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Abstract. Masonry is a heterogeneous anisotropic continuum, made up of the brick and mortar arranged in a periodic manner. Obtaining the effective elastic stiffness of the masonry structures has been a challenging task. In this study, the homogenization theory for periodic media is implemented in a very generic manner to derive the anisotropic global behaviour of the masonry, through rigorous application of the homogenization theory in one step and through a full three dimensional behavior. We have considered the periodic Eshelby self-consistent method and the finite element method. Two representative unit cells that represent the microstructure of the masonry wall exactly are considered for calibration and numerical application of the theory.

Keywords: Homogenization; Masonry; Periodic Composites, Self-Consistent Technique

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1. Introduction

Masonry is a heterogeneous anisotropic continuum, made up of the brick and mortar arranged in a periodic manner. In particular, the inhomogeneity is due to the different mechanical properties of its constituents namely the mortar and the brick. The anisotropy is due to the different masonry patterns, since the mechanical response is affected by the geometrical arrangement of the constituents. Understanding the in-plane load deformation characteristics is important for designing and retrofitting of masonry structures. The homogenization theory for periodic media allows determining global behavior of the masonry to be obtained from the behavior of its constituents.

The most detailed approach would be to make a discrete model by considering each brick and mortar joint in the masonry where linear and nonlinear material behavior of brick and mortar joint can be considered. Such an approach would be computationally expensive. The present study is on the composite behavior of masonry, in terms of determining averaged microscopic stress and strains so that the material can be assumed effectively as a homogeneous elastic continuum. Pande *et al.* (1989), Maier *et al.* (1991) and Pietruszczak and Niu (1992) introduced stepwise techniques for estimating the effective elastic properties of masonry. In these case studies the masonry was assumed to be a layered material. The homogenization procedure was performed in several steps with the head joints and bed joints being introduced successively. Such a methodology introduces several errors and the results generally depend on the order of the successive steps (Geymonat *et al.* (1987)). Also in these multi-step procedures, the geometrical arrangement is not fully taken into account, in the sense that, different bond patterns

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may lead to exactly the same result. For instance, running bond and stack bond would result in same homogenized properties. Moreover in some of these works the thickness of the masonry is not taken in to account and masonry is considered to be infinitely thin two dimensional media under plane stress assumptions being valid (Maier *et al.* (1991)).

Anthoine (1995), Urbanski *et al.* (1995) have applied the homogenization theory for periodic media rigorously to the basic cell of masonry to carry out a single step homogenization, with adequate boundary conditions and also considering exact geometry. As exact solutions are not possible, a semi analytical approach based on finite element method was used to obtain numerical solutions. A rigorous application of the homogenization theory for the non-linear behavior of the complex masonry basic cell implies solving the problem for all possible macroscopic loading histories, since the superposition principle does not apply anymore. Thus, for the complete determination of the homogenized constitutive law would require an infinite number of computations. There has also been other computationally very intensive works on use of asymptotic homogenization methods with finite element method (see Cecchi and Rizzi (2001)).

Many studies has been conducted on the homogenization of the masonry in recent years, Chang Yan (2003) conducted a study on a unified modeling approach for homogenization (forward) and de-homogenization (backward), applicable to unidirectional composite systems. Emphasis is placed on the uniqueness between the forward and the backward modeling processes. Mistler *et al.* (2007) focuses on the generalization of the homogenization procedure of the out-of-plane behavior of masonry, in such a way that the in-plane and out-of-plane characteristics of the homogeneous equivalent plate can be derived in one step. Zucchini and Lourenco (2007) contributes to the understanding of masonry under compression, using a novel non-linear homogenization tool that includes the possibility of tensile and compressive progressive damage, both in the unit and mortar. Gitman *et al.* (2007) investigated the representative volume element for different stages of the material response, including pre- and post-peak loading regimes. Gang Wang *et al.* (2007) introduced the effective elastic stiffness for periodic masonry structures via Eigen strain homogenization. Klusemann *et al.* (2010) a made a comparative study on the homogenization methods for multi-phase elastic composites based on Eshelby theory (Eshelby (1975) and Eshelby (1957)) and Mori Tanaka theory (Mori and Tanaka (1973)). The work was based on evaluation of elastic properties methodology proposed by Nemat – Nasser *et al.* (1982) for periodically distributed inclusions for spherical and cylindrical geometries. Elio Sacco (2009) presented a nonlinear homogenization procedure for periodic masonry, in which linear elastic constitutive relationship is considered for the blocks, while a new special nonlinear constitutive law is proposed for the mortar joints. This work has been extended by Daniela Addessi *et al.* (2010) to Cosserat model for periodic masonry, which accounts for the absolute size of the constituents and is derived by a rational homogenization procedure based on the transformation field analysis. A nonlinear homogenization technique to solve masonry structures problems is proposed by Quinteros *et al.* (2012) and describes the behaviour of brittle materials subjected to tension–compression cyclic loads based on the introduction of two damage variables and it assumes that the damage is due to the beginning and growth of cracks only in the mortar joints. Recently Norris *et al.* (2012) have proposed analytical formulation of three-dimensional dynamic homogenization for periodic elastic systems. Milani *et al* (2015) have estimated the in-plane failure surfaces for masonry with joints of finite thickness by a method of cells type

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approach. . Stefannou et al. (2015) provided a methodology for the estimation of overall strength of an in plane loaded masonry wall by accounting the failure of its bricks. A. Rekik et al. (2015) presented compressive behavior of Magnesia-Carbon mortar less joint using digital image correlation method. E. Reccia et al. (2014) proposed a procedure for evaluation of nonlinear behavior of masonry arch bridges. The 3D behavior of Venice trans lagoon masonry arch bridge subjected to train loads and pile foundation settlement is investigated. Alessandri et al. (2014) presented the advanced finite element homogenization strategy for failure analysis of double curvature masonry elements. D. Hu et al. (2013) using FE homogenization analysis established micro mechanics models of masonry basic cell to obtain homogenized Young's modulus and Poisson ratio. Travalusci et al. (2014) presented mechanical behavior of brick masonry through equivalent continua. G. Milani et al. (2013) presented a methodology using homogenization approach for the evaluation of out of plane strength for quasi periodic masonry. Milani et al. (2013) proposed aa simplified kinematic procedure to obtain in plane elastic moduli and macroscopic masonry strength domains in the case of Herringbone masonry. Yuan et al. (2013) presented Fourier based incremental homogenization of coupled unilateral damage plasticity model for masonry structures. Chettah et al. (2013) used transformation yield analysis for the multi scale analysis of cracking localization in masonry. G. Milani et al. (2013) presented a simple numerical model with second order effects for out of plane loaded masonry walls. Lucciano et al. (1997) presented micro mechanical approach in order to define the properties of a periodic masonry material. A. Gabor et al. (2006) presented a new finite element modelling approach for the analysis of behavior of unreinforced and FRP strengthened masonry walls subjected to a predominant shear load. R. Lucciano et al. (1998) introduced variational principles for the overall properties of composite materials with periodic micro structures. Alfano et al. (2006) presented a new method to combine interface damage and friction in a cohesive zone model. Marfia et al. (2001) developed a micromechanical investigation for the evaluation of the overall properties of the masonry material reinforced by innovative composite material. Sacco et al. (2012) presented a micro mechanical model which is able to couple damage evolution, the non-penetration conditions and the friction effect. Toti et al. (2013) presented modelling of detachment mechanisms of fiber reinforced polymers from quasi brittle materials. Fouchal et al. (2009) modelled the mechanical behavior of interfaces in masonry structures. Characteristics of the materials and interfaces are determined experimentally. Rekik et al. (2010) proposed a methodology for the identification of the representative crack length evolution in a multi-level interface model for quasi brittle masonry. Rekik et al. (2012) presented a homogenization method using asymptotic techniques and finite element method for the interface modelling in damaged masonry. Nasedkin (2015) et al have made a nonlinear homogenization procedure for the finite element analysis.

The main objective of this study is to derive the global behavior of the masonry, through rigorous application of the homogenization theory in one step and through a full three dimensional behavior. In this work, we propose to implement a one-step micromechanical homogenization technique based on the periodic Eigen strain method to model masonry structures. In this method the periodicity of the variables is imposed by Fourier series. The microstructural details of the bricks and mortar can be accurately described. Using a strain energy approach the effective properties of the masonry are derived. For validation, two different periodic unit cells models are considered and elastic properties are determined for a stress or strain prescribed analysis using finite element method. The paper is organized as follows: Section 2 gives an overview of homogenization theory of periodic media. Section 3 gives the details of equivalent Eigen strain approach. Section 4 discusses the finite element implementation together with the results and discussion. In Section 5 summary and conclusions are presented.

