the FFT will be real. Let the PDWVD kernel sequences at two successive instances n and $n + 1$ are combined as follows

$$
k_{z_iz_i}^{comb}(m) = k_{z_iz_i}^n(m) + j * k_{z_iz_i}^{n+1}(m)
$$
 (7)

Then FFT of combined kernel sequence can be computed using following equation

$$
K_{z_iz_i}^{comb}(l) = FFT(k_{z_iz_i}^{comb}(m))
$$
\n(8)

Now the individual FFT of kernel sequences $K_{z_iz_i}^n(l)$ and $K_{z_iz_i}^{n+1}(l)$ can be computed using single FFT as follows

$$
\rho_{z_iz_i}(n,l) = K_{z_iz_i}^n(l) = real(K_{z_iz_i}^{comb}(l))
$$
\n(9)

$$
\rho_{z_iz_i}(n+1,l) = K_{z_iz_i}^{n+1}(l) = imag(K_{z_iz_i}^{comb}(l))
$$
\n(10)

From (9) and (10), it is important to note that by combining successive pairs of kernel sequences $k_{z_iz_i}^n(m)$ and $k_{z_iz_i}^{n+1}(m)$, the number of FFT computations will be halved in AWVD computation.

Now we will investigate the properties of PDWVD kernel sequence when $i \neq j$ for XWVD computation. The $k_{z_iz_j}^n(m)$ cannot possess conjugate symmetry property as shown in (6). Hence, we define a virtual kernel sequence $p(m)$ having length N using $k_{z_iz_j}^n(m)$. The $k_{z_iz_j}^n(m)$ and $p(m)$ have the following relation.

$$
p(m) = \begin{cases} k_{z_iz_j}^n(m) & 1 \le m \le N - 1\\ real(k_{z_iz_j}^n(0)) & m = 0 \end{cases}
$$
 (11)

From (5), it is important to note that the $k_{z_iz_j}^n(m)$ possess conjugate symmetry property except at $m = 0$ when $i \neq j$. Therefore, the virtual kernel $p(m)$ possess conjugate symmetry property i.e, $p(m) = p^*(-m)$.

The FFT of $\hat{k}_{z_iz_j}^n(m)$ can be computed as follows:

$$
K_{z_iz_j}^n(l) = \sum_{m=0}^{m=N-1} k_{z_iz_j}^n(m) e^{-j2\Pi l m}
$$
 (12)

By substituting (5) in (12) and expanding (12), we get:

$$
K_{z_iz_j}^n(l) = k_{z_iz_j}^n(0) + \sum_{m=0}^{m=\frac{N}{2}-1} k_{z_iz_j}^n(m)e^{-j2\Pi l m} + \sum_{m=N-1}^{m=N-1} k_{z_iz_j}^n(m)e^{-j2\Pi l m}
$$
\n(13)

The $k_{z_iz_j}^n(m)$ satisfies the conjugate symmetry as shown in (6) except at $m = 0$. Then (13) can be rewritten as follows:

$$
K_{z_iz_j}^n(l) = k_{z_iz_j}^n(0) + \sum_{m=0}^{m=\frac{N}{2}-1} k_{z_iz_j}^n(m)e^{-j2\Pi l m} + \sum_{m=\frac{N}{2}-1}^{m=\frac{N}{2}-1} k_{z_iz_j}^{n^*}(m)e^{j2\Pi l m}
$$
\n(14)

From (14), the real and imaginary parts of $K_{z_1z_j}^n(l)$ can be computed as follows:

real
$$
(K_{z_iz_j}^n(l)) = real(k_{z_iz_j}^n(0)) + \sum_{m=0}^{m=\frac{N}{2}-1} k_{z_iz_j}^n(m)e^{-j2\Pi l m}
$$

 $+ \sum_{m=0}^{m=\frac{N}{2}-1} k_{z_iz_j}^{n^*}(m)e^{j2\Pi l m}$
(15)

$$
imag(K_{z_iz_j}^n(l)) = imag(k_{z_iz_j}^n(o))
$$
\n(16)

The FFT of $p(m)$ as computed as follows:

$$
P(l) = \sum_{m=0}^{m=N-1} p(m)e^{-j2\Pi l m}
$$
 (17)

