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Heavy neutrino mass hierarchy from leptogenesis in left-right symmetric models with spontaneous CP-violation

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Abstract

We consider left-right symmetric model with spontaneous CP-violation. The Lagrangian of this model is CP invariant and the Yukawa couplings are real. Due to spontaneous breaking of the gauge symmetry, some of the neutral Higgses acquire complex vacuum expectation values, which lead to CP-violation. In the model considered here, we identify the neutrino Dirac mass matrix with that of Fritzsch type charged lepton mass matrix. We assume a hierarchical spectrum of the right handed neutrino masses and derive a bound on this hierarchy by assuming that the decay of the lightest right handed neutrino produces the baryon asymmetry via the leptogenesis route. It is shown that the mass hierarchy we obtain is compatible with the current neutrino oscillation data.

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I. INTRODUCTION

Present low energy neutrino oscillation data [1, 2, 3] are elegantly explained by neutrino oscillation hypothesis with very small masses (≤ 1 eV) for the light neutrinos. These masses can be either Dirac or Majorana. See-saw mechanism is an elegant technique to generate small Majorana masses for light neutrinos without fine tuning [4]. This can be achieved by introducing right handed neutrinos into the electroweak model, which are invariant under all gauge transformations. The Majorana masses of these right handed neutrinos are free parameters of the model and are expected to be either at TeV scale [5] or at a higher scale [6, 7, 8, 9].

In the simplest scenario a right handed neutrino per generation is added. They are coupled to left handed neutrinos via Dirac mass matrix (m_D) which is assumed to be similar to charged lepton mass matrix. The consequent type-I seesaw mechanism [4] gives rise to Majorana mass matrix of the light neutrinos of the form

$$m_\nu = -m_D M_R^{-1} m_D^T. \tag{1}$$

The goal of the present neutrino oscillation experiments is to determine the nine degrees of freedom of the above equation (1). These are given by three light neutrino masses, three mixing angles and three phases which include one Dirac and two Majorana. At present the neutrino oscillation experiments are able to measure the two mass square differences, the solar and the atmospheric, and three mixing angles with varying degrees of precision, while there is no information about the phases. Moreover, it is difficult to constrain the parameters of the right handed neutrinos from low energy neutrino data. However, an early attempt [10] was made by inverting the seesaw formula (1).

Baryogenesis via leptogenesis [11] provides an attractive scenario to link the physics of right handed neutrino sector with the low energy neutrino data. In this scenario, a lepton (L) asymmetry is produced first which is then transformed to baryon (B) asymmetry of the Universe via the high temperature behavior of the $B + L$ anomaly of the Standard Model (SM) [12]. Most proposals along these lines rely on the out of equilibrium decay of heavy Majorana neutrinos to generate the L-asymmetry [11, 13, 14]. In these modifications of SM, the $B - L$ conservation is ad hoc.

An alternative is to consider leptogenesis [15, 16] within left-right symmetric model [17] where $U(1)_{B-L}$ is a gauge symmetry. Because $B - L$ is a gauge charge of the model, no primordial $B-L$ can exist. Further, the rapid violation of $B+L$ conservation by the anomaly due to high temperature sphaleron fields erases any $B+L$ generated earlier. Thus the lepton asymmetry must be produced entirely during or after the $B-L$ symmetry breaking phase transition. The Higgs sector of this model is very rich which consists of two triplets Δ_L and Δ_R and a bi-doublet Φ. In contrast to type-I models, in the present case the vacuum expectation value (VEV) of the triplet Δ_L provides an additional mass, m_L, to the light Majorana neutrino mass matrix [18, 19, 20, 21, 22]

$$m_\nu = m_L - m_D M_R^{-1} m_D^T = m_\nu^{II} + m_\nu^{I}, \tag{2}$$

where the two terms on the right hand side of above equation are called type-II and type-I respectively.