2. Homogenization theory for periodic media

The theory of homogenization allows global behavior of the periodic media to be derived from the behavior of its constituents. In this section theory of homogenization is presented in a very generic way for the three dimensions by using basic mechanics and mathematics. Since the theory will be applied to the masonry, a half brick thick wall is considered for the analysis.

2.1 Description of periodicity

Consider a portion $\mathbf{\Omega}$ of a masonry wall (see in Fig. 1). It is a two dimensional periodic composite continuum, in which brick and mortar are arranged in the running bond. This periodicity can be characterized by a frame of reference $(\alpha_1,\alpha_2,\alpha_3)$, where$\alpha_1$, α_2 and α_3 are independent vectors of a basic cell $\widehat{\Omega}$ such that property of masonry can be expressed in terms of the independent variables. The basic cell is considered such that the masonry domain can be generated by repeating the cells in e_1 and e_2 direction. Since finite element calculations are to be performed on the cell, it is preferred to choose cell with least volume and with symmetries. The choice of the cell depends on the arrangement of the brick and mortar for the masonry. For the half brick thick wall, a simplest basic cell is made up of one brick surrounded by half mortar joint, the masonry property in periodic direction can be express as $\alpha_1\beta_1 + \alpha_2\beta_2$, where α_1, α_2 having the zero component of the e_3;whereas α_3 have only e_3 i.e. thickness of basic cell $\widehat{\Omega}$

where β_1 and β_2 are integers. The reference frame for the half brick wall may be written as

$$\alpha_1 = 2le_1 \quad (1)$$

$$\alpha_2 = dle_1 + 2he_2 \quad (2)$$

$$\alpha_3 = 2we_3 \quad (3)$$

where $2l$ is equal to the length of the brick plus the thickness of the head joint, $2w$ is thickness of masonry, $2h$ is equal to the height of the brick plus the thickness of the bed joint and d is the overlapping. $d = 0$, gives stack bond and $d = 1$ gives running bond. For the more complex geometry, masonry would require larger basic cell, i.e. cell involving more than one brick.

In the boundary surface of the three-dimensional basic cell, two different regions may be separated (Fig. 1) as $\partial\widehat{\Omega}_i$ which is internal to the wall (interfaces with adjacent cells) and $\partial\widehat{\Omega}_{\boldsymbol{e}}$ which is external (lateral faces). $\partial\widehat{\Omega}_i$ can be divided into three pairs of identical sides (due to periodicity in e_1 ande_2) corresponding to each other through a translation alongα_1, α_2 or $\alpha_1 - \alpha_2$ (opposite sides), where two pairs (onlyα_1, α_2) are of identical sides in the case of stack bond pattern. As there is no periodicity in e_3 direction thus the two lateral faces of $\partial\widehat{\Omega}_{\boldsymbol{e}}$ are just opposite sides of the cell.

Now, suppose the portion $\boldsymbol{\Omega}$ of masonry is subjected to a globally (macroscopically) homogeneous stress state (see in Fig. 2). A stress state is said to be globally or macroscopically homogeneous over a domain $\boldsymbol{\Omega}$ if all basic cells within $\boldsymbol{\Omega}$ undergo the same loading conditions. This can be achieved by applying biaxial principal stress state to the domain. A cell lying nears the boundary $\partial\boldsymbol{\Omega}$ of the specimen is not subjected to the same loading as one lying in the centre. However, on account of the Saint-Venant principle, all cells lying far enough from the boundary

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are subjected to the same loading conditions and therefore deform in the same way. In particular, two joined cells must still fit together in their common deformed state. Means that, condition (i) stress compatibility and condition (ii) strain compatibility must be satisfied on the internal boundary $\partial\widehat{\Omega}_i$, whereas the external boundaries $\partial\widehat{\Omega}_{\boldsymbol{e}}$ remain stress free.

If we are passing from a cell to the next cell, which is identical to first one, this means passing from a side to the opposite one in the same cell $\widehat{\Omega}$, then the condition (i) becomes stress vectors $\boldsymbol{\sigma}.\mathbf{n}$ are opposite on opposite sides of $\partial\widehat{\Omega}_i$ because external normal $\mathbf{n}$ are also opposite. Such a stress field $\boldsymbol{\sigma}$ is said to be periodic on $\partial\widehat{\Omega}_i$, whereas the external normal $\mathbf{n}$ and the stress vector $\boldsymbol{\sigma}.\mathbf{n}$ are said to be anti-periodic on $\partial\widehat{\Omega}_i$. For the condition (ii), it is necessary that opposite sides can be superimposed in their deformed states without separation or overlapping. The displacement fields on two opposite sides must be a rigid displacement. Any strain periodic displacement field u can be written in the following way

$$u_i(x_1, x_2, x_3) = \delta_{ij} E_{jk} \delta_{kl} x_l + u_i^p(x_1, x_2, x_3) \tag{4}$$

where $\boldsymbol{E}$ is a symmetric second-order tensor for the strain; δ_{ij} is the Kronecker delta; u^p is a periodic displacement field and x_1, x_2, x_3 are the spatial parameters. In particular, the anti-symmetric part of $\boldsymbol{E}$ corresponds to a rigid rotation of the cell. And thus only the symmetric part of $\boldsymbol{E}$ is considered (rigid displacements are disregarded) with the intuitive definition of the average of a quantity on the cell. The average of strain can be written as

$$\bar{\varepsilon}_{ij}(u) = \frac{1}{|\widehat{\Omega}|} \int_{|\widehat{\Omega}|} \varepsilon_{ij}(u)\, d\widehat{\Omega} \tag{5}$$

where $|\widehat{\Omega}|$ stands for the volume of the basic cell. Similarly, for consistency of the equation-

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with the stress. The average of the stress on the cell should be given by

$$\bar{\sigma}_{ij} = \frac{1}{|\widehat{\Omega}|}\int_{|\widehat{\Omega}|} \sigma_{ij}\, d\widehat{\Omega} \qquad (6)$$

2.2 *Homogenization*

Let us still consider the problem of a masonry specimen subjected to a macroscopically homogeneous stress state $\mathbf{\Sigma}$. The above conditions (conditions (i) and (ii) in the Section 2.1) make it possible to study the problem within a single cell (unit cell) of the domain rather than on the whole domain. In order to find $\boldsymbol{\sigma}$ and $\boldsymbol{u}$ everywhere in a cell, equilibrium conditions and constitutive relationships must be added so that the problem can be solved. The required equation can be written as

$$\mathrm{div}\boldsymbol{\sigma} = 0 \; on \; \widehat{\Omega} \text{ (No body forces)} \qquad (7)$$

$$\boldsymbol{\sigma} = \mathrm{F}\big(\boldsymbol{\varepsilon}(\mathbf{u})\big) \text{ (Complete constitutive law)} \qquad (8)$$

$$\boldsymbol{\sigma}.\mathbf{n} = \mathbf{0} \text{ on } \partial\widehat{\Omega}_e \qquad (9)$$

$$\bar{\boldsymbol{\sigma}} = \mathbf{\Sigma} \quad \text{Where } \mathbf{\Sigma} \text{ is given, for stress controlled loading} \qquad (10)$$

where the constitutive law $\mathrm{F}\big(\boldsymbol{\varepsilon}(\mathbf{u})\big)$ is a periodic function of the spatial variable $\mathbf{x}$, and describes the behaviour of the different materials in the composite cell. A problem similar to Eq. (10) is obtained when replacing the stress controlled loading by a strain controlled one

$$\bar{\boldsymbol{\varepsilon}} = \boldsymbol{E} \quad \text{Where } \boldsymbol{E} \text{ is a given value for displacement and is controlled during loading} \qquad (11)$$

In both cases, the resolution of Eqs. (10) and (11) are sometimes termed *localization* because the local (microscopic) fields $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}(\mathbf{u})$ are determined from the global (macroscopic) quantities $\mathbf{\Sigma}$ or $\boldsymbol{E}$. The averaging procedure relating these quantities can be written as

$$\overline{\boldsymbol{\sigma}:\boldsymbol{\varepsilon}(u)} = \overline{\boldsymbol{\sigma}}:\bar{\boldsymbol{\varepsilon}}(u) = \boldsymbol{\Sigma}:\boldsymbol{E} \tag{12}$$

where [¯] defines the average of the quantity over the unit cell. Once we get $\boldsymbol{\sigma}$ and $\boldsymbol{u}$, then the missing macroscopic quantity $\boldsymbol{\Sigma}$ or $\boldsymbol{E}$ can be determined through the average relation given in Eq. (12). Homogenization theory can only be applied when the loads are homogeneous in nature or the variation of the loads from one unit cell to another is very small. In practice, this is satisfied, if the size of the unit cell is very small in comparison to the structure and thus any two adjacent cells, will have almost the same position, and will therefore undergo almost the same loading.