By substituting (11) in (17), we get:

$$
P(l) = real(k_{z_iz_j}^n(0)) + \sum_{m=0}^{m=\frac{N}{2}-1} k_{z_iz_j}^n(m)e^{-j2\Pi lm} + \sum_{m=0}^{m=\frac{N}{2}-1} k_{z_iz_j}^{n^*}(m)e^{j2\Pi lm}
$$
\n(18)

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From (15), (16) and (18) the relation between $K_{z_iz_j}^n(l)$ and $P(l)$ can be defined as follows

$$
real(K_{z_iz_j}^n(l)) = P(l)
$$

$$
imag(K_{z_iz_j}^n(l)) = imag(k_{z_iz_j}^n(0))
$$
 (19)

Consider the XWVD kernel sequences at two successive instances n and $n+1$ which are $k_{z_iz_j}^n(m)$ and $k_{z_iz_j}^{n+1}(m)$ respectively. From (11), their virtual kernel sequences $p^h(m)$ and $p^{n+1}(m)$ can be expressed as follows:

$$
p^{n}(m) = \begin{cases} k_{z_{i}z_{j}}^{n}(m) & 1 \leq m \leq N-1\\ real(k_{z_{i}z_{j}}^{n}(0)) & m = 0 \end{cases}
$$
 (20)

$$
p^{n+1}(m) = \begin{cases} k_{z_iz_j}^{n+1}(m) & 1 \le m \le N-1\\ real(k_{z_iz_j}^{n+1}(0)) & m = 0 \end{cases}
$$
 (21)

The XWVD kernel $k_{z_iz_j}^n(m)$ possess conjugate symmetry except at $m = 0$. Hence the defined virtual kernels possess conjugate symmetry property as shown in (6). The $p^{n}(m)$ and $p^{n+1}(m)$ can be combined as follows:

$$
p^{comb}(m) = p^n(m) + j * p^{n+1}(m)
$$
 (22)

Then the FFT of combined kernel sequence can be computed using following equation:

$$
P^{comb}(l) = FFT(p^{comb}(m)) = P^n(l) + j * P^{n+1}(l)
$$
 (23)

Now using (19) and (23) the individual FFT of XWVD kernel sequences $\widehat{K}_{z_iz_j}^n(l)$ and $K_{z_iz_j}^{n+1}(l)$ can be computed as follows:

$$
\rho_{z_iz_j}(n,l) = K_{z_iz_j}^n(l) = P^n(l) + j * imag(K_{z_iz_j}^n(l))
$$

= $real(P^{comb}(l)) + j * imag(k_{z_iz_j}^n(0))$ (24)

$$
\rho_{z_iz_j}(n,l) = K_{z_iz_j}^{n+1}(l) = P^{n+1}(l) + j * imag(K_{z_iz_j}^{n+1}(l))
$$

= $imag(P^{comb}(l)) + j * imag(k_{z_iz_j}^{n+1}(0))$ (25)

From (22) , (23) , (24) and (25) , it can be noted that by combining successive pairs of virtual kernel sequences $p^{n}(m)$ and $p^{n+1}(m)$, the number of FFT computations will be halved in XWVD computation.

Subsequently an architecture has been designed for both AWVD and XWVD computation based on (20)-(25) is depicted in Fig.1(a). Here the analytical signals are computed using re-configurable DHT presented in [25]. The functionality of architecture is explained with the help of pseudo code as shown in Algorithm1 (Please see the supplementary material). Two port memories are used in this design for storing the input signals $z_i(n)$ and $z_j(n)$. The controller module consists of counters to generate the memory indices $m, m + n$, $m - n$ as shown in pseudo code (Algorithm1) (Please see the supplementary material). This module also generates the read write signals for memories, FIFO and control signals for other sub modules based on the proposed methodology. The kernel sequence $k_{z_iz_j}^n(m)$ is computed using (3) at n and $n+1$ instants. The window coefficients are precomputed and stored in the window coefficient memory. The stored coefficients are fetched and multiplied as shown in (3). By using (20) and (21), the corresponding virtual sequence $p^{n}(m)$ and $p^{n+1}(m)$ are computed. In this virtual sequence computation, the imaginary parts of $k_{z_iz_j}^n(m)$ and $k_{z_iz_j}^{n+1}(m)$ at $m = 0$ are stored in a buffer. If $i = j$, these imaginary parts will become 0. The imaginary part selection module is designed to select the

imaginary part of WVD as shown in (24) and (25). The virtual kernel sequence $p^{n}(m)$ and $p^{n+1}(m)$ are combined using (22). Combining of the virtual kernel sequences eliminates the additional computation of FFT in the WVD design.