In the present work, we consider a left-right symmetric model in which CP-violation occurs via spontaneous symmetry breaking [23, 24, 25, 26]. The Lagrangian of the model is CP invariant which demands that all the Yukawa couplings should be real. CP violation occurs via the complex vacuum expectation values (VEVs) of the neutral Higgses in the model. In the present case, there are four complex neutral scalars, all of which can acquire complex VEVs. However, the global $U(1)$ symmetries associated with $SU(2)_L$ and $SU(2)_R$ gauge groups allow two of the phases to be set to zero. Using the remnant U(1) symmetry related to $SU(2)_R$, one phase choice is made to make the VEV of Δ_R, and hence the mass matrix of right handed neutrinos, real. The phase associated with the other $U(1)$ symmetry can be chosen to achieve two different types of simplification of neutrino mass matrix. In the *type-II choice*, the m_ν^I is made real leaving the CP-violating phase purely with m_ν^{II}. In this phase convention, we derive a lower bound on the mass scale of N_1 from the leptogenesis constraint by assuming a normal mass hierarchy in the right handed neutrino sector. In the *type-I phase choice*, only the type-I term contains CP-violating phase leaving type-II term real. This allows us to derive an upper bound on the heavy neutrino mass hierarchy from the leptogenesis constraint.

The early analyses [24, 25, 26] show that, the vacuum alignment of the most general Higgs potential in the left-right symmetric model requires both the phases to be very tiny $\mathcal{O}(m_W/v_R)$ ($v_R = \langle\Delta_R\rangle$) and hence there is no observable CP-violation. However, this analyses assumed that the VEVs of the two neutral scalars in the bidoublet Φ are of the same order of magnitude. On the other hand, requiring one of the bidoublet VEVs to be much smaller than the other [23] allows the phase associated with the triplet VEV to be large [27]. Later analyses worked out scenarios where the above conclusion [27] was explicitly demonstrated [28]. These papers also showed that the choice $v_R \geq 10^8$ GeV, suppresses flavour changing neutral currents adequately. This is in accord with our result, shown in section V, that the present B-asymmetry of the Universe also requires a similar magnitude of v_R.

Rest of the paper is organised as follows. In section II, we briefly recapitulate the left-right symmetric model with spontaneous CP-violation. In section III, we derive an upper bound on the CP-asymmetry in left-right symmetric models by keeping both type-I and type-II terms in the mass matrix of light neutrinos. We identify the neutrino Dirac mass matrix with that of charged lepton mass matrix [4]. Further we choose this matrix to be of Fritzsch type [29]. With these assumptions, in section IV, we show that a successful lepton asymmetry can be created for a reasonable mass hierarchy among right handed neutrinos and derive bounds on this hierarchy. In section V, we demonstrate that this hierarchy is compatible with the current neutrino oscillation data. Section VI contains our summary and conclusions.

II. LEFT-RIGHT SYMMETRIC MODEL AND SPONTANEOUS CP-VIOLATION

In the left-right symmetric model the right handed charged lepton of each family, which was a singlet under the SM gauge group $SU(2)_L \otimes U(1)_Y$, gets a new partner ν_R. These two form a doublet under the $SU(2)_R$ of the left-right symmetric gauge group $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$. Similarly, in the quark sector, the right handed up and down quarks of each family, which were singlets under SM gauge group, combine to form a doublet under $SU(2)_R$.

The Higgs sector of the model consists of two triplets Δ_L and Δ_R and a bidoublet Φ, which contains two copies of SM Higgs. Under $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ the field content and the quantum numbers of the Higgs fields are given as

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix} \sim (1/2, 1/2, 0) \tag{3}$$

$$\Delta_L = \begin{pmatrix} \delta_L^+/\sqrt{2} & \delta_L^{++} \\ \delta_L^0 & -\delta_L^+/\sqrt{2} \end{pmatrix} \sim (1, 0, 2) \tag{4}$$

$$\Delta_R = \begin{pmatrix} \delta_R^+/\sqrt{2} & \delta_R^{++} \\ \delta_R^0 & -\delta_R^+/\sqrt{2} \end{pmatrix} \sim (0, 1, 2). \tag{5}$$

To achieve the correct phenomenology, the various Higgs multiplets in the model should have the following VEVs,

$$\langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R e^{i\theta_R} & 0 \end{pmatrix}, \tag{6}$$

$$\langle \Phi \rangle = \begin{pmatrix} k_1 e^{i\alpha} & 0 \\ 0 & k_2 e^{i\beta} \end{pmatrix}, \tag{7}$$

and

$$\langle \Delta_L \rangle = \begin{pmatrix} 0 & 0 \\ v_L e^{i\theta_L} & 0 \end{pmatrix}. \tag{8}$$