3. Computational Homogenization

For the periodic masonry structure, a unit cell that represents a unit of periodicity in the horizontal and vertical directions is considered. The unit cell $(\widehat{\Omega})$ is a microstructured composite. Unlike the representative volume element (RVE) where in the inclusion distribution can only be accounted for in a statistically uniform manner, in case of a unit cell the microstructure can be described exactly and is shown in Fig. 3, it can be as per periodic unit cell 1 or periodic unit cell 2 and it contains one complete brick unit and mortar joints between the bricks. The bricks are considered to be inclusions (L) and the mortar is treated as a matrix (M). We denotes $\boldsymbol{C}^{\mathrm{L}}$ and $\boldsymbol{C}^{M}$ as the stiffness of inclusion and matrix, $\boldsymbol{S}^{\mathrm{L}}$ and $\boldsymbol{S}^{M}$ denotes the compliance of inclusion and matrix.

3.1 Constitutive relation for unit cell

Let a displacement field $\mathbf{u}^0$ be prescribed on the boundary of the periodic unit cell. The

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displacement field induces a constant strain field $\boldsymbol{E}$ in a homogeneous material, i.e.

$$\mathbf{u}^0 = \mathbf{x} \cdot \boldsymbol{E} \tag{13}$$

Because of the presence of the inclusions, the total strain ε within the unit cell is the addition of the constant strain ε^0 prescribed and a disturbance strain field $\varepsilon^d(\mathrm{x})$, which is unknown at present and to be determined later. Assuming both inclusions and matrix are linearly elastic, the total stress in inhomogeneous unit cell is a superposition of a homogenous state and a perturbed state as shown in Fig. 3 and is given by

$$\sigma(\mathrm{x}) = \begin{cases} \boldsymbol{C}^L : [\varepsilon^0 + \varepsilon^d(\mathrm{x})], & \mathrm{x} \in L \\ \boldsymbol{C}^M : [\varepsilon^0 + \varepsilon^d(\mathrm{x})], & \mathrm{x} \in \mathrm{M} \end{cases} \tag{14}$$

To equivalently account for the presence of the second phase (inclusion), an equivalent Eigen strain field ε^*_{ij}, is introduced, which would produce a compatible deformation field without generating stresses. The stress consistency condition requires that

$$\sigma_{ij}(\mathrm{x}) = \begin{cases} C^{\mathrm{L}}_{ijkl}\left[\varepsilon^0_{\mathrm{kl}} + \varepsilon^{\mathrm{d}}_{\mathrm{kl}}(\mathrm{x})\right] = C^M_{ijkl}\left[\varepsilon^0_{\mathrm{kl}} + \varepsilon^{\mathrm{d}}_{\mathrm{kl}}(\mathrm{x}) - \varepsilon^*_{\mathrm{kl}}(\mathrm{x})\right], & \mathrm{x} \in L \\ C^M_{ijkl}\left[\varepsilon^0_{\mathrm{kl}} + \varepsilon^{\mathrm{d}}_{\mathrm{kl}}(\mathrm{x})\right], & \mathrm{x} \in M \end{cases} \tag{15}$$

Note that the Eigen strain is only prescribed on the inclusion in Eq. (15). We can extend the definition of the Eigen strain to the whole unit cell $\widehat{\Omega}$ as

$$\varepsilon^*_{ij}(\mathrm{x}) = \begin{cases} \varepsilon^*_{ij}(\mathrm{x}), & \mathrm{x} \in \mathrm{L} \\ 0, & \mathrm{x} \in M \end{cases} \tag{16}$$

So Eq. (27) can be rewritten in a unified fashion over $\widehat{\Omega}$

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$$\sigma_{ij}(\mathrm{x}) = C^{M}_{ijkl}\left[\varepsilon^{0}_{kl} + \varepsilon^{d}_{kl}(\mathrm{x}) - \varepsilon^{*}_{kl}(\mathrm{x})\right] \qquad \mathrm{x} \in \widehat{\Omega} \tag{17}$$

Assuming no body force, the stresses satisfy the following equilibrium condition

$$\sigma_{ij,j}(\mathrm{x}) = C^{M}_{ijkl}\left[\varepsilon^{0}_{kl} + \varepsilon^{d}_{kl}(\mathrm{x}) - \varepsilon^{*}_{kl}(\mathrm{x})\right]_{,j} \qquad \mathrm{x} \in \widehat{\Omega} \tag{18}$$

3.2 Eshelby self-consistent approach

The periodic Eigen strain formulation (Nemat- Nasser *et al.* 1982) which is based on Ehselby's Eigen strain method (Eshelby (1975) and Eshelby (1957)) for solids with periodic structures is has been adopted by Wang et al (2007) to obtain the disturbance strain and displacements. Since masonry has a periodic structure, the disturbance displacement field and Eigen strain field are also periodic. These periodic fields using the Fourier series as given in Wang *et al.* (2007) and find the equivalent properties... Following Wang et al (2007), the disturbance strain field can be written as

$$\varepsilon^{d}_{ij} = \sum_{\xi\in\wedge'} f_{\mathrm{L}} \cdot g_0(\xi) g_0(-\xi) g_{ijmn}(\xi)\varepsilon^{*}_{mn} = D^{\mathrm{L}}_{ijmn}\varepsilon^{*}_{mn} \tag{19}$$

and the Eshelby tensor for periodic unit cell is determined as

$$D^{L}_{ijmn} = \sum_{\xi\in\wedge'} f_{\mathrm{L}} \cdot g_0(\xi) g_0(-\xi) g_{ijmn}(\xi) \tag{20}$$

Which plays the same role as the Eshelby tensor for an inclusion in an infinite space (Eshelby (1957) and Eshelby (1975)). However the tensor we obtained is for the unit cell, which is finite in size. The tensor conveys the microstructure details of the matrix and inclusion within the unit cell and it is represented by an infinite series.

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The effective stiffness of the masonry can be obtained as given in Wang et al (2007) as

$$\boldsymbol{C}^{hom} = \boldsymbol{C}^{M}: \left[\mathbf{1}^{(4S)} - f_L(\boldsymbol{A}^L - \boldsymbol{D}^L)^{-1}\right] \tag{21}$$

Where $\mathbf{1}^{(4S)}$ fourth order identity tensor; and f_{L} is the volume fraction of inclusions. Similarly, the effective compliance is exactly the inverse of the above.

$$\boldsymbol{S}^{hom} = \boldsymbol{S}^{M}: \left[\mathbf{1}^{(4S)} - f_{\mathrm{L}}(\boldsymbol{A}^{\mathrm{L}} - \boldsymbol{D}^L)^{-1}\right] \tag{22}$$

3.4 Example for the periodic Eigen strain model.

For validation of the above Eigen strain model we consider a test example. Here in the brick and mortar are assumed to be isotropic. The Young's modulus of the brick (E_b) and the Poisson ratio (γ_b) are 11,500 Mpa and 0.15, respectively. The ratio of the Young's moduli of the brick over the mortar E_b/E_m ranges from 1 to 12 and Poisson ratio for the mortar (γ_m) is 0.23. The brick dimensions are 225mm (length) × 50mm (height). As it is seen from Fig 4 the mortar stiffness and thickness greatly influence the overall properties of masonry. The values asymptotically decrease with increase in thickness of mortar. A comparison of the results is made with those obtained from finite element method. The results obtained from Unit cell 2 (as described in next section) are compared with the Eigen strain model and are found to match closely with the analytical results.

4. Finite element Analysis for determining the homogenous properties

In particular, the two unit cells are taken from a periodic basic cell extracted from a single leaf masonry wall in running bond, as shown in the Fig. 5. The assigned dimensions to

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both the unit cell is such that volume fraction of the mortar and brick remains same. Brick and mortar are assumed to be isotropic: the Young's moduli and Poisson ratios are 2 x 10^5 MPa and 0.15 for the brick, 2 x 10^4 MPa and 0.15 for the mortar respectively. The brick dimensions are 210 x 50 x 100 mm (length x height x thickness). Head and bed mortar joints are 10 mm.

The study of elastic response of the model is done, for a generic loading condition as linear combination of the elastic responses for six elementary loading conditions. Both stress-prescribed and displacement-prescribed analyses have been carried out in the present work. The finite element model which has been used in numerical analysis is given below in Fig. 4. 8 noded linear brick element with reduced integration is used for the simulation. The structured mesh was obtained by taking into account a maximum element size of 0.5 cm. thus the unit cell 1 and unit cell 2 have 5280 and 21120 elements respectively.