III. EXPERIMENTAL RESULTS AND DISCUSSION

A. Verification of the proposed methodology

In this subsection, before implementing the proposed methodology on hardware, we modeled in MATLAB for double precision floating point numbers to check the functionality and simulate the absolute errors. The functionality of the AWVD and XWVD is cross verified with "WVD function" and "XWVD function" in Time Frequency Tool Box (TFTB) of MATLAB [31]. The MATLAB model is simulated by supplying the non-stationary input signals such as chirp signals, ECG and EEG signals to validate the accuracy of the proposed hardware design in comparison with the software version of the WVD. For example, the AWVD computation of a linear chirp signal of bandwidth 2 MHz and 100 microseconds in length is shown in Fig.S1(a) (Please see the supplementary material). The XWVD computation between the chirp signal and Gaussian signal of bandwidth 1.4 MHz and 100 microsecond in length is shown in Fig.S1(d) (Please see the supplementary material). The AWVD and XWVD using TFTB are shown in Fig.S1(b) and Fig.S1(e) respectively (Please see the supplementary material). The "WVD function" and "XWVD function" in TFTB are implemented using the state of the art methodologies[27], [29], [30]. The absolute error between the proposed method and state of the art [27], [29], [30] is zero. Therefore, the proposed methodology does not have any negative impact on the functionality but optimizes the requirement of number of FFT computations. Similarly, the MATLAB model for the proposed methodology is tested for different input lengths and for different input signals such as non-linear chirp, ECG and EEG signal shown in Fig.S2 (ECG signal for 120 seconds, EEG signal for 100 seconds linear Chirp and logarithmic chirp signals for duration of 200 micro seconds and band width of 4 MHz) (Please see the supplementary material).

B. FPGA Implementation of the proposed methodology The proposed methodology is independent of either single

FFT or parallel FFTs implementation and saves half number of FFT computations. Therefore the proposed methodology can be implemented using either single FFT or parallel FFTs. In this paper, the proposed architecture, state of the art architectures [27], [28] and [30] are implemented using kernel computation followed by single FFT structure as shown in Fig.1(a) to compare the design performance on a uniform platform. The proposed architecture and state of the art architectures [28]-[30] are coded in VHDL for signal length $T = 1024$, FFT length $N = 512$ (minimum frequency resolution required is $\frac{T}{2} = 512$). The proposed methodology is independent of technology node, fixed or floating point implementation and saves the half number of FFT computations when compared with the state of the art methodology. However, to provide more insight to the reader the proposed methodology is implemented for both fixed point (FxP) and floating point (FP). The FPGA prototype for the proposed architecture has been done on VC707 evaluation platform which is having the

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Fig. 1. (a) WVD Architecture for AWVD and XWVD computation based on the proposed methodology (b) Experimental setup for verification of the proposed methodology using Xilinx Virtex-7 FPGA (XC7vx485tffg) and FMC 144ADC/DAC card

Xilinx Virtex-7 FPGA (XC7vx485tffg). The implementation results are shown in Table I. The FPGA implementation for proposed methodology is performed for 16 bit fixed point and 32 bit single precision floating point.

As shown in the Table I, the proposed method requires 0.904% higher LUT requirement than [28] but the methodology in [28] performs only AWVD computation whereas proposed methodology performs both AWVD and XWVD computations. On the other hand, the proposed methodology requires 0.904% higher LUT requirement than [30]. However, it is found that the proposed methodology saves 50% FFT computations resulting 49% of reduction in the energy consumption. It can be noted that if the proposed methodology and state of the architectures are implemented using kernel computation followed by parallel FFT structure, the state of the methodology requires more FPGA resources than proposed methodology. In this example, the proposed methodology requires the resources for 512 FFTs whereas state of the art methodology requires the resources for 1024 FFTs. The parallel FFTs structure implementation requires more area, energy consumption and faster than the single FFT implementation. In order to provide more insight on the FPGA implementation for different signal lengths, the proposed methodology is implemented for signal length $T = 2048$, FFT lengths $N = 1024$ and $N = 2048$. The synthesis results are tabulated in Table SI. As signal dimension increases, the resource requirement will increases however the proposed methodology saves the half number of FFT computations irrespective of signal length and FFT length.

