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Implementation of lossy FTRs for perfect risk hedging under the marginal loss pricing

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Abstract: In this study, a market framework is proposed for the practical implementation of lossy financial transmission rights (FTRs). The advantage of lossy FTRs over conventional FTRs is that the lossy FTRs can be settled directly according to locational marginal prices (LMPs) without requiring any LMP decomposition. Therefore, the price risk for a forward contract can be perfectly hedged if the power transaction involved perfectly matches the corresponding FTR. Although proposed long back, lossy FTRs still did not find an entry to the market because of the prejudice of market complexity and inefficiency. The principal aim of this study is, thus, to create the necessary environment so as to make those fine risk-hedging tools available in the market. First of all, a suitable format for forward contracts is prescribed to enable proper utilisation of lossy FTRs. The detailed lossy FTR auction model is prepared based upon a suitable optimal power flow (OPF) formulation. In addition, the implementation of lossy FTRs is shown for an AC–DC system by appropriately modelling the DC-line power flow behaviour according to the chosen OPF framework. The lossy FTR auction model prepared is thoroughly verified for the FTR issuance as per the market expectations.

Nomenclature

Indices

c network topology index (0 for base topology)
fr index indicating the 'From' end bus of a line
j bus index
k FTR (financial transmission right) bid index
l line index
r auction round index
s line flow segment index
to index indicating the 'To' end bus of a line

Numbers

η_{ftr} total number of bids in the lossy FTR auction
L total number of transmission lines
L_{ac} number of AC transmission lines
L_{dc} number of HVDC transmission lines
C number of line outage scenarios
N_{B} number of AC buses in the network
R number of rounds in an FTR auction
S number of segments in each half of the piecewise linear loss curve

Variables

$\boldsymbol{\alpha}_{\text{fr}}$ $(L_{\text{ac}} \times 1)$ vector of 'From' end phase shifter angles
$\boldsymbol{\alpha}_{\text{to}}$ $(L_{\text{ac}} \times 1)$ vector of 'To' end phase shifter angles
$\boldsymbol{\delta}$ $(N_{\text{B}} \times 1)$ AC bus voltage angle vector
ϑ_{c_1} DC side terminal voltages of Converter 1 in an HVDC line (in per unit)
ϑ_{c_2} DC side terminal voltages of Converter 2 in an HVDC line (in per unit)
I_{dc} current through an HVDC line (in per unit)
$\boldsymbol{P}_{\text{fl}}$ $(L \times 1)$ vector representing terminal-averaged power flows over transmission lines
$\boldsymbol{P}_{\text{fl(fr)}}$ $(L \times 1)$ vector representing 'From' end power flows over transmission lines
$\boldsymbol{P}_{\text{fl(to)}}$ $(L \times 1)$ vector representing 'To' end power flows over transmission lines
$\boldsymbol{P}_{fsg,s}^{(f)}$ $(L \times 1)$ vector representing absolute values of line flow components corresponding to the sth segment on the right half of piecewise linear loss curve
$\boldsymbol{P}_{fsg,s}^{(r)}$ $(L \times 1)$ vector representing absolute values of line flow components corresponding to the sth segment on the left half of piecewise linear loss curve
$\boldsymbol{P}_{\text{inj}}$ $(N_{\text{B}} \times 1)$ vector representing net nodal power injections
$\boldsymbol{P}_{\text{loss}}$ $(L \times 1)$ vector representing line losses
V voltage magnitude of an AC bus
$\boldsymbol{X}_{\text{ftr}}^{\text{lo}}$ $(\eta_{\text{ftr}} \times 1)$ vector representing the cleared amounts towards loss components of lossy FTR bids
$\boldsymbol{X}_{\text{ftr}}^{\text{tr}}$ $(\eta_{\text{ftr}} \times 1)$ vector representing the cleared amounts towards transportation components of lossy FTR bids
$\boldsymbol{z}_s^{\sigma}$ $(\eta_{\text{ftr}} \times 1)$ vector representing the cleared LCFs for the lossy FTR bids

Parameters/constants

σ loss contribution factor of a lossy FTR
$\boldsymbol{\Psi}$ $(L_{\text{ac}} \times N_{\text{B}})$ DC power flow-based lossless AC line flow sensitivity matrix without the phase-shifting control
$\boldsymbol{A}_{\text{tl}}$ $(N_{\text{B}} \times L)$ element-node incidence matrix
β_{lo} price quoted for the loss component in a lossy FTR bid
β_{tr} price quoted for the transportation component in a lossy FTR bid
b AC line susceptance (in per unit)
g line conductance (in per unit)
$m_s^{(f)}$ slope of the sth segment on the right half of the piecewise linear loss curve
$m_s^{(r)}$ slope of the sth segment on the left half of the piecewise linear loss curve
P_{B} base MVA
P_{ftr} MW specification of a lossy FTR
$\boldsymbol{P}_{\text{ld}}^{\text{ftr}}$ $(N_{\text{B}} \times 1)$ vector representing the net nodal inelastic loads (physical equivalent) in the FTR auction

$\boldsymbol{p}$ — $(N_B \times 1)$ vector representing the point of loss contribution of a lossy FTR
$\mathbf{0}_x$ — $(x \times 1)$ vector of all zeros
$(.)_{max}$ — specified maximum limit of variable (.)
$(.)_{min}$ — specified minimum limit of variable (.)

Market results

$\boldsymbol{\lambda}_{dam}$ — $(N_B \times 1)$ nodal LMP vector in the DA market
$\boldsymbol{\lambda}_{ftr}$ — $(N_B \times 1)$ nodal LMP vector in the FTR auction
$(.)^*$ — optimal solution of a variable

1 Introduction

Financial transmission rights (FTRs) are useful tools to guard forward contracts from the volatility of spatial locational marginal price (LMP) variation [1]. FTRs, in effect, provide the option to the forward contracts for paying network usage charges to the independent system operator in advance. However, the owner of an FTR neither receives any exclusive right for the actual network usage nor needs to make any commitment for the execution of a physical transaction. Thus, compared to physical transmission rights, FTRs enable more efficient utilisation of the network capacity along with less market power issue [2]. Apart from risk hedging, FTRs can also be issued to speculators as market derivatives to invest in [3, 4]. Such a provision, in effect, helps in keeping the FTR market more active. Traditional FTRs are perfectly balanced with equal source and sink quantities.

The original version of FTRs entitles full reimbursement of the day-ahead (DA) network usage payment up to the MW amount specified in an FTR [5]. However, the particular FTR version requires lossless DC optimal power flow (DCOPF) based LMP calculation so as to enable revenue adequate issuance of FTRs [5, 6]. With the promotion of marginal loss pricing, it became necessary to decompose the LMP vector into energy, loss and congestion components [7, 8] for the FTR settlement. With decomposed LMP vector, only the congestion components of network usage charges are reimbursed to the FTR owners. The revenue adequate FTR issuance can still be made by satisfying the same set of simultaneous feasibility constraints [9] as in the lossless DCOPF case. However, the shares of loss and congestion components in a network usage charge are not fixed. Moreover, the loss component is not always negligible [10]. Therefore, there may still be significant price risks for forward contracts. In addition, the LMP decomposition is not well defined in the presence of controlled transmission lines.