4.1 Stress prescribed analysis

In the stress-prescribed analysis, the overall compliance tensor is to be obtained by means of six numerical analysis i.e. XX-compression, YY-compression, ZZ-compression, XY-shear, XZ-shear, and YZ-shear. Periodic boundary conditions for all the six numerical analysis are applied as per the Section 2.2. An anisotropic mechanical behaviour is considered, thus stress strain relationship can be written in the following form

$$\begin{bmatrix}\overline{\varepsilon_1}\\\overline{\varepsilon_2}\\\overline{\varepsilon_3}\\\overline{\varepsilon_4}\\\overline{\varepsilon_5}\\\overline{\varepsilon_6}\end{bmatrix} = \begin{bmatrix}\bar{S}_{11} & \bar{S}_{12} & \bar{S}_{13} & \bar{S}_{14} & \bar{S}_{15} & \bar{S}_{16}\\ \bar{S}_{21} & \bar{S}_{22} & \bar{S}_{23} & \bar{S}_{24} & \bar{S}_{25} & \bar{S}_{26}\\ \bar{S}_{31} & \bar{S}_{32} & \bar{S}_{33} & \bar{S}_{34} & \bar{S}_{35} & \bar{S}_{36}\\ \bar{S}_{41} & \bar{S}_{42} & \bar{S}_{43} & \bar{S}_{44} & \bar{S}_{45} & \bar{S}_{46}\\ \bar{S}_{51} & \bar{S}_{52} & \bar{S}_{53} & \bar{S}_{54} & \bar{S}_{55} & \bar{S}_{56}\\ \bar{S}_{61} & \bar{S}_{62} & \bar{S}_{63} & \bar{S}_{64} & \bar{S}_{65} & \bar{S}_{66}\end{bmatrix} \cdot \begin{bmatrix}\bar{\sigma}_1\\\bar{\sigma}_2\\\bar{\sigma}_3\\\bar{\sigma}_4\\\bar{\sigma}_5\\\bar{\sigma}_6\end{bmatrix} \tag{23}$$

Where the superscript [‾] means that the above written quantities refer to the average values within the considered unit cell.

Compliance tensor is obtain by applying the six loading conditions one at a time, only a single column of the compliance tensor is obtain by applying one loading condition out of the six. By applying the average theorem to the unit cell and using the Eqs. (5)- (6), the following relation is obtained for the Compliance tensor and the average stress value in the unit cell

$$\bar{S}_{ij} = \frac{\bar{\varepsilon}_i}{\bar{\sigma}_j} \qquad (24)$$

$$\bar{\sigma}_i = \frac{1}{|\widehat{\Omega}|}\int_{|\widehat{\Omega}|} \sigma_i \, d\widehat{\Omega} = \boldsymbol{\Sigma} \qquad (25)$$

Where i, j =1, 2, 3, 4, 5, 6; and $|\widehat{\Omega}|$ stands for the volume of the unit cell and $\boldsymbol{\Sigma}$ is the generic stress-prescribed component. The average value of strain within the unit cell is obtained as

$$\bar{\varepsilon}_i = \sum_{e=1}^{n} \frac{\bar{\varepsilon}_i^{(e)}}{n} \qquad (26)$$

Where n = number of elements in the unit cell (uniformly discretized); $\bar{\varepsilon}_i^{(e)}$ = the average value of i^{th} strain component for generic element. The average value of stress within the unit cell is obtained as

$$\bar{\sigma}_j = \sum_{e=1}^{n} \frac{\bar{\sigma}_j^{(e)}}{n} \qquad (27)$$

Where $\bar{\sigma}_j^{(e)}$ = the average value of j^{th} strain component for generic element. Hence, all six stress states are applied one by one to both the unit cell and the single columns of Compliance tensor is obtained by using the Eq. (51) and the corresponding coefficient of Compliance tensor are given below

a) Unit Cell 1

$$\bar{S}_{ij} = 10^{-6}\begin{bmatrix} 8.37 & -1.17 & -0.99 & 0 & 0 & 0 \\ -1.17 & 13.01 & -1.08 & 0 & 0 & 0 \\ -0.99 & -1.08 & 7.16 & 0 & 0 & 0 \\ 0 & 0 & 0 & 15.88 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8.17 & 0 \\ 0 & 0 & 0 & 0 & 0 & 15.9 \end{bmatrix}$$

b) Unit Cell 2

$$\bar{S}_{ij} = 10^{-6}\begin{bmatrix} 7.97 & -1.11 & -0.96 & 0 & 0 & 0 \\ -1.11 & 12.78 & -1.06 & 0 & 0 & 0 \\ -0.96 & -1.06 & 7.11 & 0 & 0 & 0 \\ 0 & 0 & 0 & 15.82 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8.65 & 0 \\ 0 & 0 & 0 & 0 & 0 & 14.82 \end{bmatrix}$$

4.2 Displacement prescribed Analysis

In the displacement-prescribed analysis, the aim is to find out stiffness tensor by means of six numerical analysis i.e. XX-compression, YY-compression, ZZ-compression, XY-shear, XZ-shear, and YZ-shear. Periodic boundary conditions for the six numerical analysis are applied. An anisotropic mechanical behaviour is considered, thus stress strain relationship can be written in the following form

$$\begin{bmatrix}\bar{\sigma}_1\\ \bar{\sigma}_2\\ \bar{\sigma}_3\\ \bar{\sigma}_4\\ \bar{\sigma}_5\\ \bar{\sigma}_6\end{bmatrix} = \begin{bmatrix}\bar{C}_{11} & \bar{C}_{12} & \bar{C}_{13} & \bar{C}_{14} & \bar{C}_{15} & \bar{C}_{16}\\ \bar{C}_{21} & \bar{C}_{22} & \bar{C}_{23} & \bar{C}_{24} & \bar{C}_{25} & \bar{C}_{26}\\ \bar{C}_{31} & \bar{C}_{32} & \bar{C}_{33} & \bar{C}_{34} & \bar{C}_{35} & \bar{C}_{36}\\ \bar{C}_{42} & \bar{C}_{42} & \bar{C}_{43} & \bar{C}_{44} & \bar{C}_{45} & \bar{C}_{46}\\ \bar{C}_{52} & \bar{C}_{52} & \bar{C}_{53} & \bar{C}_{54} & \bar{C}_{55} & \bar{C}_{56}\\ \bar{C}_{62} & \bar{C}_{62} & \bar{C}_{63} & \bar{C}_{64} & \bar{C}_{65} & \bar{C}_{66}\end{bmatrix} . \begin{bmatrix}\bar{\varepsilon}_1\\ \bar{\varepsilon}_2\\ \bar{\varepsilon}_3\\ \bar{\varepsilon}_4\\ \bar{\varepsilon}_5\\ \bar{\varepsilon}_6\end{bmatrix} \qquad (28)$$

Where the superscript [‾] means that the above written quantities refer to the average values within the unit cell. Stiffness tensor is obtain by applying the six loading conditions one at a time, only a single column of the Stiffness tensor is obtain by applying one loading condition out of the six. By applying the average theorem to the unit cell and using the Eqs. (5)- (6), the following relation is obtained for the average strain value in the unit cell

$$\bar{\varepsilon}_i = \frac{1}{|\widehat{\Omega}|}\int_{|\widehat{\Omega}|} \varepsilon_i \, d\widehat{\Omega} = \boldsymbol{E} \qquad (29)$$

Where i =1, 2, 3, 4, 5, 6; and $|\widehat{\Omega}|$ stands for the volume of the unit cell and $\boldsymbol{E}$ is the generic strain component, such that $\bar{\varepsilon}_i . x = u_i^0$, Where u_i^0 is a prescribed displacement on the boundary of the unit cell. The average theorem yields the following relation to obtain the stiffness tensor

$$\bar{C}_{ij} = \frac{\bar{\sigma}_i}{\bar{\varepsilon}_j} \qquad (30)$$

Where i, j =1, 2, 3, 4, 5, 6. The average value of stress within the unit cell is obtained as

$$\bar{\sigma}_i = \sum_{e=1}^{n} \frac{\bar{\sigma}_i{}^{(e)}}{n} \qquad (31)$$

Where n = number of element in the basic cell (uniformly discretized), $\bar{\sigma}_i^{(e)}$ = the average value of i^{th} strain component for generic element. The average value of strain within the unit cell volume is obtained as:

$$\bar{\varepsilon}_j = \sum_{e=1}^{n} \frac{\bar{\varepsilon}_j^{(e)}}{n} \quad (32)$$

