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Abstract

State estimation plays an important role in real time security monitoring and control of power systems. There are many problems in the implementation of state estimator for large scale networks due to measurement errors, weights given and the numerical ill-conditioning associated with the solution techniques. In this paper a new formulation using linear programming approach is presented. The formulation is devoid of weights and errors associated with the measurements are taken care of in constraints. The non linear problem is linearized at previous operating state and constraints are set up using flow mismatches. The implementation of the formulation exploits sparse features of the network matrices and avoids matrix inversions. Upper bound optimization technique is employed to solve the linear programming problem. Illustration of the proposed approach on sample 3-bus and 6-bus systems and a practical Indian Southern grid 72 bus equivalent system are presented.

KEYWORDS: Linear Programming, LP, Normal Equations, NE, Power System State Estimation, PSSE, upper bound optimization, Weighted Least Squares, WLS

1 Introduction

State estimation is an essential precursor to all network security applications in modern Energy Control Centers (ECC). Implementation of online security-oriented functions in Energy Management System (EMS) requires prior information of system operating states provided by the state estimator.

Although different versions of state estimators are widely adopted in the EMS, difficulties are being reported for its effective implementation to practical systems. In analyzing the causes of the 2003 blackout [1] the NERC outage task force final report indicates the ineffectiveness of one of the EMS state estimators to assess system conditions for most of the outage period.

Development of efficient Power System State Estimator (PSSE) for handling large power systems has been a topic of research since 1970's. Tinney *et al.* [2, 3] presents a methodology for processing measured data from electric power networks in order to obtain the best possible estimates of the system variables. This approach is popularly known as the Normal Equations (NE) approach or Weighted Least Squares (WLS) approach. Problems show up in implementation of WLS approach for large scale systems. Monticelli *et al.* [4] describes the ill conditioning associated with NE approach when dealing with networks having large number of injection measurements, networks with long line connected to short line, assigning weights to virtual measurements etc. It also suggests solving of normal equations by orthogonal transforms to overcome the problems associated with the ill conditioning arising in gain matrix.

Various methods have been suggested to overcome the numerical ill conditioning problem [5, 6, 7, 8, 9]. Holten *et. al* [10] compares various methods for state estimation in terms of numerical stability, computational efficiency and implementation complexity. Monticelli [11] discusses the state of art of electric power system state estimation along with alternative formulations for solving Power System State Estimation (PSSE) .

State estimation problem formulation suitable for employing Linear Programming (LP) approach has been attempted in the past. Irving *et al.* [12] were the first to propose the Weighted Least Absolute Value (WLAV) approach for PSSE. Although different approximations [13, 14] were considered to improve computational efficiency, the objective function still dependent on measurement weights, which is one of the major causes of ill-conditioning. Clewer *et al.* [15] proposed an variant of LP approach for estimating the voltage magnitudes and phase angles utilizing four sets of linear equations.

The primary objective of this paper is to develop a new formulation for PSSE that is free from ill-conditioning caused due to weightages. The objective function of the proposed approach is devoid of weights but the errors associated

with the measurements are taken care of in constraints. As decoupled technique is employed, the voltage magnitudes and angles are obtained by solving sequentially two sets of LP problem as against four in method proposed by Clewer *et al.*. Also the proposed approach tries to make use of the previous state information for effective implementation.

The paper is organized as follows. Sections 2 and 3 describes the proposed approach and issues regarding implementation. Network modeling, actual algorithm and computational requirements are described in Sections 4 and 5. Section 6 gives the illustrative results of the proposed approach tested on sample 3 bus, 6 bus system and also a practical 72 bus equivalent 400/230/220kV Indian southern grid system. Section 7 discusses the advantages associated with the proposed approach and concludes the discussion.

2 Mathematical formulation for the proposed approach

The general estimation consists of estimating the state vector x based on a set of measurements z in the presence of an error ε . A mathematical model describing the functional relations between z , x and ε is described by (1). This model is expressed in the form of a set of non-linear equations which relates the measurements z and the true state vector x .

$$z_j = f_j(x) + \varepsilon_j \quad (1)$$

where f_j is a nonlinear scalar function relating the j^{th} measurement to states and ε_j is the measurement error.