There are a few works reported in the literature [11–13] to minimise the price risk caused by the loss components of LMP differentials. However, the methodology proposed in [11] has several practical limitations that are discussed in [12]. On the other hand, the loss hedging FTR concept proposed in [12] requires the information of linearised loss parameters from the DA market to make the FTR issuance. This, in turn, makes the implementation of loss hedging FTRs difficult since the linearised loss parameters are not stable quantities. In [13], the proportion of the congestion component in the LMP differential is maximised by optimally adjusting the slack weight vector. However, it may still not be possible to fully nullify the loss component. In addition, the optimal LMP decomposition framework is built upon some fairness measures that can lead to controversy.

In [14], a concept of lossy FTRs was proposed. A lossy FTR is, basically, a point-to-point unbalanced FTR with source end power higher than the sink end power. The lossy FTRs are to be settled directly according to locational marginal prices. Thus, the LMP decomposition and the associated complexities can be completely avoided by using lossy FTRs instead of conventional balanced FTRs. However, even after 13 years since its birth, the lossy FTR mechanism is still not adopted in any of the power markets. The probable cause for the same may be the complexities perceived with regard to the proper utilisation and proper issuance of lossy FTRs. The proper utilisation of lossy FTRs remains questionable since it is practically not simple to obtain the perfect physical match for a lossy FTR. In addition, it is necessary to formulate a suitable auction model for the issuance of lossy FTRs according to the existing market norms. These issues are not well addressed in the literature. Therefore, the practical implementation of lossy FTRs could not be possible till date.

Although lacked attention, lossy FTRs seem to be better instruments than conventional FTRs because of their perfection in risk hedging. Therefore, revival of the lossy FTR mechanism may be of practical importance. The same thought has led to the motivation behind the present work. Thus, it is, in essence, attempted to create a market system so that the lossy FTR mechanism can be brought into the practice by appropriately serving the needs of the market participants. The salient contributions in this regard are as follows:

(i) Restructuring the format of a forward contract to enable proper utilisation of lossy FTRs for the purpose of perfect risk hedging.
(ii) Developing the lossy FTR auction model in consistence with security-constrained dispatch scheduling and other market standards.
(iii) Implementation of the lossy FTR mechanism for an AC–DC system by taking into account the power losses occurring in DC lines.
(iv) Detailed experimental validation of the practical feasibility of lossy FTRs.

The market standards that need to be satisfied while preparing the lossy FTR auction model are as follows:

(i) Revenue adequate FTR issuance.
(ii) Different modes of participation of market players.
(iii) Multiple FTR auctions.
(iv) Multi-round FTR auction.

As a supportive work, the network power flow modelling had to be redefined. The revenue adequate issuance of lossy FTRs requires a linear power flow model with stable loss parameters, which should be used both in the dispatch scheduling as well as in the FTR issuance. The particular issue is resolved by proposing a novel DC power flow (DCPF) model with piecewise linear loss approximation. It is to be noted that a similar power flow model was also employed in the earlier New Zealand market [15, 16] for evaluating network loss and congestion in dispatch scheduling. In this paper, a more accurate version of the respective power flow modelling is prepared by addressing the true non-linear power flow behaviour of a transmission line.

The rest of the paper is organised as follows: The risk hedging functionality of lossy FTRs along with the proposed new format of the forward contract is discussed in Section 2. The network power flow modelling proposed for the implementation of lossy FTRs is elaborated in Section 3. The auction of lossy FTRs is discussed in Section 4. In Section 5, a detailed case study is performed to verify the market efficiency with lossy FTRs. Finally, the paper is concluded in Section 6.

2 Risk hedging functionality of lossy FTRs

Conventionally, an FTR is specified with a source, a sink, a MW amount, a validity period and the time of use. In the case of a lossy FTR, two additional specifications are needed. Those are the loss contribution factor (LCF) and the point of loss contribution (PLC). A lossy FTR essentially comprises of a transportation component and a loss component. The transportation component, in effect, represents a balanced FTR. The MW amount specified for the lossy FTR actually corresponds to its transportation component. On the other hand, the loss component represents a locational FTR of injection type [12] at the PLC. The PLC can be lumped at a particular bus or can be distributed over several buses. In general, the PLC can be represented by a vector $\boldsymbol{p}$ that gives the ratio by which the MW amount of the loss component is distributed over different buses. In the case of the lumped PLC, there should be only one non-zero entry (that is 1) in $\boldsymbol{p}$ corresponding to the specific bus at which the entire loss contribution is made. The MW amount of the loss component is derived from the MW amount of the transportation component by means of the LCF. The hourly settlement of a lossy FTR is carried out according to the settlement rules of balanced and locational FTRs. For example, consider a lossy FTR of $P_{\rm ftr}$ MW specified between Buses 1 and 2 with an LCF σ. The target payment (TP) to the FTR for a particular hour is given by the following equation

$$\mathrm{TP} = \left\{(\lambda_{\mathrm{dam},2} - \lambda_{\mathrm{dam},1}) - \sigma \sum_{j=1}^{N_{\mathrm{B}}} p_j \lambda_{\mathrm{dam},j}\right\} P_{\mathrm{ftr}}. \tag{1}$$

For perfect risk hedging, it should be convenient to lump the entire loss contribution either at the source bus or at the sink bus. Based upon whether the loss contribution is made at source or sink bus, a lossy FTR can be referred to as source bus contributed lossy FTR or sink bus contributed lossy FTR, respectively. Therefore, in order to be perfectly at the LMP neutral position, the owner of the above lossy FTR should ensure $(1 + p_1\sigma)P_{\rm ftr}$ MW power injection at the source bus and $(1 - p_2\sigma)P_{\rm ftr}$ MW power drawal at the sink bus in the DA dispatch scheduling.

The deployment of lossy FTRs in mitigating the price risks for forward contracts is apparently simple. The entity that executes (i.e. is responsible for participating in the DA market) a bilateral transaction needs to get an FTR with transportation quantum equal to the MW amount involved in the respective power transaction. In the case the bilateral transaction is executed by the generating entity, it should choose the source bus as the loss contribution point of its FTR. This in turn enables the entity to counterbalance the FTR loss payment by self-scheduling equivalent power generation from its own generator. The cost of this additional power generation should be recovered from the FTR auction. The entity needs to maintain consistent (with the loss quantum offered in the FTR request) margin in its power generation while signing the bilateral contract in the forward market. In the other case, the load entity may have the responsibility to execute the bilateral transaction. In that case, the sink bus should be chosen as the loss contribution point of the FTR and the FTR loss payment can be counterbalanced by load decrement.