Where $\bar{\varepsilon}_j^{(e)}$ = the average value of j^{th} strain component for generic element. Hence, all six displacement states are applied one by one to both the unit cell and the single columns of Compliance tensor is obtained by using the Eq. (57) and the corresponding coefficient of Compliance tensor are given below

a) Unit cell 1

$$\bar{C}_{ij} = 10^6 \begin{bmatrix} 0.14 & 0.01 & 0.02 & 0 & 0 & 0 \\ 0.01 & 0.08 & 0.01 & 0 & 0 & 0 \\ 0.02 & 0.01 & 0.17 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.07 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.14 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.07 \end{bmatrix}$$

b) Unit cell 2

$$\bar{C}_{ij} = 10^6 \begin{bmatrix} 0.14 & 0.01 & 0.02 & 0 & 0 & 0 \\ 0.01 & 0.08 & 0.01 & 0 & 0 & 0 \\ 0.02 & 0.01 & 0.17 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.07 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.13 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.07 \end{bmatrix}$$

4.3 Results and discussion

To study the effects of mortar moduli on the homogenized equivalent material properties of the unit cells, various stiffness ratios are considered for the analysis. This allows assessing the nonlinear behavior of the model for various ratios. The material properties of the unit are kept constant and the properties of the mortar are varied. In particular, for the unit, the Young's modulus $\mathrm{E_b}$ is taken as 20GPa and the Poisson's ratio is taken as 0.15. For the mortar, the Young's modulus is varied to yield a ratio of $\mathrm{E_b/E_m}$ value ranging from 1 to 1000. A graph has been plotted for different stiffness ratios between mortar and unit, to study the effects of mortar moduli on the homogenized equivalent material properties of the unit cells. The results are shown in Fig. 6, Fig. 7 and Fig. 8 for a stress prescribed analysis and in Fig. 9 and Fig. 10 for a displacement prescribed analysis.

To study the effect of different configurations of unit cells, two different basic cells of same masonry having the same volume of its constituent are considered for the analysis. By comparing the stiffness and compliances matrices of two basic cells it is observed that the values are all most similar and indicate that the unit cell approach can exactly represent the microstructure and similar equivalent properties are obtained in both cases. Moreover, the plot for stress and displacement prescribed analysis are also observed to be similar in both the cases. In particular it is observed that the equivalent material properties such as Young's modulus, shear modulus and Poisson's ratio decrease with increase in the ratio of Young's modulus of unit and mortar (i.e. with decrease in Young's modulus of mortar).

It is observed that the equivalent Young's modulus E_x , E_y of masonry decreases asymptotically and reaches a constant state with increase in the ratio of Young's modulus of unit and mortar. The equivalent Young's modulus E_z (for out of plane direction) reduces rapidly till the stiffness ratio of 10 and subsequently becomes almost a constant. It is also

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observed that the shear modulus of the masonry also decreases with increase in the ratio of shear moduli and reaches a constant value. The rate of shear modulus G_{yz} deceases when mortar strength gets exhausted and the softening rate of equivalent shear modulus G_{yz} is governed by the softening of shear modulus of the unit only. Finally, a similar behaviour is observed for the Poisson's ratio, that equivalent Passion's ratio also decreases with increase in the ratio of moduli of brick and mortar.

5. Conclusions

The homogenization theory for periodic media has been applied in a rigorous way for deriving the elastic characteristics of masonry that is in one step. In particular, the real geometry has been taken into account (bond pattern and finite thickness of the wall). The microstructural details of the bricks and mortar can be accurately described. Using a strain energy approach the effective properties of the masonry are derived. For validation two different periodic unit cells models are considered and elastic properties are determined for a stress or strain prescribed analysis using Finite element method. A one-step micromechanical homogenization technique based on the periodic Eigen strain method to model masonry structures available from literature has been implemented and the results are compared with finite element solution. The analytical solutions match very well with the numerical solution.

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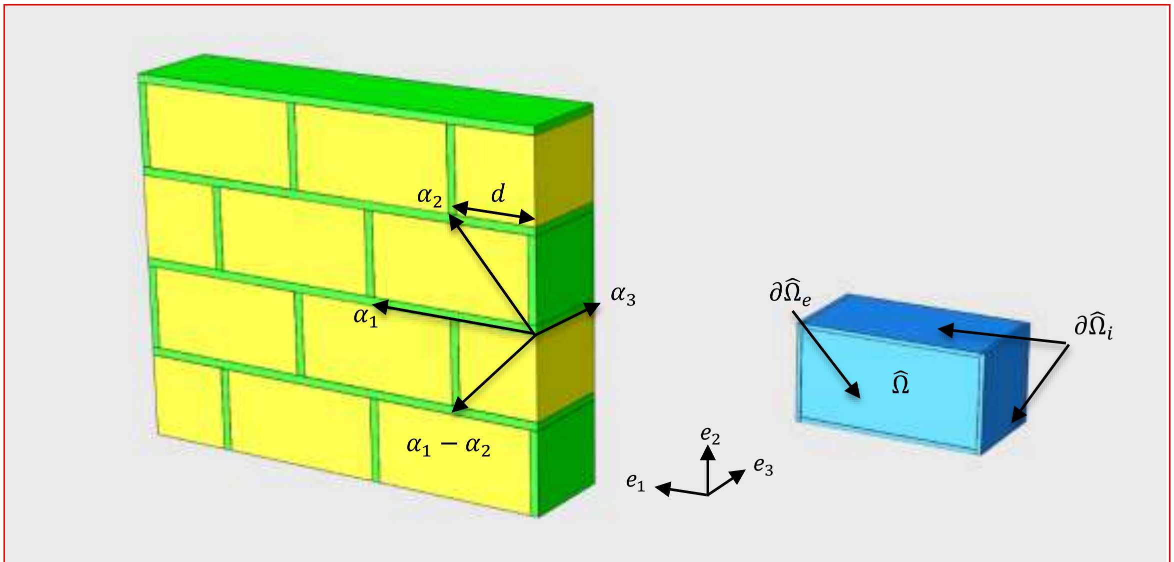


Fig. 1 Half brick thick masonry wall in running bond with frame of reference (left) and corresponding three dimension basic cell (right)

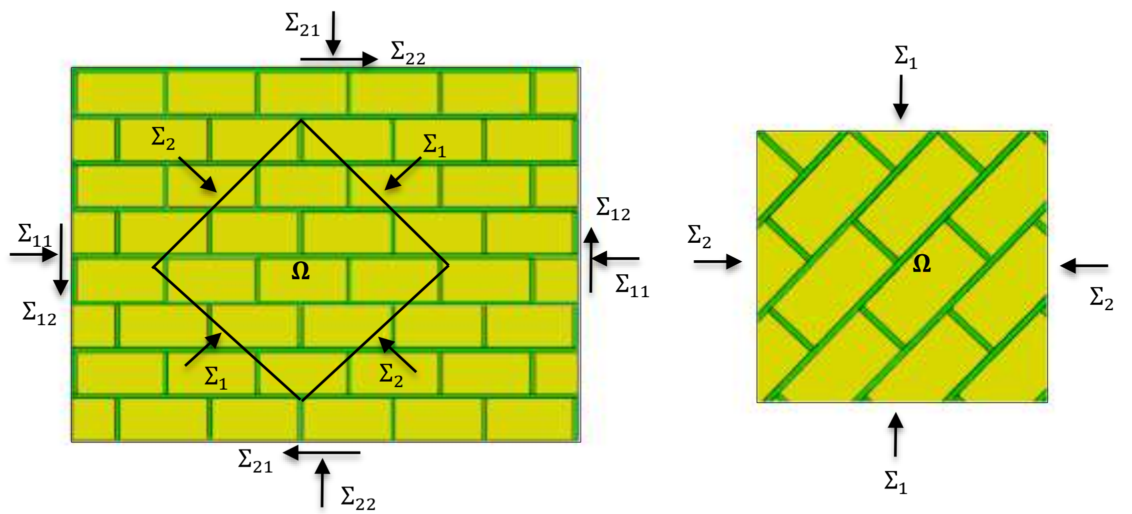


Fig. 2 Half brick thick masonry wall subjected to macroscopically homogeneous stress state $\mathbf{\Sigma}$