The initial objective for performing state estimation is to determine the state x such that the difference between the measured value and the estimated value is minimum. i.e.

$$J(x) = \sum_{i=1}^{n_m} (z_i - f_i(x))^2 \quad (2)$$

Where, n_s =number of states or unknowns, n_m =number of measurements.

In this approach, we start with an initial value of x and determine correction Δx so that $J(x)$ is minimized. The correction Δx can be obtained by solving the linear optimization problem given by (3).

$$\begin{aligned}
 & \min \quad v_s(x) = c^T [\Delta x] \\
 & \text{sub to } b^{\min} \leq [A][\Delta x] \leq b^{\max} \\
 & \text{and } \Delta x^{\min} \leq \Delta x \leq \Delta x^{\max}
 \end{aligned} \tag{3}$$

For state estimation, the initial value of x is chosen as the voltage magnitude and angles during the previous state (i.e. voltage magnitudes and angles at the instant when state estimation was last performed) or flat start (in case previous state information is not available).

In the optimization problem (3), the control variables (decision variables) are the deviations in state variables (Δv or $\Delta \delta$) i.e. deviations in voltage magnitude and angle. As decoupled technique is employed, the control variables are $\Delta \delta$ during the $p - \delta$ process and Δv during the $q - v$ process.

$$\begin{aligned}
 \Delta x_{p\delta} &= [\Delta \delta_2 \ \Delta \delta_3 \ \dots \ \Delta \delta_n]^T \\
 \Delta x_{qv} &= [\Delta v_1 \ \Delta v_2 \ \dots \ \Delta v_n]^T
 \end{aligned} \tag{4}$$

The objective function coefficients $[c]^T$, is obtained by linearizing (2) at previous state or at flat start.

$$c^T = \left[\frac{\partial J(x)}{\partial x_1} \ \frac{\partial J(x)}{\partial x_2} \ \dots \ \frac{\partial J(x)}{\partial x_n} \right] \tag{5}$$

upon finding the derivative the expression for c^T is given by (6)

$$\begin{aligned}
 c^T &= (-2) [d]^T [H] \\
 \text{where } [d] &= [z] - [f(x)] \\
 [H] &= \frac{\partial f(x)}{\partial x}
 \end{aligned} \tag{6}$$

The constraints are formulated in such way so that they take care of the mismatches and errors associated with the measurements. The constraints are given by (7)

$$\{z - \epsilon_z - f(x^0)\} \leq H[\Delta x] \leq \{z + \epsilon_z - f(x^0)\} \tag{7}$$

The factor $H[\Delta x]$ gives the change in measurement value for a given change in Δx . Here ϵ_z is the error associated with the meter providing measurement z . The measurement uncertainty is handled by adding the meter errors to the measurements. $z + \epsilon_z$ and $z - \epsilon_z$ are the measurement values with the associated errors. If

these errors are not known, conventional weights can also be used to scale each row of the constraints. In such a case, the constraints are given as,

$$[W][H][\Delta x] \leq [W]\{z - f(x^0)\}$$

However, in this paper, we consider constraints given by (7).

The reason for having limits on Δx is to set the region where the linearizing approximation holds good. This is generally chosen as 5-6%. In case if Δx hits the limits, the c^T is again calculated at this updated state and optimization is again performed. This is employed to get a better accuracy.

The following features of the power system make this approach feasible

1. Under normal operating conditions the voltage magnitude and phase angle deviations from instant to instant will be marginal or around 5-6%. So linearizing it around the previous operating state is a good approximation.
2. Effectiveness of the proposed approach when previous operating state information is not available is demonstrated in sections 6.1 and 6.2.
3. Application of the proposed approach when the system is not under normal operating state or the deviation is more than 5-6% is described in section 6.3.

3 Issues regarding implementation

Techniques required for efficient implementation of linear optimization given by (3) are discussed in this section. Number of constraints (flow and injection measurements) is generally 1.5 to 2.5 times the number of states. In the optimization problem (3), the values of control variables (i.e. Δx) are more important than the minimum value of objective function.