However, the simple approach discussed above does not provide any appropriate solution for the lossy FTR utilisation in the case the bilateral transaction is executed by a third-party trader. Moreover, designating sink bus as the loss contribution point is not recommended since this may cause unnecessary load reduction. In order to enable the usage of source bus contributed lossy FTRs by traders and load entities, the format of a bilateral contract is to be redesigned. Conventionally, a bilateral contract is signed with a firm component and a flexible component. The firm component of the power transaction contract is to be fully executed under any circumstances, whereas, the flexible component is adjustable on hourly basis by bidding in the DA market according to an 'upto network usage charge' agreement. In order to facilitate the use of lossy FTRs by load entities and traders, a floating component can additionally be introduced as an intermediate provision between firm and flexible contracts. The final status of the floating component is to be confirmed periodically. The aforementioned status confirmation should be made by converting a portion of the floating contract into a firm contract and releasing the remaining portion for the respective time period. For example, consider a bilateral contract signed for five years. The MW amount that is finally selected from its floating component can be confirmed and renewed at the beginning of each year. A load entity or trader should offer loss contribution (in its FTR request) only up to the MW amount signed as floating in the corresponding bilateral contract. The offer price should be equal to the price contracted for the respective floating amount.

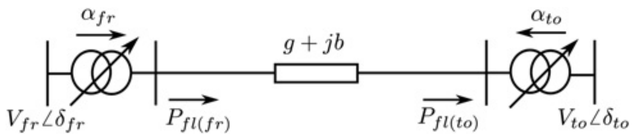

Fig. 1 *AC transmission line*

The loss component of the FTR awarded will decide the MW amount to be confirmed from the floating component. In the DA market, a load entity or trader needs to self-schedule the particular MW only on the generation side. It is to be noted that the floating component of a bilateral contract is not intended to serve any load. Furthermore, there is no mandate on the floating contract, and the same is only subjected to the mutual agreement.

In the case an entity fails to sign a floating bilateral contract, the entity may opt for a distributed PLC in its lossy FTR request. The net loss payment to be made by the entity in the DA market for the respective FTR holding is analogous to the exclusion of the loss component of an LMP difference in the conventional FTR settlement. However, with lossy FTRs, an entity receives the flexibility to better predict its loss payment by suitably choosing the PLC vector. This is because, although individual LMPs are quite volatile, there can be less volatility in a linear combination of locational marginal prices. In the same way, lossy FTRs can be more beneficial to speculators compared with the conventional FTRs.

3 Proposed network power flow modelling

This section presents the line flow model developed for the issuance of lossy FTRs. The representations of an AC transmission line and an HVDC transmission line (equivalent monopole) are shown in Figs. 1 and 2, respectively. The phase shifter provision helps in controlling the power flow over an AC line. Initially, some convenient forms of the accurate non-linear power flow equations are investigated. The non-linear power flow equations thus obtained are subsequently approximated into suitable linear forms. As in the standard DCPF practice, each bus is assumed to have adequate reactive power support so that its voltage magnitude can be maintained constant at 1 pu [17].

3.1 Non-linear power flow modelling

Irrespective of an AC line or an HVDC line, the terminal-averaged line flow and the line loss are defined as follows

$$P_{\mathrm{fl}} = 0.5\{P_{\mathrm{fl(fr)}} + P_{\mathrm{fl(to)}}\} \tag{2}$$

$$P_{\mathrm{loss}} = \{P_{\mathrm{fl(fr)}} - P_{\mathrm{fl(to)}}\}. \tag{3}$$

Therefore

$$P_{\mathrm{fl(fr)}} = P_{\mathrm{fl}} + 0.5P_{\mathrm{loss}} \tag{4}$$

$$P_{\mathrm{fl(to)}} = P_{\mathrm{fl}} - 0.5P_{\mathrm{loss}}. \tag{5}$$

The expressions of $P_{\rm fl}$ and $P_{\rm loss}$ differ between an AC line and an HVDC line.

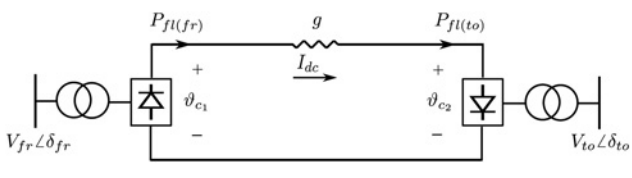

Fig. 2 *HVDC transmission line*

3.1.1 AC line flow modelling: By assuming one per unit terminal voltages, the following expressions are obtained for the terminal-averaged power flow and the power loss in an AC transmission line

$$P_{\rm fl} = -P_{\rm B} b \sin\theta \tag{6}$$

$$P_{\rm loss} = 2P_{\rm B} g\{1 - \cos\theta\}. \tag{7}$$

Here $\theta = (\delta_{\rm fr} - \delta_{\rm to} + \Delta\alpha)$ and $\Delta\alpha = \alpha_{\rm fr} - \alpha_{\rm to}$. For a given $P_{\rm fl}$, the value of θ can be determined from (6). Therefore, $P_{\rm loss}$ can directly be expressed as a function of $P_{\rm fl}$ as follows

$$P_{\rm loss} = 2P_{\rm B} g\left\{1 - \sqrt{1 - \left(\frac{P_{\rm fl}}{bP_{\rm B}}\right)^2}\right\}. \tag{8}$$

3.1.2 DC line flow modelling: The expressions of the terminal-averaged line flow and the line loss for an HVDC transmission line are shown in (9) and (10), respectively,

$$P_{\rm fl} = 0.5P_{\rm B}(\vartheta_{c_1} + \vartheta_{c_2})I_{\rm dc} = 0.5gP_{\rm B}(\vartheta_{c_1}^2 - \vartheta_{c_2}^2) \tag{9}$$

$$P_{\rm loss} = P_{\rm B} I_{\rm dc}^2/g = gP_{\rm B}(\underbrace{\vartheta_{c_1} - \vartheta_{c_2}}_{\Delta\vartheta_{\rm dc}})^2. \tag{10}$$

From (9) and (10), the following relationships can be obtained

$$(2P_{\rm fl} - P_{\rm loss})^2 = (2gP_{\rm B}\vartheta_{c_2}\Delta\vartheta_{\rm dc})^2 = 4\vartheta_{c_2}^2 gP_{\rm B}P_{\rm loss} \tag{11}$$

$$(2P_{\rm fl} + P_{\rm loss})^2 = (2gP_{\rm B}\vartheta_{c_1}\Delta\vartheta_{\rm dc})^2 = 4\vartheta_{c_1}^2 gP_{\rm B}P_{\rm loss}. \tag{12}$$

