

INTERSECTION DIMENSION AND GRAPH INVARIANTS

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Abstract

We show that the intersection dimension of graphs with respect to several hereditary properties can be bounded as a function of the maximum degree. As an interesting special case, we show that the circular dimension of a graph with maximum degree Δ is at most $O\left(\Delta \frac{\log \Delta}{\log \log \Delta}\right)$. It is also shown that permutation dimension of any graph is at most $\Delta(\log \Delta)^{1+o(1)}$. We also obtain bounds on intersection dimension in terms of treewidth.

Keywords: circular dimension, dimensional properties, forbidden-subgraph colorings.

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