Generally constraints such as $b^{min} \leq A[\Delta x] \leq b^{max}$ are handled by splitting the constraint into two constraints. i.e.

$$[A][\Delta x] \leq b^{max} \text{ and } -[A][\Delta x] \leq -b^{min}$$

Employing such technique doubles the number of constraints which increases complexity. Even if b^{min} is positive, a two phase simplex method will be required in order to handle $-A[\Delta x] \leq -b^{min}$. Also limits on Δx can cause infeasibility of the constraint $b^{min} \leq A[\Delta x] \leq b^{max}$.

An efficient way to handle such a constraint from power system prospective is

$$\begin{aligned}
& \text{repeat until : } A[\Delta X] \geq b^{\min} \\
& \min \quad v_s(x) = c^T [\Delta x] \\
& \text{sub to} \quad [A][\Delta x] \leq b^{\max} \\
& \text{and} \quad \Delta x^{\min} \leq \Delta x \leq \Delta x^{\max}
\end{aligned} \tag{8}$$

Initially $x = x^0$ and after each iteration of solving the bounded optimization problem, x is updated as $x = x + \Delta x$. Suppose 'l' being number of times the optimization problem is solved in order to satisfy $[A][x - x^0] \geq b^{\min}$, the new state x^1 is given as

$$x^1 = x^0 + \Delta x^{01} + \Delta x^{02} + \dots + \Delta x^{0l} \tag{9}$$

The additional advantage of such an approach is detection of no change in states (control variables) without actually proceeding to the optimization phase. This possibility arises as PSSE is run repeatedly in the order of seconds.

Simplex method [16] developed for handling upper bounds on control variables is employed to solve the optimization problem. Bland's rule for avoiding cycling and presolving [17] for reducing the number of constraints are employed. The general technique to handle optimization problems having generic form as in (8) is outlined in Appendix A

4 Decoupled model for PSSE

In Power System State Estimator (PSSE), the measurement vector z comprises of active and reactive power injection measurements at buses and flows in the lines. The control variables (Δx) are deviations in magnitude of bus voltages and angles. The function matrix that relates the deviations in estimated values to the deviations in states is the Jacobian H .

Active/reactive decoupling introduces into power system state estimators the same computational advantages as in the load-flow calculations. It reduces the complexity or dimensionality of the problem, computing time per iteration and storage. Only zeroing effect of decoupling is considered without major approximations. $P - \delta$ and $Q - V$ are solved as two separate linear programming sub-problems. The results from one sub problem are used to modify the power mismatches of other.

4.1 $P - \delta$ sub-problem

In decoupled model the voltage angles are estimated from active power measurements. Mathematically the $P - \delta$ sub-problem can be stated as

“Given a set of noisy active power measurements and load angles at all the buses in the previous state, find the deviation in load angles from present state to previous state such that the difference between the measurements and the computed values is minimum or zero ideally”.

The voltage angles can be computed from active power mismatches by solving optimization problem (10).

$$\begin{aligned} \min \quad & v_s(\delta) = (-2) [d_p]^T [H_{p\delta}] [\Delta\delta] \\ \text{sub to} \quad & [H_{p\delta}] [\Delta\delta] \leq \{z_p + \epsilon_z - f(\delta^0, v^0)\} \\ \text{and} \quad & \Delta\delta^{\min} \leq \Delta\delta \leq \Delta\delta^{\max} \end{aligned} \quad (10)$$

$$\text{where} \quad d_p = z_p - f_p(\delta^0) \quad (11)$$

and $H_{p\delta}$ is the Jacobian defined as

$$\begin{bmatrix} \Delta P_i \\ \Delta P_{ij} \end{bmatrix} = [H_{p\delta}] [\Delta\delta] \quad (12)$$

The vector z_p comprises of active power injection and flow measurements (13) and f_p comprises of computed values of active power injection and flow measurements,

$$z_p = \begin{bmatrix} P_i^{\text{measured}} \\ P_{ij}^{\text{measured}} \end{bmatrix} \text{ and } f_p = \begin{bmatrix} P_i^{\text{calculated}} \\ P_{ij}^{\text{calculated}} \end{bmatrix} \quad (13)$$

4.2 $Q - V$ sub-problem

The voltage magnitude at all the buses are estimated from the reactive power mismatches. The updated voltage angles obtained from (10) are used to compute the reactive power mismatches. The $Q - V$ sub-problem can be stated as

“Given a set of noisy reactive power measurements and voltages at all the buses in the previous state, find the deviation in voltages from previous state to present state such that the difference between the measurements and the computed values is minimum or zero ideally”.