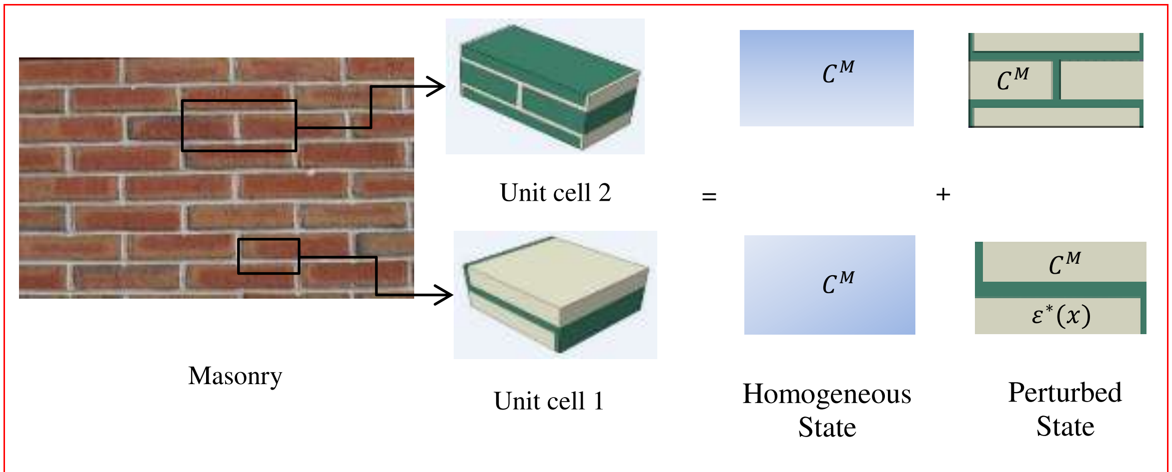
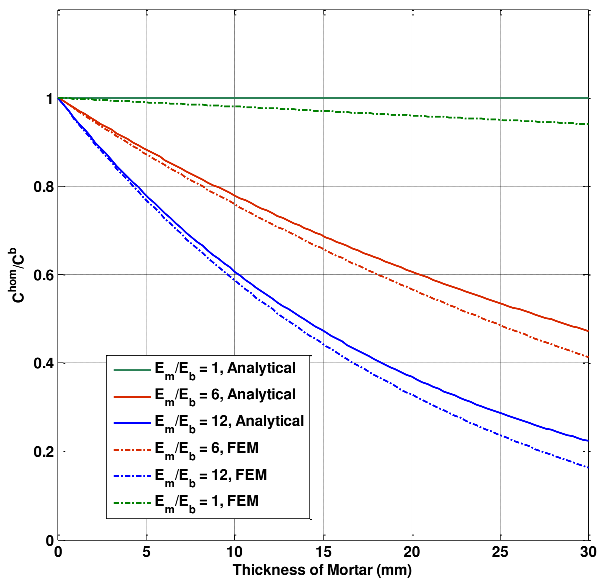
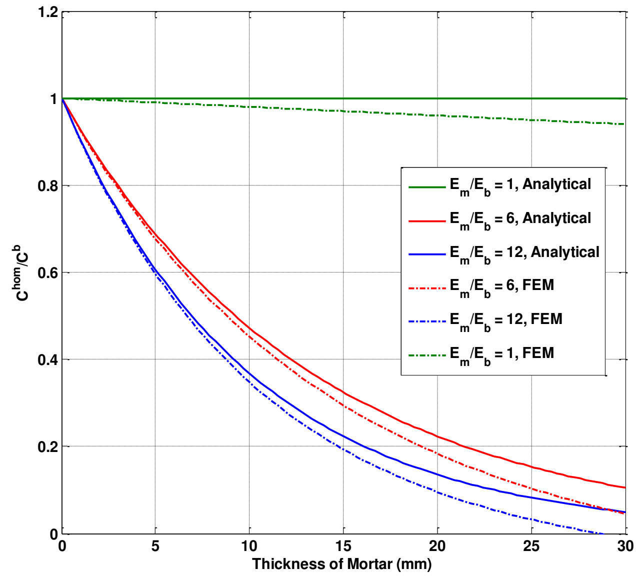


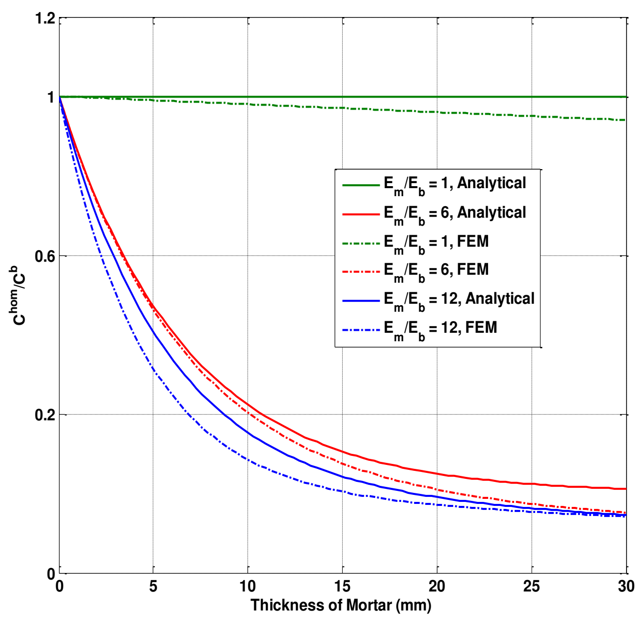
Fig. 3 Chosen micro mechanical models indicating the Homogeneous and Perturbed State