The electric charge of the fields is given by

$$Q = T_L^3 + T_R^3 + \frac{1}{2}(B - L). \tag{9}$$

In the above v_L, v_R, k_1 and k_2 are real parameters and the electroweak symmetry breaking scale $v = 174$ GeV is given by $v^2 = k_1^2 + k_2^2$. Further we require that $v_L \ll v \ll v_R$. The requirement of the spontaneous breakdown of parity gives rise to

$$v_L v_R = \gamma(k_1^2 + k_2^2) = \gamma v^2, \tag{10}$$

where γ is parameter which is a function of the quartic couplings in the Higgs potential.

The minimisation of the most general Higgs potential involving Δ_L, Δ_R and Φ was studied in refs. [24, 28]. The relations between the various couplings, for which the above set of VEVs are generated, were derived. In this scenario, the gauge symmetry $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ is broken to $U(1)_{em}$ in a single step. Thus the CP-violating phases come into existence at the same scale where the left-right symmetry is broken. Since $v \ll v_R$, the SM

symmetry is present as an approximate symmetry at the scale where symmetry breaking occurs.

An attractive alternative to the single step symmetry breaking is the phenomenon of inverse symmetry breaking [30, 31]. This phenomenon usually occurs in models with multiple HIggs representations. The zero temperature Higgs potential of the model can be chosen so that all the neutral Higgses acquire non-zero VEVs. To consider the pattern of symmetry breaking at high temperatures, a temperature dependent correction is added to the Higgs potential and the potential is minimized. At high temperatures, it was shown that some of the VEVs grow with the temperature [31, 32]. In ref. [32], this mechanism was demonstrated for a left-right symmetric model with two Higgs bidoublets. The relevance of this mechanism to the model considered here is being studied and will be reported soon.

The fermions get their masses via Yukawa couplings. The Lagrangian for one generation of quarks and leptons is

$$\begin{aligned} -\mathcal{L}_{yuk} &= \tilde{h}_q \bar{q}_L \Phi q_R + \tilde{g}_q \bar{q}_L \tilde{\Phi} q_R + \tilde{h}_l \bar{\ell}_L \Phi l_R + \tilde{g}_l \bar{\ell}_L \tilde{\Phi} l_R \\ & \quad + i f (\ell_L^T C \tau_2 \Delta_L \ell_L + \ell_R^T C \tau_2 \Delta_R \ell_R) + H.c. \end{aligned} \tag{11}$$

where q and ℓ are quark and lepton doublets, $\tilde{\Phi} = \tau_2 \Phi^* \tau_2$ and C is the Dirac charge conjugation matrix. Further the Majorana Yukawa coupling f is the same for both left and right handed neutrinos to maintain the discrete $L \leftrightarrow R$ symmetry.

Substituting the complex VEVs (6), (7) and (8) in (11) we obtain fermion masses to be

$$\begin{aligned} M_f &= (\tilde{h}_q k_1 e^{i\alpha} + \tilde{g}_q k_2 e^{i\beta}) \bar{u}_L u_R + (\tilde{h}_q k_2 e^{i\beta} + \tilde{g}_q k_1 e^{i\alpha}) \bar{d}_L d_R \\ & \quad + (\tilde{h}_l k_1 e^{i\alpha} + \tilde{g}_l k_2 e^{i\beta}) \bar{\nu}_L \nu_R + (\tilde{h}_l k_2 e^{i\beta} + \tilde{g}_l k_1 e^{i\alpha}) \bar{e}_L e_R \\ & \quad + f(\nu_L^T C v_L e^{i\theta_L} \nu_L + \nu_R^T C v_R e^{i\theta_R} \nu_R) + H.C. \end{aligned} \tag{12}$$

Generalising the above equation (12) for three generation of matter fields we get the up and down quark mass matrices to be

$$(M_u)_{ij} = (\tilde{h}_q)_{ij} k_1 e^{i\alpha} + (\tilde{g}_q)_{ij} k_2 e^{i\beta} \quad \text{and} \quad (M_d)_{ij} = (\tilde{h}_q)_{ij} k_2 e^{i\beta} + (\tilde{g}_q)_{ij} k_1 e^{i\alpha}. \tag{13}$$