The voltage magnitude can be estimated from the reactive power measurements by solving the optimization problem (14).

$$\begin{aligned}
\min \quad & v_s(v) = (-2) [d_q]^T [H_{qv}] [\Delta v] \\
\text{sub to} \quad & [H_{qv}][\Delta v] \leq \{z_q + \varepsilon_z - f(v^0, \delta^1)\} \\
\text{and} \quad & \Delta v^{\min} \leq \Delta v \leq \Delta v^{\max}
\end{aligned} \tag{14}$$

$$\text{where} \quad d_q = z_q - f_q(v^0) \tag{15}$$

and H_{qv} is the Jacobian defined as

$$\begin{bmatrix} \Delta Q_i \\ \Delta Q_{ij} \end{bmatrix} = [H_{qv}][\Delta v] \tag{16}$$

The vector z_q comprises of reactive power injection and flow measurements (17) and f_q comprises of computed values of reactive power injection and flow measurements,

$$z_q = \begin{bmatrix} Q_i^{\text{measured}} \\ Q_{ij}^{\text{measured}} \end{bmatrix} \text{ and } f_q = \begin{bmatrix} Q_i^{\text{calculated}} \\ Q_{ij}^{\text{calculated}} \end{bmatrix} \tag{17}$$

5 Computational algorithm

The real time modeling of power network usually follows a six-step procedure. Data gathering, network topology processing and observability analysis are essential before actual state estimation algorithm begins. Inability of state estimator to yield estimates with a desired degree of accuracy indicates the presence of bad data. An ideal state estimator should detect and eliminate bad data.

The proposed approach can be summarized by the following algorithm:

- Step 1: Process the raw data obtained from various meters to form z_p (real power measurements) and z_q (reactive power measurements) matrices.
- Step 2: Check if the previous operating state of the system is available. If the states of the system are known use them as initial condition else use flat start.
- Step 3: Compute d_p , $H_{p\delta}$ and $f_p(\delta)$.
- Step 4: Form the optimization problem as defined in (10) and solve to get an estimate of $\Delta\delta$.
- Step 5: Update δ .
- Step 6: Compute d_q , H_{qv} and $f_q(v)$ using updated values of δ .
- Step 7: Solve the optimization problem as defined in (14) to get estimate of Δv
- Step 8: Update v .
- Step 9: Check if $[H_{p\delta}][\Delta\delta] \geq b_p^{\min}$ and $[H_{qv}][\Delta V] \geq b_q^{\min}$, if so the current solution is optimal else goto step 3.

Step 10: Compute the *injected powers* and *line flows* using estimated v and δ and compute the residue.

Step 11: Print the estimated values of voltages, load angles, power flows, power injections and exit.

This formulated approach has been developed in “C” programming language. The sparse features of the matrices are exploited for efficient implementation. Sparsity not only reduces memory requirements but brings down the computation time significantly. Compact row scheme for storing sparse matrices has been employed for the elements of Y_{bus} and Jacobian H .

The popular WLS algorithm is implemented by taking into account the sparse features. One of the major advantages of the proposed approach is that the memory requirement is relatively low as compared for the WLS method as there are no inversions.

6 System studies

The proposed approach has been demonstrated on three sample systems. Two sample systems [18] i.e. 3-bus system, and a 6-bus system are used to compare the proposed approach with the WLS approach. Practical 72 bus equivalent of Indian Southern grid 400 kV/230 kV system is used to demonstrate the application of the proposed approach in situations where the previous state information is available. The input data for the state estimator has been obtained by performing the load flow and introducing random errors for measurement data.

6.1 3 bus system

Sample 3-bus system as shown in Fig.1 comprising of 2 generators and a load is presented for analysis. The parameters of the system along with the injections are indicated in the figure. Injection measurement at all the buses is given as an input to state estimator. There are 3 measurements, giving a redundancy of 1.0.