The Hardware implementation of the proposed methodology is tested using the experimental setup shown in Fig.1(b). The SMW200A is a dual channel vector signal generator which consists of arbitrary waveform memory which provides an option to generate any waveform. For example, the chirp signals, ECG and EEG signals generated in MATLAB simulation section III-A can be stored and playback. The SMW200A is stored with chirp signals for pulsed of 100 microseconds. The SMW200A is connected to VC707 board through FMC144 ADC/DAC quad channel daughter card. THE FMC 144 is tuned to 10 MHz sampling clock using FMC analyzer software. The VHDL reference design of FMC144 is integrated with proposed VHDL design and generated a bit file. The functionality is verified using chip scope.

Accuracy of the proposed design is determined by comparing outputs from FPGA with modeled function in MATLAB. The HDL outputs for the proposed AWVD and XWVD are shown in Fig.S1(c) and Fig.S1(f) respectively for the same test bench used in MATLAB simulation. The signature of spectrogram in the MATLAB simulation and VHDL simulations is same for the respective AWVD and XWVD computations. However, to provide insight into the accuracy of the proposed architecture, Mean Square Error (MSE) and signal to noise ratio (SNR) are computed using the approach in [32]-[33]. The SNR and MSE variations with respect to word length is shown in Fig.2. From Fig.2, the observed MSE is 1.2×10^{-12} and SNR is $120dB$ for both proposed and state of the art AWVD and XWVD computations. The MSE and SNR for the proposed and state of the art architecture are same due to both followed similar methodology but the proposed method saves 50% FFT computations.

TABLE I PERFORMANCE OF PROPOSED ALGORITHM ON FPGA

Parameter	Proposed	AWVD	AWVD	XWVD	Proposed
		$[27]$	[28]	$[30]$	
AWVD /	AWVD	AWVD	AWVD	AWVD&	AWVD&
XWVD	&			XWVD	XWVD
	XWVD				
Word	FxP 16	Fxp 16	FxP 16	FxP 16	FP ₃₂
length	bit	bit	bit	bit	bit
Number of	512	512	512	1024	$\overline{512}$
FFT com-					
putations					
(F)					
Maximum	$\overline{280MHz}$				172MHz
clock					
frequency					
(f)					
Energy	68.2423	68.2423	67.9954	135.99	330.13
consump-					
tion					
(nano)					
Joule)					
Slice	7521	7453	7453	7453	34740
LUTs					
DSP slices	$\overline{35}$	$\overline{35}$	$\overline{35}$	$\overline{35}$	83
18k	$\overline{12}$	12	$\overline{12}$	12	$\overline{25}$
BRams					
36k	2	$\overline{2}$	$\overline{2}$	$\overline{2}$	$\overline{6}$
BRams					
SNR	120	119.89	120	120.1	123.25

The timing of the proposed methodology is analyzed in terms of the latency and execution time in the supplementary material. For example, for $T = 1024$, the time required for kernel computation (T_{kernel}) is 1024 clock cycles. From (S3), the latency of the proposed methodology is 1024 clock cycles more than the state of the art methodology [30]. However, from

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TCSII.2020.2992514, IEEE Transactions on Circuits and Systems II: Express Briefs

(S6), $ETS = 512 * T_{FFT}$ so that the proposed methodology saves the ET of 512 number of FFTs when compared with the state of the art methodology [30].

Fig. 2. SNR and MSE vs Word length for the proposed methodology IV. CONCLUSION

In this paper, we proposed an architecture design methodology for low complex Wigner Ville Distribution (WVD) computation which performs both AWVD and XWVD and saves 50% FFT computations when compared with the state of the art methodologies. It also significantly reduces the energy consumption by approximately 49%. The architecture based on the proposed methodology has been designed and implemented on FPGA (Xilinx Virtex-7 FPGA (XC7vx485tffg)) platform and performance has been evaluated in terms of area, energy consumption and accuracy of the algorithm. The proposed methodology presents the details for implementation of basic WVD which suffers with cross frequency terms. The approaches introduced in [18]-[22] require the WVD, the proposed methodology can be used for the implementation of the WVD and the approaches [18]-[22] can be applied to reduce the cross frequency terms.

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