We assume [26, 33] $k_1/k_2 \sim m_t/m_b$. In the see-saw mechanism, the Dirac mass matrix of the neutrinos is assumed to be similar to the mass matrix of the charged leptons. For $k_2 \ll k_1$, and assuming $\tilde{h}_l \sim \tilde{g}_l$ in equation (12), the Dirac mass matrix of the neutrinos to a good approximation becomes $\tilde{h}_l k_1 e^{i\alpha}$. Thus neglecting k_2 terms, the masses of three generations of neutrinos are given by

$$(M_\nu)_{ij} = \bar{\nu}_{L_i} k_1 e^{i\alpha} (\tilde{h}_l)_{ij} \nu_{R_j} + f_{ij} (v_L e^{i\theta_L} \nu^T_{L_i} C \nu_{L_j} + v_R e^{i\theta_R} \nu^T_{R_i} C \nu_{R_j}) + H.C. \tag{14}$$

The Majorana Yukawa coupling matrix f_{ij} is real and symmetric and hence can be diagonalized by an orthogonal transformation on ν_R

$$N_R = O_R^T \nu_R. \tag{15}$$

In this basis, we have

$$O_R^T f O_R \; = \; f_{dia}, \tag{16}$$

$$h \; = \; \tilde{h} O_R. \tag{17}$$

In the transformed basis we get the mass matrix for the neutrinos to be

$$\begin{pmatrix} f v_L e^{i\theta_L} & k_1 e^{i\alpha} h \\ k_1 e^{i\alpha} h^T & f_{dia} v_R e^{i\theta_R} \end{pmatrix}. \tag{18}$$

Diagonalising the neutrino mass matrix into 3×3 blocks we get the light neutrino mass matrix to be

$$m_\nu = f v_L e^{i\theta_L} - \frac{k_1^2}{v_R} (h f_{dia}^{-1} h^T) e^{i(2\alpha - \theta_R)} \tag{19}$$

Notice that the Lagrangian (11) is invariant under the following unitary transformations of the fermion and Higgs fields,

$$\psi_L \longrightarrow U_L \psi_L \quad \text{and} \quad \psi_R \longrightarrow U_R \psi_R, \tag{20}$$

$$\Phi \longrightarrow U_L \Phi U_R^\dagger \quad \text{and} \quad \tilde{\Phi} \longrightarrow U_L \tilde{\Phi} U_R^\dagger \tag{21}$$

$$\Delta_L \longrightarrow U_L \Delta_L U_L^\dagger \quad \text{and} \quad \Delta_R \longrightarrow U_R \Delta_R U_R^\dagger, \tag{22}$$

where $\psi_{L,R}$ is a doublet of quark or lepton fields. The invariance under U_L is the result of the remnant global $U(1)$ symmetry which remains after the breaking of the gauge symmetry $SU(2)_L$ and similarly for U_R. The matrices U_L and U_R can be parametrized as

$$U_L = \begin{pmatrix} e^{i\gamma_L} & 0 \\ 0 & e^{-i\gamma_L} \end{pmatrix} \quad \text{and} \quad U_R = \begin{pmatrix} e^{i\gamma_R} & 0 \\ 0 & e^{-i\gamma_R} \end{pmatrix}. \tag{23}$$

By redefining the phases of the fermion fields we can rotate away two of the phase degrees of freedom from the scalar sector of the theory. Thus only two of the four phases of Higgs VEVs have phenomenological consequences. Under these unitary transformations, the VEVs (6), (7) and (8) become

$$\langle\Delta_R\rangle = \begin{pmatrix} 0 & 0 \\ v_R e^{i(\theta_R - 2\gamma_R)} & 0 \end{pmatrix}, \tag{24}$$

$$\langle\Phi\rangle = \begin{pmatrix} k_1 e^{i(\alpha+\gamma_L-\gamma_R)} & 0 \\ 0 & k_2 e^{i(\beta-\gamma_L+\gamma_R)} \end{pmatrix}, \tag{25}$$

and

$$\langle\Delta_L\rangle = \begin{pmatrix} 0 & 0 \\ v_L e^{i(\theta_L - 2\gamma_L)} & 0 \end{pmatrix}. \tag{26}$$