Flat start ($V \angle \delta = 1 \angle 0$) is used as the initial operating condition as previous state information is not available. The estimated values of injections along with the true and measured value is given in Table 1.

It has been observed that the application of the WLS approach for the given sample system did not converge.

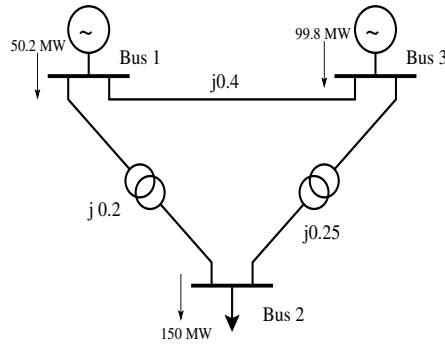


Figure 1: Sample 3 bus system

Table 1: Estimated values for Sample 3 bus System

Meter	True Value		Measured Value		Estimated Value	
	MW	MVAR	MW	MVAR	MW	MVAR
M1	50.2	111.3	48.2	105.4	48.6	110.9
M2	-150.0	-70.0	-155.0	-68.3	-147.7	-68.8
M3	99.8	94.8	97.3	93.8	99.1	94.8

6.2 6 bus system with flat start

A sample 6 bus system (Fig. 2) is chosen for performing comparative studies with the WLS approach. Injection measurements at all the buses and flow measurements at either ends of the lines are given as an input to state estimator.

In the developed approach, the $p - \delta$ optimization problem has 23 control variables (i.e. voltage angles) governed by 28 constraints 28 (i.e. the real power measurements), while the $q - v$ problem has 24 control variables (i.e. voltage magnitudes) governed by 28 constraints.

A comparison between the estimated voltage magnitude and angles of the proposed approach and WLS approach is given in Table 2. It can be inferred from the results that the proposed approach has a much better estimate for voltage magnitudes than the WLS approach. For example, the estimated voltage at bus 1 is 1.043 p.u by proposed approach and 0.98 by the WLS approach. The estimated values of the flows by the proposed approach along with the the true values are given in Table 3.

The proposed approach took 4 iterations to converge while the WLS approach took 7 iterations.

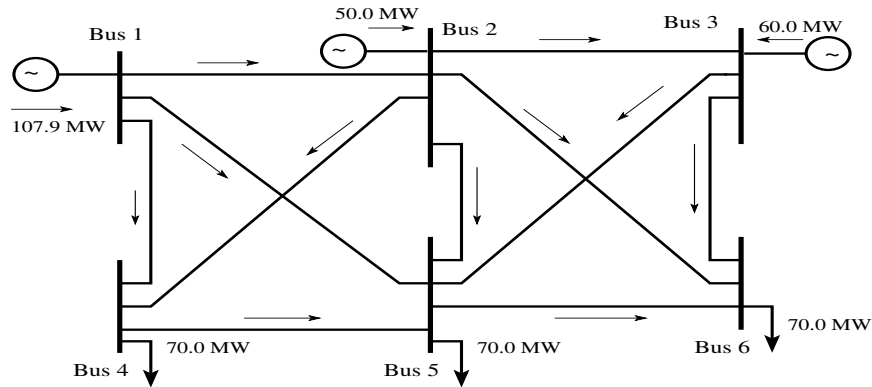


Figure 2: Sample 6 bus system

Table 2: Comparison of estimated voltages for 6 bus system

Bus	Load flow		LP Approach		WLS Approach	
	v	δ (Deg)	v	δ (Deg)	v	δ (Deg)
1	1.050	0.00	1.043	0.00	0.98	0.00
2	1.050	-3.67	1.044	-3.58	0.98	-4.38
3	1.070	-4.0	1.067	-3.88	1.00	-5.11
4	0.989	-4.2	0.980	-4.52	0.91	-4.98
5	0.985	-5.28	0.985	-5.20	0.91	-6.31
6	1.000	-5.90	1.004	-5.27	0.93	-7.05