Each of the above quadratic equations is to be solved for $P_{\rm loss}$. Irrespective of current-source or voltage-source converters, both ϑ_{c_1} and ϑ_{c_2} should be of the same polarity as per the normal operational practice. This, in turn, implies that $P_{\rm loss}$ must be zero for zero $P_{\rm fl}$. The final solution of a quadratic equation (out of two possibilities) is to be chosen accordingly. The alternative solutions of $P_{\rm loss}$ thus obtained from (11) and (12) are shown in (13) and (14), respectively,

$$P_{\rm loss} = 2P_{\rm fl} + 2P_{\rm B}\left\{g\vartheta_{c_2}^2 - \sqrt{g^2\vartheta_{c_2}^4 + 2g\vartheta_{c_2}^2\frac{P_{\rm fl}}{P_{\rm B}}}\right\} \tag{13}$$

$$P_{\rm loss} = -2P_{\rm fl} + 2P_{\rm B}\left\{g\vartheta_{c_1}^2 - \sqrt{g^2\vartheta_{c_1}^4 - 2g\vartheta_{c_1}^2\frac{P_{\rm fl}}{P_{\rm B}}}\right\}. \tag{14}$$

From (9), it is obvious that $|\vartheta_{c_1}| \geq |\vartheta_{c_2}|$ if $P_{\rm fl} \geq 0$ and $|\vartheta_{c_2}| > |\vartheta_{c_1}|$ if $P_{\rm fl} < 0$. Therefore, a combined representation of (13) and (14) appears as follows

$$P_{\rm loss} = 2|P_{\rm fl}| + 2P_{\rm B}\left\{g\vartheta_{iv}^2 - \sqrt{g^2\vartheta_{iv}^4 + 2g\vartheta_{iv}^2\frac{|P_{\rm fl}|}{P_{\rm B}}}\right\}. \tag{15}$$

Here, ϑ_{iv} indicates the inverter-end voltage and $|\vartheta_{iv}| = \min\{|\vartheta_{c_1}|, |\vartheta_{c_2}|\}$. In the practical operation, the inverter-end voltage magnitude is usually maintained constant at the rated value [18]. Thus, $P_{\rm loss}$ becomes a function of only $P_{\rm fl}$.

3.2 Linear power flow approximation

Equations (4)–(6), (8) and (15) describe the exact power flow behaviours of AC and HVDC transmission lines. The non-linearity involved in (6) can be removed by means of the standard DCPF approximation as follows

$$P_{\rm fl} \simeq -P_{\rm B} b\theta = -P_{\rm B} b(\delta_{\rm fr} - \delta_{\rm to} + \Delta\alpha). \tag{16}$$

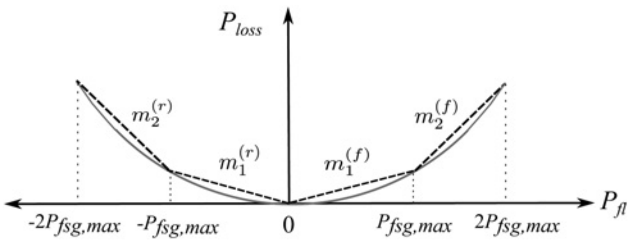

Fig. 3 *Piecewise linearisation of loss curve*

The loss expressions (8) and (15) exhibit parabolic shapes; therefore, are suitable for piecewise linearisation. The piecewise loss linearisation is illustrated in Fig. 3. The actual and approximated loss curves are shown by solid and dotted curves, respectively. For the sake of simplicity, all the flow interval lengths are taken to be equal. However, more accurate representation is possible through optimal selection of straight line segments [16]. It is to be noted that (9) is no longer required since the power flow over an HVDC line can be independently controlled and the voltage or current limit constraint can be reflected into the line loadability limit.

With the above piecewise linear representation, the power loss in a line can be determined through the following equation

$$P_{\rm loss} = \sum_{s=1}^{S}(m_s^{(f)}P_{fsg,s}^{(f)} - m_s^{(r)}P_{fsg,s}^{(r)}). \tag{17}$$

Equation (17) is subjected to the following conditions

$$P_{\rm fl} = \sum_{s=1}^{S}(P_{fsg,s}^{(f)} - P_{fsg,s}^{(r)}) \tag{18}$$

$$0 \leq P_{fsg,s}^{(f)} \leq P_{fsg,\max} \tag{19}$$

$$\left.\begin{array}{l} P_{fsg,s+1}^{(.)} = 0 \quad \text{if } P_{fsg,s}^{(.)} < P_{fsg,\max}, \quad \forall s < S \\ \sum_{s=1}^{S} P_{fsg,s}^{(r)} = 0 \quad \text{if } \sum_{s=1}^{S} P_{fsg,s}^{(f)} > 0. \end{array}\right\} \tag{20}$$

Here $P_{fsg,\max} = P_{\rm fl,max}/S$. Condition (20) can also be put down in a linear form by using additional binary variables.

4 Lossy FTR auction

Traditional FTRs are available both as obligations and options [5]. In principle, it is also possible to issue option lossy FTRs. However, the issuance of option lossy FTRs may be difficult because of certain computational issues. Practically, option FTRs are useful only to speculators. Although an option FTR can provide perfect hedge for the transaction in which the MW quantity involved is variable over time, the similar benefit can alternatively be obtained by procuring cheaper time-differentiated obligation FTRs. In practical systems, even, option FTRs are usually issued in very limited volumes [19]. Furthermore, because of certain market performance issues, the issuance of option FTRs is restricted only to a few paths in the PJM market [20]. Therefore, sacrificing the option feature for the benefit of perfect risk hedging seems to be rational policy. The auction model for the issuance of obligation lossy FTRs is explained below.

4.1 Objective function

In a lossy FTR auction, each participant needs to provide separate price quotes for the transportation and loss components in its lossy FTR request portfolio. The prices quoted indicate the maximum and minimum prices that the concerned market player is willing to pay and receive for the transportation component and the loss component, respectively. The auction is cleared by minimising the social cost function according to the lossy FTR bids. Mathematically, the problem objective can be formulated as

$$\text{minimise}\ \underbrace{\left\{\sum_{k=1}^{\eta_{\text{ftr}}}(\beta_{\text{lo},k}X^{\text{lo}}_{\text{ftr},k}-\beta_{\text{tr},k}X^{\text{tr}}_{\text{ftr},k})\right\}}_{f(\boldsymbol{X}^{\text{tr}}_{\text{ftr}},\boldsymbol{X}^{\text{lo}}_{\text{ftr}})}. \tag{21}$$

The sale offer for an existing obligation lossy FTR can be visualised as a purchase bid for a new obligation lossy FTR on the opposite path with negative price quote for transportation component and zero price quote for the loss component. The FTR sold must have the same LCF and PLC as those of the parent FTR. In the equivalent purchase bid, the LCF should be shown as negative.