We choose $\gamma_R = \theta_R/2$ so that the masses of the right handed neutrinos are real. The light neutrino mass matrix (19) then becomes

$$m_\nu = f v_L e^{i(\theta_L-2\gamma_L)} - \frac{k_1^2}{v_R}(h f_{dia}^{-1} h^T) e^{i(2\alpha+2\gamma_L-\theta_R)} \tag{27}$$

$$= m_\nu^{II} + m_\nu^{I} \tag{28}$$

Conventionally, in equation (27), γ_L was chosen to be $-\alpha + \theta_R/2$ [26, 34]. This makes m_ν^I real leaving the imaginary part purely in m_ν^{II}. We call this *type-II phase* choice. The light neutrino mass matrix, with this phase choice, is

$$m_\nu = f v_L e^{i\theta'_L} - \frac{k_1^2}{v_R}(h f_{dia}^{-1} h^T), \tag{29}$$

where $\theta'_L = (\theta_L - \theta_R + 2\alpha)$. On the other hand, by choosing $\gamma_L = \theta_L/2$ in equation (27) m_ν^{II} can be made real, with the phase occuring purely in m_ν^I. We call this *type-I phase* choice. Consequently the light neutrino mass matrix (27) becomes

$$m_\nu = f v_L - \frac{k_1^2}{v_R} e^{i\theta'_R}(h f_{dia}^{-1} h^T) \tag{30}$$

where $\theta'_R = (\theta_L - \theta_R + 2\alpha)$. Note that the authors in ref. [34] displayed the possibility of leptogenesis through the type-II choice only. With this choice, they related the magnitude of CP violation in leptogenesis to the magnitude of CP violation possible in neutrino oscillations in certain models. However, in the present work we consider two distinct phase choices, i.e. type-I and type-II, and consider the implication of each choice to leptogenesis. The CP-violating parameter ϵ_1 which gives rise to the lepton asymmetry is independent of the phase choice. However, the theoretical upper bound on ϵ_1 is not a physical parameter of the theory and can depend on the choice of phases as we see in the next section. In numerical calculations, we take into account the consistency of the bounds coming from the different phase choices.

Diagonalization of the light neutrino mass matrix m_ν, through lepton flavour mixing matrix U_{PMNS} [35], gives us three light Majorana neutrinos. Its eigenvalues are

$$U_L^T m_\nu U_L = dia(m_1, m_2, m_3), \tag{31}$$

where m_1, m_2 and m_3 are the absolute masses of light Majorana neutrinos and are chosen to be real.

III. UPPER BOUND ON CP-ASYMMETRY IN LEFT-RIGHT SYMMETRIC MODELS

We assume that the lepton asymmetry of the Universe is produced by the CP-violating decay of the heavy right handed Majorana neutrinos to standard model leptons (l) and Higgs (ϕ). We also assume a normal mass hierarchy for the heavy Majorana neutrinos. In this scenario while the heavier neutrinos, N_2 and N_3, decay, the lightest of heavy Majorana neutrinos is still in thermal equilibrium. Any asymmetry, thus, produced by the decay of N_2 and N_3 will be erased by the lepton number violating interactions mediated by N_1. Therefore, the final lepton asymmetry is given only by the CP-violating decay of N_1. The CP-asymmetry, thus, is given by

$$\epsilon_1 = \epsilon_1^I + \epsilon_1^{II}, \tag{32}$$

where the contribution to ϵ_1^I comes from the interference of tree level, self-energy correction and the one loop radiative correction diagrams involving the heavier Majorana neutrinos N_2 and N_3. This contribution is same as in type-I models [6, 7] and is given by

$$\epsilon_1^I = \frac{3M_1}{16\pi v^2} \frac{\sum_{i,j} Im\left[h_{1i}^T (m_\nu^I)_{ij} h_{j1}\right]}{(h^T h)_{11}}. \tag{33}$$