Table 3: Estimated Line flows for 6 bus system

Meter	True Value		Measured Value		Estimated Value	
	MW	MVAR	MW	MVAR	MW	MVAR
MG1	107.9	16	113.1	20.2	107.75	14.7
M12	28.7	-15.4	31.5	-13.2	27.35	-14.2
M14	43.6	20.1	38.9	21.2	46.09	21.2
MG2	50	74.4	48.4	71.9	49.93	71.7
M21	-27.8	12.8	-34.9	9.7	-26.5	12.9
M25	15.5	15.4	17.4	22	14.92	13.56
ML5	-70	-70	-71.8	-71.9	70.41	62.52
M41	-42.5	-19.9	-40.1	-14.3	-44.86	-20.38
M15	35.6	11.3	35.7	9.4	34.32	9.03
M53	-18.0	-26.1	-25.1	-29.9	-19.42	-24.35
M52	-15.0	-18.0	-11.7	-22.2	-14.49	-16.39

6.3 72 bus equivalent Indian Southern grid

For most of the practical systems, the information from the previous state will be available. Such information can be used as initial point for performing state estimation. In this case study, we have presented the results for a practical 72 bus equivalent 400/230/220kV Indian southern grid system comprising of 15 generating stations, 36 transformers, 49 transmission lines and 18 shunt reactors. The single line diagram along with the geographical map is shown in Fig.3. Total load on the system is 7480MW and 1834MVAR with generation capacity of 9420MW and 4250MVAR.

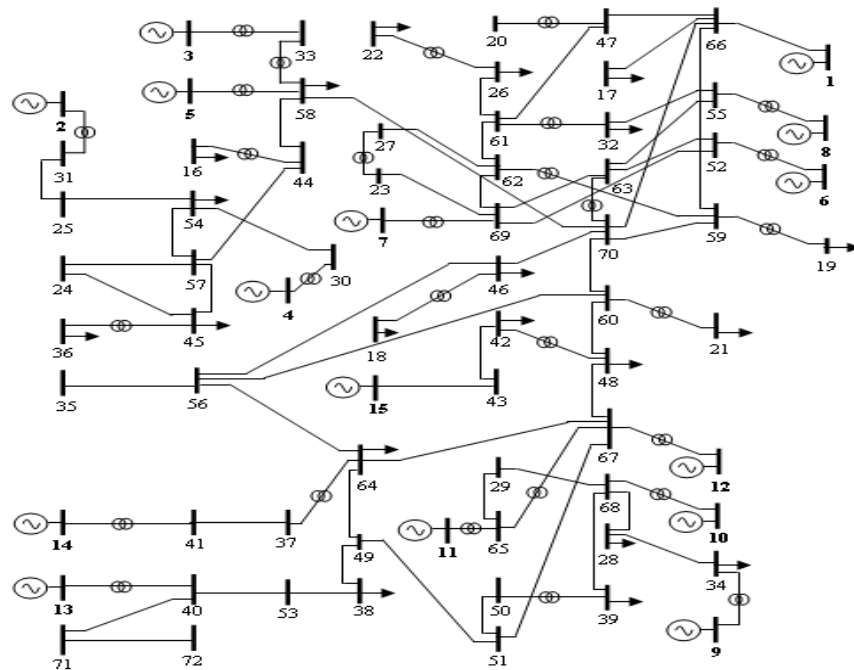


Figure 3: Practical 72-bus equivalent 400/220 kV Indian southern grid system

The proposed approach was tested for different cases and it was observed to converge successfully. One of the main reasons for indicating this set of results is to show the accuracy when there are no limits on Δx and performing only one iteration. Such a technique can be helpful for performing dynamic security analysis where computational time is a major constraint. Results obtained by the proposed approach under such condition are also found to be reasonably accurate. Injection and flow measurements are given as inputs to state estimator with a random error in each measurement. The results are indicated in Table 4 and Table 5 for those measurements which have maximum and minimum deviations.