4.2 Bidders' constraints

The bidder of a lossy FTR needs to specify the required transportation quantity as well as the loss contribution that it can make. The request for the transportation quantity should be made with a specific MW number. The loss contribution, on the other hand, should be linked to the transportation quantity awarded. Otherwise, the auction may end up by assigning only the loss responsibility to a market participant. Therefore, instead of offering the loss contribution through a specific MW limit, a market player may specify an LCF range that is acceptable to it. This is in accord with the Design A-1 of [14]. Thus, the lossy FTR issuance is subjected to the following bid limit constraints

$$\boldsymbol{q}_1\colon \boldsymbol{X}^{\text{lo}}_{\text{ftr}}-\hat{\boldsymbol{z}}_{\sigma,\max}\boldsymbol{X}^{\text{tr}}_{\text{ftr}}\le \boldsymbol{0}_{\eta_{\text{ftr}}} \tag{22}$$

$$\boldsymbol{q}_2\colon -\boldsymbol{X}^{\text{lo}}_{\text{ftr}}+\hat{\boldsymbol{z}}_{\sigma,\min}\boldsymbol{X}^{\text{tr}}_{\text{ftr}}\le \boldsymbol{0}_{\eta_{\text{ftr}}} \tag{23}$$

$$\boldsymbol{q}_3\colon \boldsymbol{X}^{\text{tr}}_{\text{ftr}}-\boldsymbol{X}^{\text{tr}}_{\text{ftr},\max}\le \boldsymbol{0}_{\eta_{\text{ftr}}} \tag{24}$$

$$\boldsymbol{q}_4\colon -\boldsymbol{X}^{\text{tr}}_{\text{ftr}}\le \boldsymbol{0}_{\eta_{\text{ftr}}}. \tag{25}$$

Matrix $\hat{\boldsymbol{z}}_{\sigma,\max}$ and matrix $\hat{\boldsymbol{z}}_{\sigma,\min}$ are $(\eta_{\text{ftr}}\times\eta_{\text{ftr}})$ diagonal matrices with $\hat{z}_{\sigma,(.),k,k}=z_{\sigma,(.),k}$. Typically, in a regular purchase bid, the minimum LCF limit specified is expected to be zero. In the case of the equivalent purchase bid for a sale offer, the LCF must be set to a fixed value. This is, in effect, equivalent to setting the maximum and minimum LCF limits equal.

Equations (22) and (23) can alternatively be written as follows

$$\boldsymbol{X}^{\text{lo}}_{\text{ftr}}=\hat{\boldsymbol{z}}_{\sigma}\boldsymbol{X}^{\text{tr}}_{\text{ftr}} \tag{26}$$

$$\boldsymbol{z}_{\sigma,\min}\le\boldsymbol{z}_{\sigma}\le\boldsymbol{z}_{\sigma,\max}. \tag{27}$$

In (26) and (27), the LCF variables are explicitly considered. However, the direct inclusion of LCF variables introduces non-linearity in the problem formulation. Therefore, those are eliminated to obtain the linear forms as in (22) and (23). After the auction is over, the LCF of a cleared lossy FTR can be determined by using the relationship shown in (26).

4.3 Network constraints

The network constraints should be addressed in an FTR auction so as to ensure revenue adequate FTR issuance. The same power flow model should be employed both in the DA dispatch scheduling and in the FTR auction. For the implementation of lossy FTRs, the dispatch scheduling problem is to be reformulated based upon the power flow model developed in Section 3. However, it may not be required to explicitly incorporate (20) in the dispatch scheduling problem. Consequently, the particular constraint should also be relaxed in the FTR auction. Usually, this does not cause any serious violation of the particular condition in the final solution of the dispatch scheduling problem [15]. This is because, for the line flow solutions obtained, line losses should be minimum to ensure minimum possible social cost at each bus. It is to be noted that the above argument is particularly true if the addition of an infinite load of very low bid price does not affect the optimal solution.

The lossy FTR auction problem is formulated on the background of the corrective security-constrained dispatch scheduling. The post-contingency corrections in phase shifter angles and HVDC line flows are included. For the sake of simplicity in representation, the power flow equations of a tripped line are also included in the formulation. The outage status of the respective line is reflected through the element-node incidence matrix for the corresponding network topology. With regard to the piecewise linearisation, the number of linear segments in the approximated loss curve is taken to be the same for all lines. Without losing generality, the first L_{ac} lines are taken as AC lines and the next $L-L_{\text{ac}}$ (or L_{dc}) lines are taken as HVDC lines. Consequently, the line flow vector and the element-node incidence matrix can be decomposed as follows

$$\boldsymbol{P}_{\text{fl}}=[\boldsymbol{P}^{(\text{ac})^{\text{T}}}_{\text{fl}}\quad \boldsymbol{P}^{(\text{dc})^{T}}_{\text{fl}}]^{\text{T}} \tag{28}$$

$$\boldsymbol{A}_{\text{tl}}=[\boldsymbol{A}^{(\text{ac})}_{\text{tl}}\quad \boldsymbol{A}^{(\text{dc})}_{\text{tl}}]. \tag{29}$$

Superscripts 'ac' and 'dc' indicate AC lines and HVDC lines, respectively.

To begin with, the net nodal injections caused by the physical equivalents of lossy FTRs can be obtained through the following equation

$$\boldsymbol{h}_1\colon \boldsymbol{P}_{\text{inj}}-(\boldsymbol{A}^{\text{tr}}_{\text{ftr}}\boldsymbol{X}^{\text{tr}}_{\text{ftr}}+\boldsymbol{A}^{\text{lo}}_{\text{ftr}}\boldsymbol{X}^{\text{lo}}_{\text{ftr}}-\boldsymbol{P}^{\text{ftr}}_{\text{ld}})=\boldsymbol{0}_{N_{\text{B}}}. \tag{30}$$

Here, the $(N_{\text{B}}\times\eta_{\text{ftr}})$ matrix $\boldsymbol{A}^{\text{tr}}_{\text{ftr}}$ defines the incidences of the transportation paths of lossy FTRs (with '+1' for the source and '−1' for the sink) onto the network buses. Similarly, matrix $\boldsymbol{A}^{\text{lo}}_{\text{ftr}}$ defines the incidences of the PLCs of requested lossy FTR. Matrix $\boldsymbol{A}^{\text{lo}}_{\text{ftr}}$ is obtained just by juxtaposing the PLC vectors of the requested lossy FTRs. Vector $\boldsymbol{P}^{\text{ftr}}_{\text{ld}}$ pertains to the self-scheduled and already existing (i.e. previously issued) FTRs.