On the other hand the contribution to ϵ_1^{II} in equation (32) comes from the interference of tree level diagram and the one loop radiative correction diagram involving the virtual triplet Δ_L. It is given by [8, 36]

$$\epsilon_1^{II} = \frac{3M_1}{16\pi v^2} \frac{\sum_{i,j} Im\left[h_{1i}^T (m_\nu^{II})_{ij} h_{j1}\right]}{(h^T h)_{11}}. \tag{34}$$

The total CP-asymmetry is therefore given by

$$\epsilon_1 = \frac{3M_1}{16\pi v^2} \frac{\sum_{i,j} Im \left[h^T_{1i}(m^I_\nu + m^{II}_\nu)_{ij} h_{j1}\right]}{(h^T h)_{11}}. \tag{35}$$

From equation (35), we see that the physical observable ϵ_1 is not affected by the choice of phases. In the following, we use bound on ϵ_1 from the observed baryon asymmetry to obtain bounds on right-handed neutrino masses for the two different phase choices.

A. The type-II choice of phases

In this choice of phases the type-I mass term is real. The only source of CP-violation in the light neutrino mass matrix m_ν lies in the type-II mass term. Thus in this case $\epsilon_1^I = 0$ because of both h and m^I_ν are real. The total CP-asymmetry in this choice of phases is therefore given by

$$\begin{aligned} \epsilon_1 &= \epsilon_1^{II} \\ &= \frac{3M_1 v_L}{16\pi v^2} \frac{(h^T f h)_{11}}{(h^T h)_{11}} Im(e^{i\theta'_L}). \end{aligned} \tag{36}$$

Using (16) and (17) in equation (36) we get

$$\epsilon_1 = \frac{3M_1 v_L}{16\pi v^2} \frac{\sum_i f_i (O_R^T h)^2_{i1}}{\sum_i (O_R^T h)^2_{i1}} sin\theta'_L, \tag{37}$$

where $f_i = (M_i/v_R)$. Up to a first order approximation it is reasonable to assume that $\sum_i f_i \approx 1$. In this approximation the theoretical upper bound on the CP-asymmetry (37) is given by [6, 7, 8, 9]

$$\epsilon_{1,max} = \frac{3M_1 v_L}{16\pi v^2}. \tag{38}$$

Thus, for type-II phase choice, a bound on ϵ_1 leads to a bound on M_1.

B. The type-I choice of phases

In the type-I choice of phases the type-II mass term is real. Hence the CP-violation comes through the type-I mass term only. The total CP-asymmetry in this case is therefore given by

$$\begin{aligned} \epsilon_1 &= \epsilon_1^{I} \\ &= \frac{3M_1 k_1^2}{16\pi v^2 v_R} \frac{(h^T h f^{-1}_{dia} h^T h)_{11}}{(h^T h)_{11}} Im(e^{-i\theta'_R}). \end{aligned} \tag{39}$$

Let us consider the type-I term of the light neutrino mass matrix

$$\begin{aligned} m_\nu^I &= m_\nu - m_\nu^{II} \\ &= -\frac{k_1^2}{v_R x'} h f_{dia}^{-1} h^T e^{-i\theta'_R}. \end{aligned} \tag{40}$$

We can find a diagonalising matrix $U = \mathcal{O} U_{phase}$ for m_ν^I such that

$$U^T m_\nu^I U \equiv -D_{m_I} = -dia(m_{I_1}, m_{I_2}, m_{I_3}) \tag{41}$$

where $(m_{I_1}, m_{I_2}, m_{I_3})$ are real by choosing $U_{phase} = e^{i\theta'_R/2}$. Therefore, from equation (41) we have

$$D_{m_I} = \frac{k_1^2}{v_R} \mathcal{O}^T \left(h f_{dia}^{-1} h^T \right) \mathcal{O}. \tag{42}$$

Using (42) in equation (39) the CP-asymmetry ϵ_1 can be rewritten as

$$\begin{aligned} \epsilon_1 &= \frac{3M_1}{16\pi v^2} \frac{\sum_i \left[(h^T \mathcal{O})_{1i} D_{m_{I_{ii}}} (\mathcal{O}^T h)_{i1} \right]}{\sum_i \left[(h^T \mathcal{O})_{1i} (\mathcal{O}^T h)_{i1} \right]} Im(e^{-i\theta'_R}) \\ &= \frac{3M_1}{16\pi v^2} \frac{\sum_i m_{I_i} (\mathcal{O}^T h)_{i1}^2}{\sum_i (\mathcal{O}^T h)_{i1}^2)} Im(e^{-i\theta'_R}). \end{aligned} \tag{43}$$