Table 4: Results for 72 bus system after single iteration

Meter	True Value		Measured Value		Estimated Value	
	MW	MVAR	MW	MVAR	MW	MVAR
1	1548.6	81.3	1502.142	79.674	1548.6	82.6
2	550	26.5	533.5	25.97	538.9	27.4
3	550	86.2	533.5	84.476	545.2	91.1
4	390	5.4	378.3	5.292	382.1	17.2
5	190	51.3	184.3	50.274	186.2	55.3
6	500	-64.4	485	-63.112	490.2	-64.7
7	200	-150.2	194	-147.196	196.1	-155.9
8	600	-35	582	-34.3	600	-31.8
9	400	76.7	388	75.166	397.6	76.5
10	180	13.8	174.6	13.524	179.8	13.7
11	570	10.8	552.9	10.584	573.8	10.6
12	760	293.8	737.2	287.924	760.3	293.3
13	380	76.9	368.6	75.362	389.3	74.6
14	380	204.8	368.6	200.704	379.9	205
15	380	124.5	368.6	122.01	372.7	125.2
16	-71	15.7	-68.87	15.386	-69.7	15.6
17	-336	-75.5	-325.92	-73.99	-329.2	-76.2
18	-3.3	-20	-3.201	-19.6	-3.3	-20
19	-323.6	-165.1	-313.892	-161.798	-323.6	-165.1
20	-62.2	-43.2	-60.334	-42.336	-62.3	-43.2
21	-107.2	-34.9	-103.984	-34.202	-107.2	-34.9
28	-77.4	-53.3	-75.078	-52.234	-76.7	-54.6
29	-40.8	-25.3	-39.576	-24.794	-40.8	-25.3
30	-225.2	-5.2	-218.444	-5.096	-224.1	-6.1
31	-494.4	2.5	-479.568	2.45	-494	10.1
35	-266.1	-55.3	-258.117	-54.194	-263.5	-53.7
36	-263.8	-169.9	-255.886	-166.502	-265.5	-171.9
37	-154.2	-88	-149.574	-86.24	-154.1	-88.1
38	-446.4	-72.1	-433.008	-70.658	-445.3	-73.2
39	-210.7	-87.8	-204.379	-86.044	-210	-87.6
40	-230	-62.5	-223.1	-61.25	-237.4	-56.7
41	-326	-137.7	-316.22	-134.946	-326	-137.6
42	-39.7	-12.5	-38.509	-12.25	-36.9	-13.7
43	-309.1	-79.3	-299.827	-77.714	-303.2	-79.9

Table 5: Results for 72 bus system after single iteration

Meter	True Value		Measured Value		Estimated Value	
	MW	MVAR	MW	MVAR	MW	MVAR
48	-94.2	-57.8	-91.374	-56.644	-94.3	-57.9
49	-165	-26	-160.05	-25.48	-165.9	-24.2
52	-170	-10	-164.9	-9.8	-170	-10.3
53	-251.8	-80	-244.246	-78.4	-247.3	-89.7
54	-162.3	-116.7	-157.431	-114.366	-172.1	-129.4
65	-412.1	-82.2	-399.737	-80.556	-415.7	-82.5
66	-172.8	-45	-167.616	-44.1	-185.9	-47.6
68	-122.8	-53.2	-119.116	-52.136	-76.8	-61.5
69	-314.3	80.6	-304.871	78.988	-313.5	86.4
71	-7.3	12.8	-7.081	12.544	-7.3	12.8
72	0	0	0	0	0.3	-0.1

Using the previous state information helps in significantly reducing the execution time. From the practical 72 bus system results, it can be observed that reasonable estimates are obtained after single iteration. Hence, this approach can be useful for dynamic security analysis where execution time is a major constraint. Sample 3 bus system presented indicates that the proposed approach is efficient in handling those cases for which WLS does not converge.

7 Conclusion

An efficient PSSE formulation using linear programming approach is presented. The formulation is devoid of weights which cause ill conditioning. The present approach has the following advantages:

- The minimization problem is free from weightages, which is the main cause for the numerical instability in conventional methods.
- Applications to large systems shows consistent convergence.
- Computationally efficient as the approach converges in only a few iterations.
- As the case with most LP approaches, this approach is also efficient in dealing with bad data.

Simplex method was adopted to solve the linear programming problem. Further efficient methods can be developed to solve the LP problem using techniques such as interior point and the push-pull technique.