From (4) and (5), the nodal power balance equation can be formulated as follows

$$\boldsymbol{h}_{2,c}\colon \boldsymbol{A}_{\text{tl},c}\boldsymbol{P}_{\text{fl},c}+0.5|\boldsymbol{A}_{\text{tl},c}|\boldsymbol{P}_{loss,c}-\boldsymbol{P}_{\text{inj}}=\boldsymbol{0}_{N_{\text{B}}}\forall c. \tag{31}$$

Note that the value of $\boldsymbol{P}_{\text{inj}}$ does not vary with contingency. Equations (16)–(18) can be put down in matrix–vector form over all the transmission lines as follows

$$\boldsymbol{h}_{3,c}\colon \sum_{s=1}^{S}\{\hat{\boldsymbol{m}}^{(f)}_{s}\boldsymbol{P}^{(f)}_{fsg,s,c}-\hat{\boldsymbol{m}}^{(r)}_{s}\boldsymbol{P}^{(r)}_{fsg,s,c}\}-\boldsymbol{P}_{\text{loss},c}=\boldsymbol{0}_{L}\forall c \tag{32}$$

$$\boldsymbol{h}_{4,c}\colon \sum_{s=1}^{S}\{\boldsymbol{P}^{(f)}_{fsg,s,c}-\boldsymbol{P}^{(r)}_{fsg,s,c}\}-\boldsymbol{P}_{\text{fl},c}=\boldsymbol{0}_{L}\forall c \tag{33}$$

$$\boldsymbol{h}_{5,c}\colon P_{\text{B}}\{\hat{\boldsymbol{b}}\boldsymbol{A}^{(\text{ac})^{T}}_{\text{tl},c}\boldsymbol{\delta}_c+\hat{\boldsymbol{b}}\boldsymbol{\Delta\alpha}_c\}+\boldsymbol{P}^{(\text{ac})}_{\text{fl},c}=\boldsymbol{0}_{L_{\text{ac}}}\forall c \tag{34}$$

where $\hat{\boldsymbol{m}}^{(.)}_{s}$ is an $(L\times L)$ diagonal matrix with $\hat{m}^{(.)}_{s,l,l}=m^{(.)}_{s,l}$. Similarly, $\hat{\boldsymbol{b}}$ is an $(L_{\text{ac}}\times L_{\text{ac}})$ diagonal matrix containing the susceptances of AC transmission lines.

An FTR auction may consist of multiple rounds [20]. The network capacity is evenly released over different rounds. For the power flow model employed, the network capacity is defined by vectors $\boldsymbol{P}_{fsg,\max}$, $\boldsymbol{\Delta\alpha}_{\max}$ and $\boldsymbol{\Delta\alpha}_{\min}$. Therefore, the network capacity constraints for the rth round of an R round auction can be formulated as

$$\boldsymbol{q}_{5,s,c}\colon \boldsymbol{P}^{(f)}_{fsg,s,c} - \left(\frac{r}{R}\right)\boldsymbol{P}_{fsg,\max} \le \boldsymbol{0}_L \forall s,\ \forall c \tag{35}$$

$$\boldsymbol{q}_{6,s,c}\colon -\boldsymbol{P}^{(f)}_{fsg,s,c} \le \boldsymbol{0}_L \forall s,\ \forall c \tag{36}$$

$$\boldsymbol{q}_{7,s,c}\colon \boldsymbol{P}^{(r)}_{fsg,s,c} - \left(\frac{r}{R}\right)\boldsymbol{P}_{fsg,\max} \le \boldsymbol{0}_L \forall s,\ \forall c \tag{37}$$

$$\boldsymbol{q}_{8,s,c}\colon -\boldsymbol{P}^{(r)}_{fsg,s,c} \le \boldsymbol{0}_L \forall s,\ \forall c \tag{38}$$

$$\boldsymbol{q}_{9,c}\colon \boldsymbol{\Delta\alpha}_c - \left(\frac{r}{R}\right)\boldsymbol{\Delta\alpha}_{\max} \le \boldsymbol{0}_{L_{\mathrm{ac}}} \forall c \tag{39}$$

$$\boldsymbol{q}_{10,c}\colon -\boldsymbol{\Delta\alpha}_c + \left(\frac{r}{R}\right)\boldsymbol{\Delta\alpha}_{\min} \le \boldsymbol{0}_{L_{\mathrm{ac}}} \forall c. \tag{40}$$

It is to be noted that the FTRs issued in previous rounds must be included in the $\boldsymbol{P}^{\mathrm{ftr}}_{\mathrm{ld}}$ vector of the present round.

Contingency constraints (i.e. for $c > 0$) of the form (31)–(40) demand recalculation of the network loss for each possible network topology. Alternatively, in line of [21], a lossless network model can be employed to represent the contingency constraints as follows

$$-\boldsymbol{P}^{(\mathrm{ac})}_{\mathrm{fl,\max}} \le \boldsymbol{\Psi}_c\{P_{\mathrm{B}}\boldsymbol{A}^{(\mathrm{ac})}_{\mathrm{tl},c}\hat{\boldsymbol{b}}^{\mathrm{T}}\boldsymbol{\Delta\alpha}_c - \boldsymbol{A}^{(\mathrm{dc})}_{\mathrm{tl},c}\boldsymbol{P}^{(\mathrm{dc})}_{\mathrm{fl},c} + \boldsymbol{P}_{\mathrm{inj}}\} \le \boldsymbol{P}^{(\mathrm{ac})}_{\mathrm{fl,\max}} \quad \text{for} \quad c = 1 \text{ to } C \tag{41}$$

$$-\boldsymbol{P}^{(\mathrm{dc})}_{\mathrm{fl,\max}} \le \boldsymbol{P}^{(\mathrm{dc})}_{\mathrm{fl},c} \le \boldsymbol{P}^{(\mathrm{dc})}_{\mathrm{fl,\max}} \quad \text{for} \quad c = 1 \text{ to } C. \tag{42}$$

Here, the network loss calculated for the base case topology is distributed as additional loads over different nodes in the contingency model. The same is equivalent to the selection of a suitable slack reference for the calculation of the $\boldsymbol{\Psi}$ matrix [10].

Table 1 Lossy FTR requests

FTR id.	Transportation bid		Loss offer	
	Price ($/MW)	Quantity (MW)	Price ($/MW)	Max. LCF
FTR_1	3.1	60	8	0.041
FTR_2	3	70	8.5	0.048
FTR_3	1	90	9.5	0.061
FTR_4	3	80	10	0.054
FTR_5	2	65	11	0.044
FTR_6	2	100	9	0.068
FTR_7	1.4	58	10.5	0.039
FTR_8	2.5	50	8	0.034
FTR_9	2	90	10	0.061
FTR_10	4	75	8	0.051
FTR_11	2	95	11	0.065
FTR_12	3	55	7	0.037
FTR_13	3.6	50	10	0.034
FTR_14	2	45	11	0.031
FTR_15	3.1	65	9	0.044
FTR_16	3	35	9	0.024
FTR_17	3.1	70	9	0.048
FTR_18	2	50	9	0.034
FTR_19	3.1	95	8	0.065
FTR_20	6	35	8	0.024
FTR_21	4	77	11	0.052
FTR_22	4	69	9.5	0.047
FTR_23	4	64	11	0.044
FTR_24	2.5	76	6	0.052
FTR_25	4	55	11	0.037
FTR_26	4.2	26	8.5	0.018
FTR_27	4	56	10	0.038
FTR_28	5	100	8	0.068
FTR_29	4	79	10	0.054
FTR_30	2	50	5	0.034