In the above equation (43) the theoretical upper bound on CP-asymmetry is thus given by [6, 7]

$$|\epsilon_{1,max}| = \frac{3M_1}{16\pi v^2} \sum_i m_{I_i}. \tag{44}$$

In the equation (44) m_Is are the eigenvalues of the matrix m_ν^I and *are not the physical light neutrino masses*. As we saw above, the relation between neutrino masses and the theoretical upper bound on ϵ_1 is phase choice dependent.

It is desirable to express the $\epsilon_{1,max}$ in terms of physical parameters. In order to calculate the m_Is we will assume a hierarchical texture of Majorana coupling

$$f_{dia} = \frac{M_1}{v_R} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \alpha_A & 0 \\ 0 & 0 & \alpha_B \end{pmatrix}, \tag{45}$$

where $1 \ll \alpha_A = (M_2/M_1) \ll \alpha_B = (M_3/M_1)$. We identify the neutrino Dirac Yukawa coupling h with that of charged leptons [4]. We assume h to be of Fritzsch type [29]

$$h = \frac{(m_\tau/v)}{1.054618} \begin{pmatrix} 0 & a & 0 \\ a & 0 & b \\ 0 & b & c \end{pmatrix}. \tag{46}$$

We make this assumption because Fritzsch mass matrices are well motivated phenomenologically. By choosing the values of a, b and c suitably one can get the hierarchy for charged leptons and quarks. In particular [29]

$$a = 0.004, \quad b = 0.24 \quad \text{and} \quad c = 1 \tag{47}$$

can give the mass hierarchy of charged leptons. For this set of values the mass matrix h is normalized with respect to the τ-lepton mass. The set of values of a, b and c are roughly in geometric progression. They can be expressed in terms of the electro-weak gauge coupling $\alpha_w = \frac{g^2}{4\pi} = \frac{\alpha}{sin^2\theta_w}$. In particular $a = 2.9\alpha_w^2$, $b = 6.5\alpha_w$ and $c = 1$. Here onwards we will use these set of values for the parameters of h. Using equation (45) and (46) in equation (42), we now get

$$\begin{aligned} D_{mI} &= \frac{v^2}{v_R}\left(h f_{dia}^{-1} h^T\right)_{dia} \\ &\simeq \frac{v^2}{M_1}\frac{(m_\tau/v)^2}{(1.054618)^2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & B \end{pmatrix}, \end{aligned} \tag{48}$$

where the eigenvalues A and B are functions of α_A and α_B and their sum is given by

$$A + B = \frac{1}{2}\left[a^2 + \frac{1}{\alpha_A}(a^2 + b^2) + \frac{1}{\alpha_B}(b^2 + c^2)\right]. \tag{49}$$

Using equation (48) we can write the maximum value of CP-asymmetry (44)

$$\begin{aligned} \epsilon_{1,max} &= \frac{3M_1}{16\pi v^2}(m_{I2} + m_{I3}) \\ &= \frac{3}{16\pi}\frac{(m_\tau/v)^2}{(1.054618)^2}(A + B). \end{aligned} \tag{50}$$

Thus we see that, in type-I choice of phases, the leptogenesis parameter ϵ_1 constrains the hierarchy parameters α_A and α_B. In the following two sections, we will obtain numerical bounds on α_A and α_B in a manner consistent with the bound M_1 coming from the type-II phase choice.