(a) For component 11

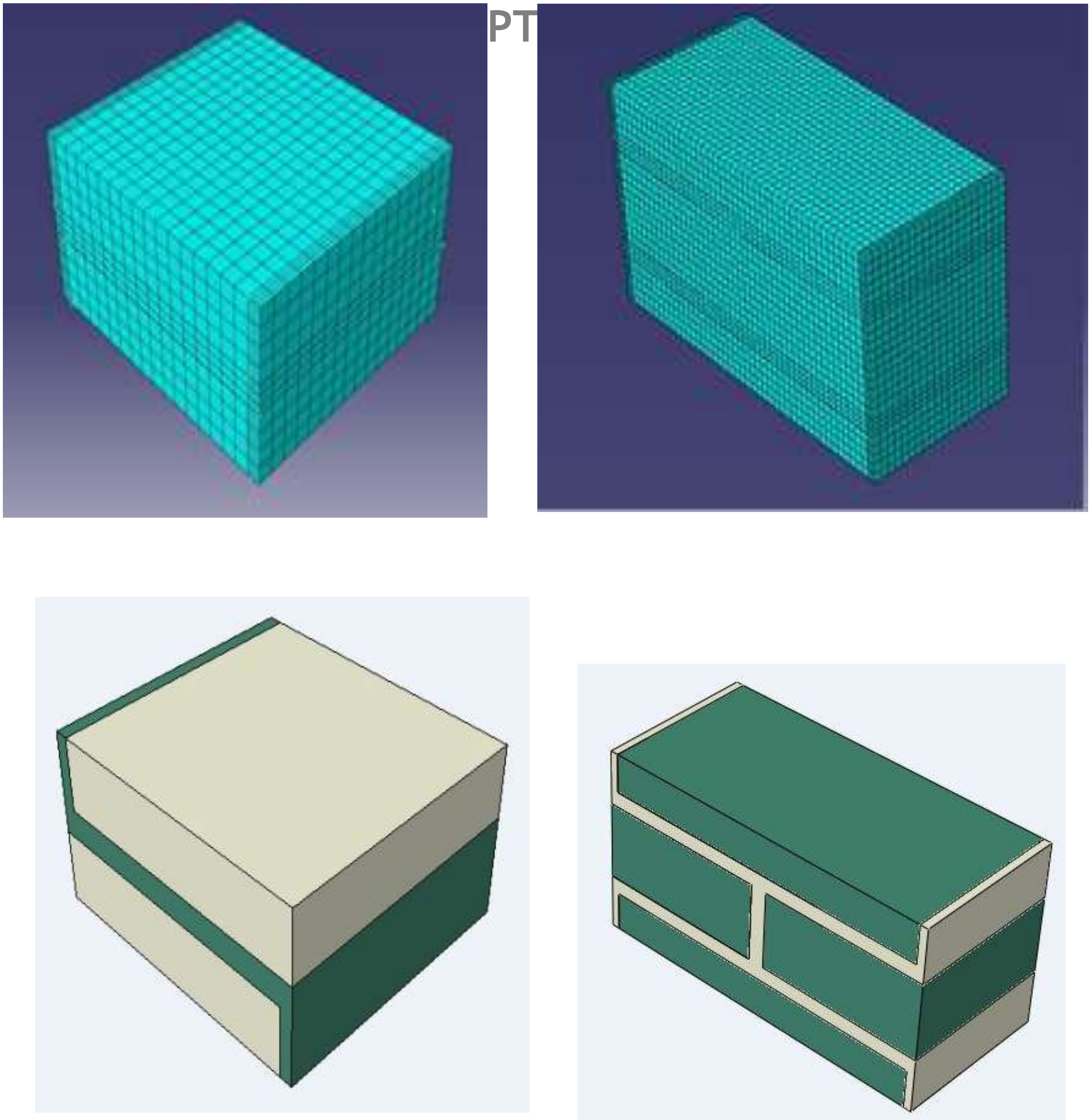


(b) For component 22



(b) For component 33

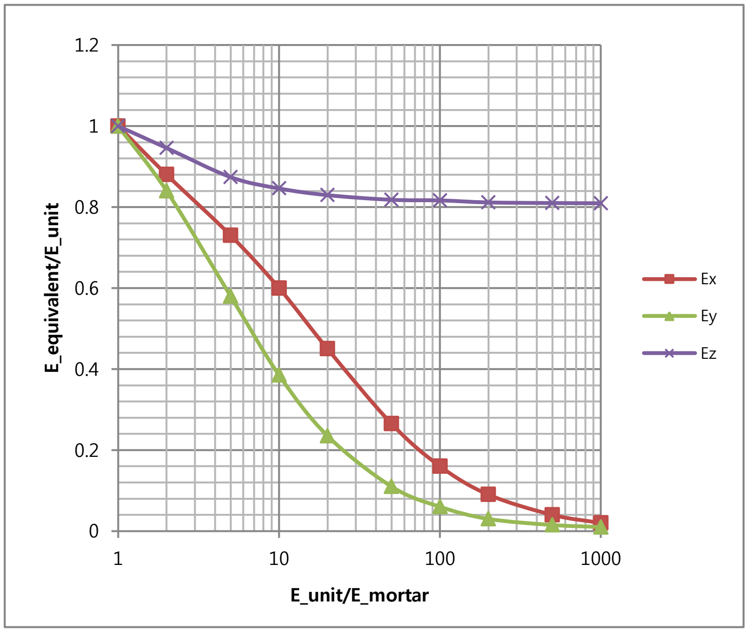
Fig. 4. Ratio between components of effective masonry properties and unit brick properties.



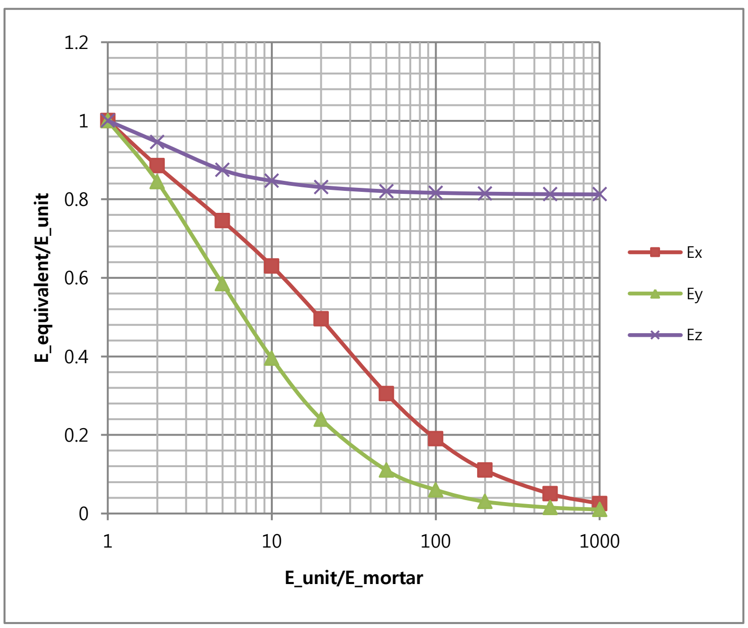
(a) Unit cell 1

(b) Unit cell 2

Fig. 5 Finite element model of the representative volume element (a) Unit cell 1 (b) Unit cell 2

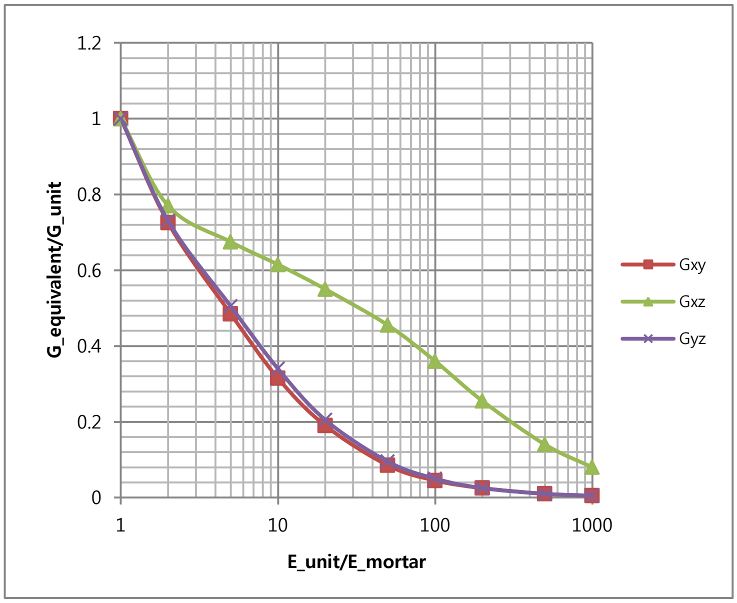


(a) Unit cell 1

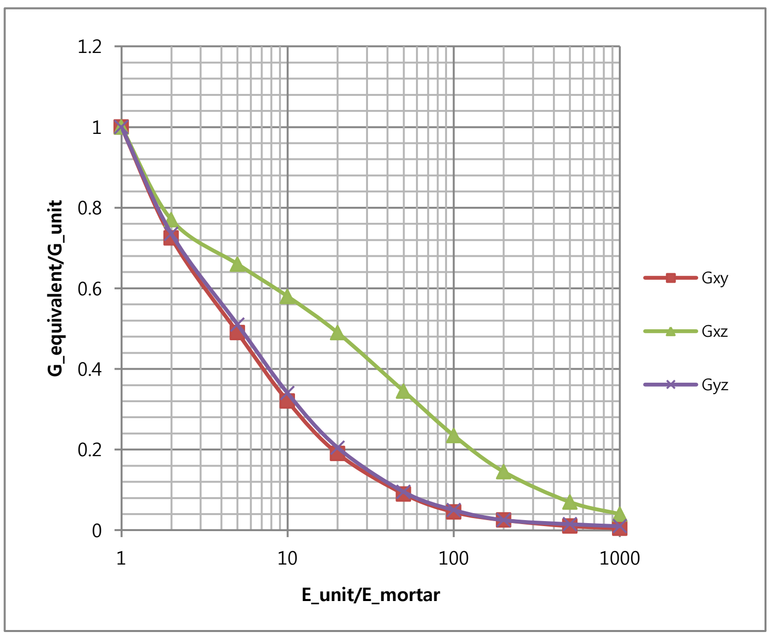


(b) Unit cell 2

Fig. 6. Variation of normalized Young's Modulus with modular ratio (Stress prescribed analysis homogenized value for different stiffness ratios) a) for Unit cell 1 b) for unit cell 2.