4.4 Auction pricing

The Lagrangian function of the optimisation problem (21)–(25) and (30)–(40) can be written as follows:

$$\Lambda = f + \boldsymbol{\kappa}_1^{\mathrm{T}}\boldsymbol{h}_1 + \sum_{i=2}^{5}\sum_{c=0}^{C}\{\boldsymbol{\kappa}_{i,c}^{\mathrm{T}}\boldsymbol{h}_{i,c}\} + \sum_{i=1}^{4}\boldsymbol{\mu}_i^{\mathrm{T}}\boldsymbol{q}_i + \sum_{i=5}^{8}\sum_{s=1}^{S}\sum_{c=0}^{C}\{\boldsymbol{\mu}_{i,s,c}^{\mathrm{T}}\boldsymbol{q}_{i,s,c}\} + \sum_{i=9}^{10}\sum_{c=0}^{C}\{\boldsymbol{\mu}_{i,c}^{\mathrm{T}}\boldsymbol{q}_{i,c}\}. \tag{43}$$

Here all the κ and μ terms are Lagrangian multipliers. The nodal LMP vector in the lossy FTR auction is obtained as

$$\boldsymbol{\lambda}_{\mathrm{ftr}} = \left\{\frac{\partial\Lambda}{\partial\boldsymbol{P}^{\mathrm{ftr}}_{\mathrm{ld}}}\right\}^{\mathrm{T}} = \boldsymbol{\kappa}_1^*. \tag{44}$$

Similarly to a bilateral transaction in the DA market, the auction price charged to the transportation component of a newly issued lossy FTR is given by the LMP difference between the sink and source points in the corresponding FTR auction. For the loss component, the bidder receives payment according to the LMP at the PLC. For a distributed PLC, the LMP at the PLC is given by the weighted average of nodal LMPs according to the PLC vector. In principle, $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$ should also be included in the FTR pricing since each of the respective constraints couples two different requests. However, it can be easily shown that the net cost of a lossy FTR remains the same irrespective of whether or not $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$ are considered in pricing. No additional charge is applied to a previously issued FTR. The use of a linear power flow model ensures net non-negative auction collection so far as the previously issued FTRs are simultaneously feasible.

5 Case study

It is still required to carry out certain performance validations to justify the merit of the lossy FTR mechanism. First of all, there should not be any drastic reduction in the overall (issued) volume of FTRs because of the introduction of a loss component. The particular possibility arises since, without power flow control, the FTR auction appears similar to a preventive security-constrained OPF calculation over a lossy system. Second, the issuance of lossy FTRs should not be overburdened by the requirement of loss contributions. The particular issue arises since the transportation component of a lossy FTR is of actual interest for the purpose of risk hedging, and the loss component is undesirable. Therefore, an entity should not be burdened to make significant loss contribution to earn the eligibility for the award of sizable transportation MW. However, in the case there is no loss contribution from any FTR, no FTR can be issued. Therefore, the other concern is to have some clear financial force to encourage auction participants for contributing towards network loss.

Case studies are performed on a modified IEEE 118-bus system to verify the market efficiency of lossy FTRs with the proposed auction model. The original IEEE 118-bus system data is obtained from [22]. In the modified IEEE 118-bus system, a DC line of resistance 0.05 pu, voltage rating 1.3505 pu and capacity (in terms of average line flow) 200 MW is added between Buses 15 and 107. The per unit representation of the DC system quantities is explained in [23]. The X/R ratio of each AC transmission line is taken as 5. The line reactances are maintained at the original values. The capacity of each AC transmission line is taken as the minimum of 200 MW and 0.7071 times of its steady-state stability limit. The number of flow segments is taken to be three on each side of the piecewise linear curve. Table 1 shows the sample FTR requests that are submitted in a particular auction. Only source bus contributed lossy FTRs are considered. No self-scheduled FTR request or previous FTR issuance is considered in the present auction and the

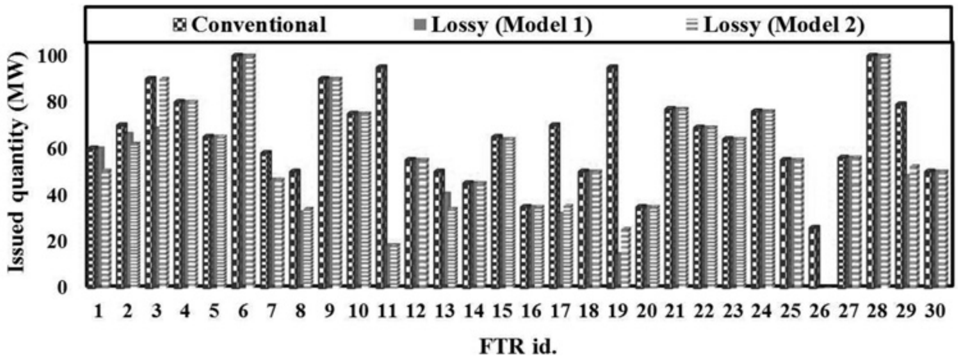

Fig. 4 *Auction results for issued FTR (transportation) quantities*

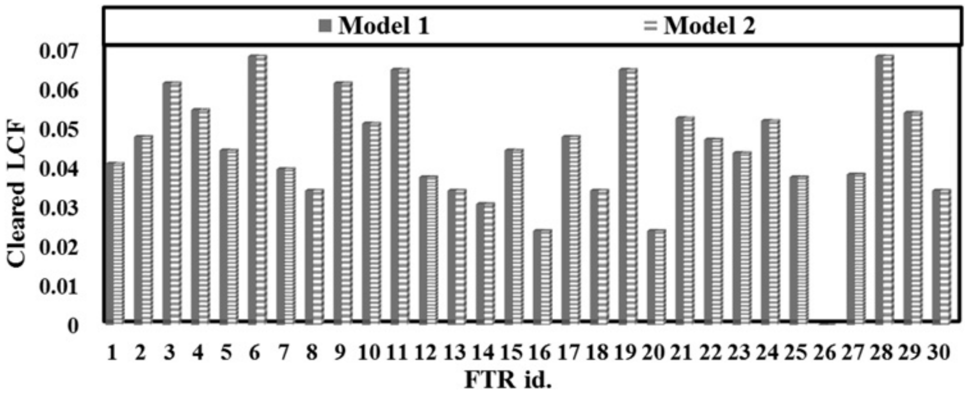

Fig. 5 *Auction results for the loss contribution requirements*

auction consists of only one round. The minimum LCF limit is taken to be zero.