Appendix A

The bounded optimization problem in (8) can be converted into standard LP form as follows

$$\begin{aligned} \min \quad & v_s(x) = c^T [\Delta x] \\ \text{sub to} \quad & H[\Delta x] \leq b^{max} \\ \text{and} \quad & \Delta x^{min} \leq \Delta x \leq \Delta x^{max} \end{aligned} \quad (18)$$

$$\begin{aligned} \text{let} \quad & \Delta x_k^{min} - \Delta x_k^{max} = h_k \\ \text{and} \quad & \Delta x_k - \Delta x_k^{min} = \Delta y_k \end{aligned} \quad (19)$$

The new problem becomes

$$\begin{aligned} \min \quad & v_s(x) = c^T [\Delta y] + c^T [\Delta x^{min}] \\ \text{sub to} \quad & H[\Delta y] \leq b^{max} - g_m \\ \text{and} \quad & 0 \leq \Delta y \leq h_k \end{aligned} \quad (20)$$

$$\text{where } g_m = H[\Delta x^{min}] \quad (21)$$

Appendix B

The expressions for each of the above types of measurements are given in this section. The general two-port π -model for the network branches (transformers and transmission lines) is shown in Figure 4.

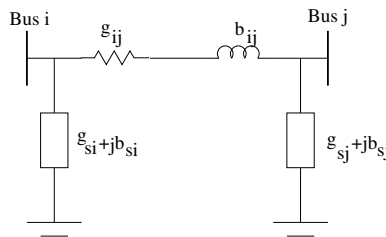


Figure 4: π -model of a network branch

From the available admittances of network elements, Y_{bus} is formed. G_{ij} and B_{ij} represent the ij^{th} elements of real and imaginary parts of Y_{bus} respectively.

The real and reactive power injected at bus i is given by (22)

$$\begin{aligned} P_i^{calculated} &= V_i \sum_{j \in i} V_j (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}) \\ Q_i^{calculated} &= V_i \sum_{j \in i} V_j (G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij}) \end{aligned} \quad (22)$$

The real and reactive power flow between bus i and j is given by (23)

$$\begin{aligned} P_{ij}^{calculated} &= V_i^2 (g_{si} + g_{ij}) - V_i V_j (g_{ij} \cos \delta_{ij} + b_{ij} \sin \delta_{ij}) \\ Q_{ij}^{calculated} &= -V_i^2 (b_{si} + b_{ij}) - V_i V_j (g_{ij} \sin \delta_{ij} - b_{ij} \cos \delta_{ij}) \end{aligned} \quad (23)$$

The elements of the Jacobian H corresponding to injection measurements has the structure of Y_{bus} and are given by (24) and (25)

$$\begin{aligned} \frac{dP_i}{d\delta_i} &= -V_i^2 B_{ii} + \sum_{j=1}^n V_i V_j (-G_{ij} \sin \delta_{ij} + B_{ij} \cos \delta_{ij}) \\ \frac{dP_i}{d\delta_j} &= V_i V_j (G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij}) \end{aligned} \quad (24)$$

$$\begin{aligned} \frac{dQ_i}{dV_i} &= -V_i B_{ii} + \sum_{j=1}^n V_j (G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij}) \\ \frac{dQ_i}{dV_j} &= V_i (G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij}) \end{aligned} \quad (25)$$

The elements of the Jacobian H corresponding to flow measurements has the structure of y_{prim} and are given by (26) and (27)

$$\begin{aligned} \frac{dP_{ij}}{d\delta_i} &= V_i V_j (g_{ij} \sin \delta_{ij} - b_{ij} \cos \delta_{ij}) \\ \frac{dP_{ij}}{d\delta_j} &= -V_i V_j (g_{ij} \sin \delta_{ij} - b_{ij} \cos \delta_{ij}) \end{aligned} \quad (26)$$

$$\begin{aligned} \frac{dQ_{ij}}{dV_i} &= -V_j (g_{ij} \sin \delta_{ij} - b_{ij} \cos \delta_{ij}) - 2V_i (b_{ij} + b_{sj}) \\ \frac{dQ_{ij}}{dV_j} &= -V_i (g_{ij} \sin \delta_{ij} - b_{ij} \cos \delta_{ij}) \end{aligned} \quad (27)$$

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