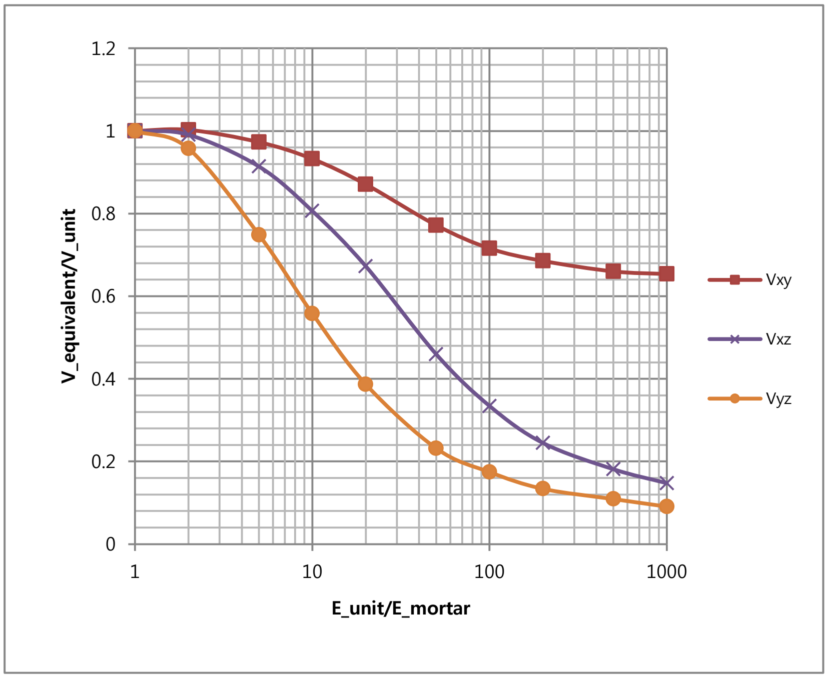


(a)Unit cell 1

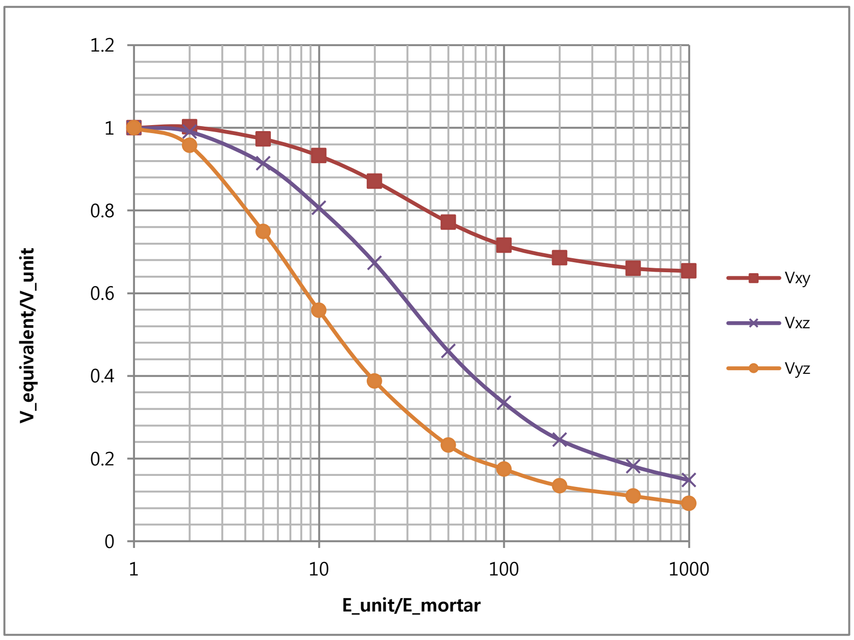


(b)Unit cell 2

Fig. 7. Variation of normalized Shear Modulus with modular ratio (Stress prescribed analysis homogenized value for different stiffness ratios) a) for Unit cell 1 b) for unit cell 2.

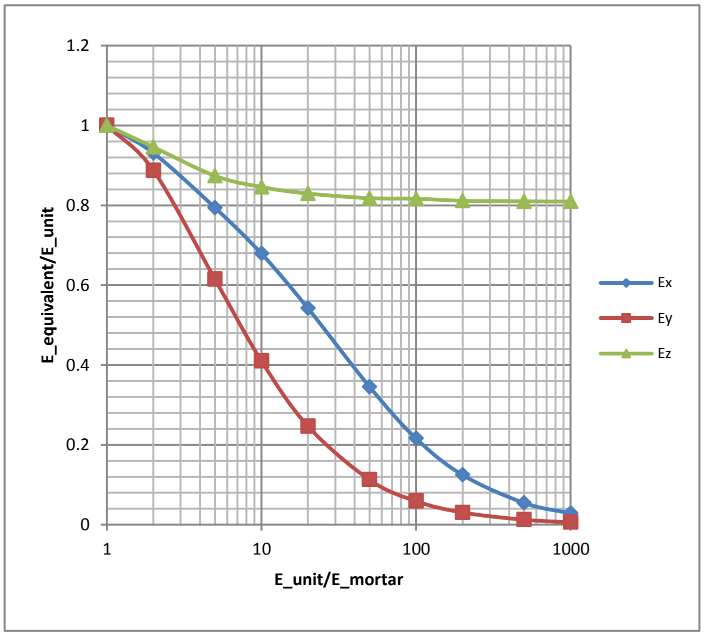


(a)Unit cell 1

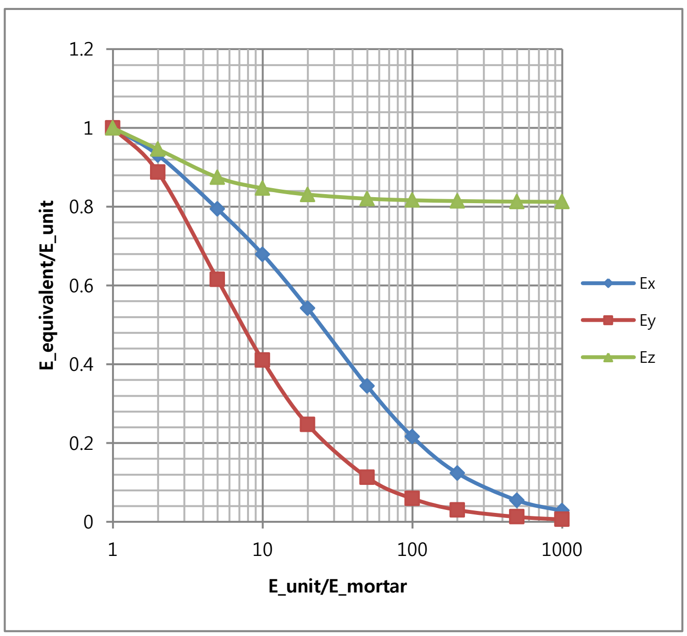


(b)Unit cell 2

Fig. 8. Variation of normalized Poisons ratio with modular ratio (Stress prescribed analysis homogenized value for different stiffness ratios) a) for Unit cell 1 b) for unit cell 2.

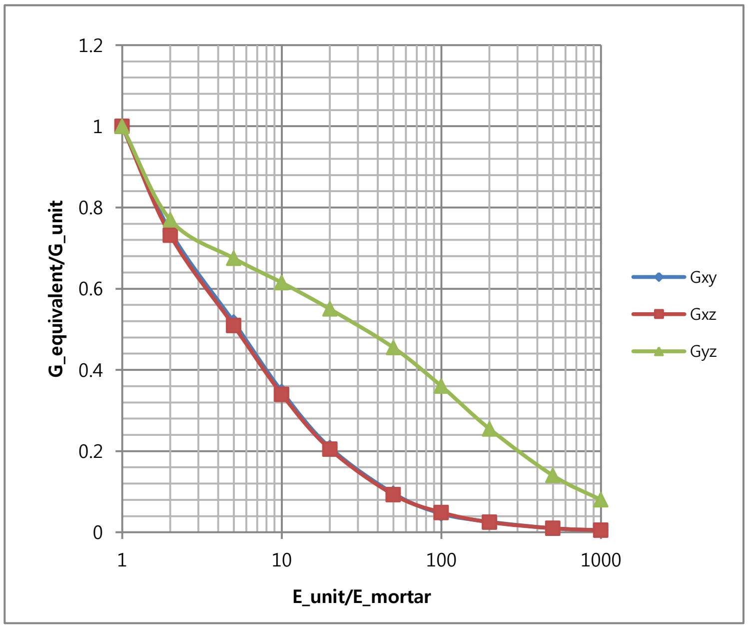


(a) Unit Cell 1

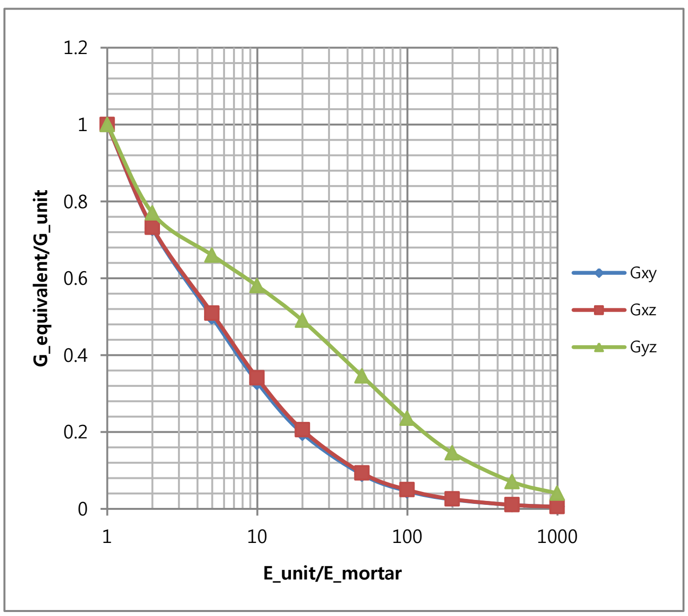


(b) Unit Cell 2

Fig.9. Variation of normalized Young's Modulus with modular ratio (Displacement prescribed analysis homogenized value for different stiffness ratios) a) for Unit cell 1 b) for Unit cell 2



(a) Unit Cell 1



(b) Unit Cell 2

Fig.10. Variation of normalized Shear Modulus with modular ratio (Displacement prescribed analysis homogenized value for different stiffness ratios) a) for Unit cell 1 b) for unit cell 2.