Three different FTR auctions are conducted. The first auction is conducted for the issuance of conventional FTRs by employing the lossless network model. Only the transportation component bids are considered in the particular auction. The second and third auctions are conducted for the issuance of lossy FTRs with different representations of contingency constraints. The auction models corresponding to contingency representations according to (31)–(40) and (41), (42) are referred to as Model 1 and Model 2, respectively. For Model 2, the sensitivity matrix is calculated by considering equal slack weights for all the nodes. Three $n-1$ contingency cases are considered corresponding to outages of Lines 11, 51 and 101, respectively.

The results of FTR auctions are shown in Figs. 4 and 5. It is interesting to see that both the models of the lossy FTR auction yield almost similar results. The particular phenomenon happens at the cost of multiple violations of Condition (20) in Model 1 because of enforcing equality of loss calculations between different network topologies. However, the issue of power flow accuracy is insignificant in the context of FTR issuance since FTRs do not yield physical power flows in the network. It is just sufficient to match the mathematical models of network power flow behaviour between dispatch scheduling and FTR issuance so as to ensure revenue adequacy. It is to be noted that the violation of Condition (20) is automatically controlled in dispatch scheduling because of relaxing the aforementioned loss equality condition through a corrective approach with reserve variables.

The difference between the results of conventional and lossy FTR auctions is a natural consequence of recognising the effect of a power injection onto the network loss in the lossy FTR auction. However, the issuance of lossy FTRs is still found to be comparable to the issuance of conventional FTRs. In the conventional FTR auction, a total of 1985 MW FTR is issued. The total FTR volume issued

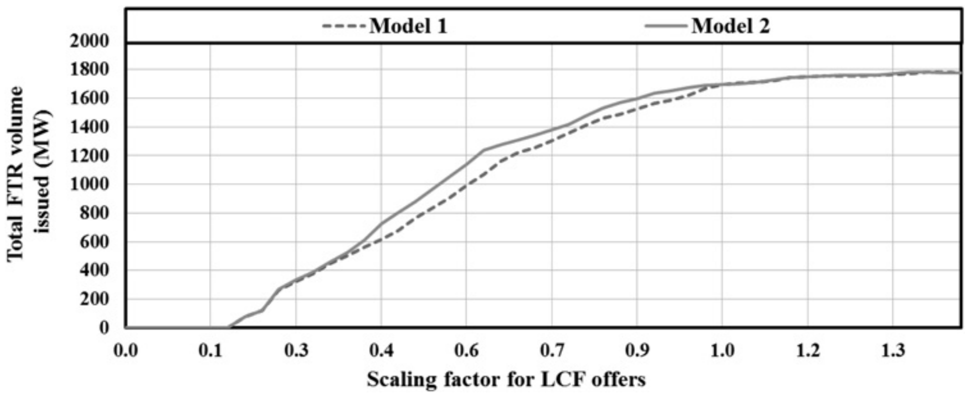

Fig. 6 *Effect of loss offers on the FTR issuance*

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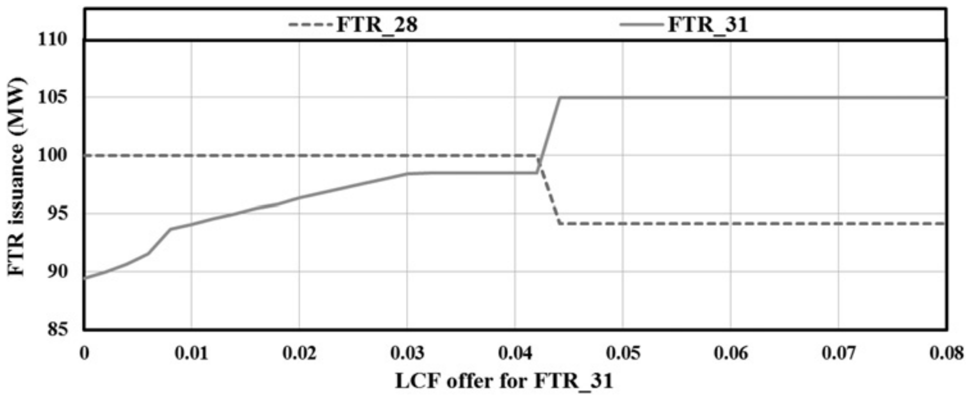

Fig. 7 *Illustration of the incentive for loss contribution*

in the lossy FTR auction is 1668.92 MW for Model 1 and 1689.72 MW for Model 2. Therefore, the reduction in FTR issuance is only 15.92% for Model 1 and 14.88%, for Model 2. Fig. 5 also reveals minimal need for the loss contribution.

In order to have a clearer view of the effect of loss offers on the total FTR volume issued, successive FTR auctions are conducted by gradually increasing the maximum LCF specifications of all the FTR requests in proportion to the reference specifications provided in Table 1. The result is plotted in Fig. 6. It is interesting to see that the FTR volume issued is highly sensitive to the LCF offer only at the low loss contribution levels. The sensitivity soon comes down as the LCF specification slightly moves away from zero.

To illustrate the incentive for loss contribution, a new lossy FTR request (referred to as FTR_31) of 105 MW is considered on Path 4-92. The bid price of the transportation component for the particular FTR request is set to 5.2 \$/MWh, which is higher than that of the other lossy FTR request on the same path (i.e. FTR_28). The offer price of the loss component is kept the same as that for FTR_28. The specifications of all other lossy FTR requests remain unaltered (as shown in Table 1). Fig. 7 presents the comparison between the cleared volumes of FTR_31 and FTR_28 for different maximum LCF specifications of FTR_31. Note that the bidder of FTR_28 wins over the bidder of FTR_31 even with a lower price quote in the case the latter is highly reluctant to provide loss contribution. The bidder of FTR_31 can, however, win over the bidder of FTR_28 with lower loss contribution because of its higher price quotation.

6 Conclusion

The proposition of lossy FTRs is reinvestigated in this paper for the possible practical implementation. Lossy FTRs retain the capability to provide perfect hedge against uncertain network usage charges under the marginal loss pricing. However, those are more complex instruments compared to conventional FTRs; therefore, no use of lossy FTRs is reported till date. This paper contributes towards designing a market platform for availing the benefit of lossy FTRs through a practical implementation. The complexities associated with the implementation of the lossy FTR mechanism are effectively dealt with. A suitable format for bilateral contracts is prescribed so that a physical power transaction can be perfectly matched to a lossy FTR irrespective of whether the transaction is executed by a load or generator or trader. It is recommended that a bilateral contract can be signed with a floating component that can be finalised periodically based upon the status of the FTR procurement. The detailed formulation of the lossy FTR auction problem is presented. A linear power flow model with stable loss parameters is prepared for the revenue adequate issuance of lossy FTRs. Both the AC and HVDC line flows are addressed in the linear power flow model prepared. The lossy FTR auction problem is suitably formulated in line of the existing FTR practice and in consistence with security-constrained dispatch scheduling. Different approaches for the representation of contingency constraints are addressed. Case studies confirm no market inefficiency with lossy FTRs.

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