IV. ESTIMATION OF LEPTON ASYMMETRY

A net $B - L$ asymmetry is generated when the gauge symmetry is broken. A partial $B - L$ asymmetry then gets converted to B-asymmetry by the high temperature sphaleron transitions. However these sphaleron fields conserve $B-L$. Therefore, the produced $B-L$ will not be washed out, rather they will keep on changing it to B-asymmetry. In a comoving volume a net B-asymmetry is given by

$$Y_B = \frac{n_B}{s} = \frac{28}{79}\epsilon_1 Y_{N1}\delta, \tag{51}$$

where the factor $\frac{28}{79}$ in front [37] is the fraction of $B-L$ asymmetry that gets converted to B-asymmetry. Here ϵ_1 is given by equation (50). Further Y_{N_1} is density of lightest right handed neutrino in a comoving volume given by $Y_{N_1} = n_{N_1}/s$, where $s = (2\pi^2/45)g_* T^3$ is the entropy density of the Universe at any temperature T. Finally δ is the wash out factor at a temperature just below the mass scale of N_1. The value of Y_{N_1} depends on the source of N_1. For example, the value of Y_{N_1} estimated from topological defects [38] can be different from thermal scenario [7, 14]. In the present case we will restrict ourselves to thermal scenario only.

Recent observations from WMAP show that the matter-antimatter asymmetry in the present Universe measured in terms of (n_B/n_γ) is [39]

$$\left(\frac{n_B}{n_\gamma}\right)_0 \equiv \left(6.1^{+0.3}_{-0.2}\right) \times 10^{-10}, \tag{52}$$

where the subscript 0 presents the asymmetry today. Therefore, rewriting equation (51) we get

$$\left(\frac{n_B}{n_\gamma}\right)_0 = 7(Y_B)_0 = 2.48(\epsilon_1 Y_N \delta). \tag{53}$$

Substituting the type-II phase choice relation (38) in (53) and comparing with the observed value (52) of the baryon asymmetry we get the bound

$$M_1 \geq 1.25 \times 10^8 GeV \left(\frac{10^{-2}}{Y_{N_1}\delta}\right)\left(\frac{0.1eV}{v_L}\right). \tag{54}$$

On the other hand, substitution of $\epsilon_{1,max}$ from the type-I phase choice (50) in equation (53) and then comparison with the observed value (52) gives the constraint

$$A + B \geq 3.46 \times 10^{-3}(10^{-2}/Y_N\delta)\left(\frac{(n_B/n_\gamma)_0}{6.1\times 10^{-10}}\right)\left(\frac{2GeV}{m_\tau}\right)\left(\frac{v}{174GeV}\right)^2, \tag{55}$$

where the physical quantities are normalized with respect to their observed values. The above equation, for the values of a, b and c from (47), gives only one constraint on the two hierarchy parameters α_A and α_B. We will determine the individual parameters α_A and

α_B by demanding that their values should reproduce the low energy neutrino parameters correctly, while satisfying the inequalities $M_1 > O(10^8)$ GeV and $\alpha_B > \alpha_A >> 1$. Individual bounds on α_A and α_B can also be obtained if we assume that the α_A term and the α_B term in the sum $A + B$ from equation (49) are roughly equal. We then get

$$\alpha_A = (M_2/M_1) \leq 17 \ \text{ and } \ \alpha_B = (M_3/M_1) \leq 289. \tag{56}$$

V. CHECKING THE CONSISTENCY OF F-MATRIX EIGENVALUES

The solar and atmospheric evidences of neutrino oscillations are nicely accommodated in the minimal framework of the three-neutrino mixing, in which the three neutrino flavours ν_e, ν_μ, ν_τ are unitary linear combinations of three neutrino mass eigenstates ν_1, ν_2, ν_3 with masses m_1, m_2, m_3 respectively. The mixing among these three neutrinos determines the structure of the lepton mixing matrix [35] which can be parameterized as

$$U_{PMNS} = \begin{pmatrix} c_1c_3 & s_1c_3 & s_3e^{i\delta} \\ -s_1c_2 - c_1s_2s_3e^{i\delta} & c_1c_2 - s_1s_2s_3e^{i\delta} & s_2c_3 \\ s_1s_2 - c_1c_2s_3e^{i\delta} & -c_1s_2 - s_1c_2s_3e^{i\delta} & c_2c_3 \end{pmatrix} dia(1, e^{i\alpha}, e^{i(\beta+\delta)}), \tag{57}$$

where c_j and s_j stands for $cos\theta_j$ and $sin\theta_j$. Here we are interested only in the magnitudes of elements of U_{PMNS}. Hence for simplicity, we neglect all phases in it. The best fit values of the neutrino masses and mixings from a global three neutrino flavors oscillation analysis are [40]

$$\theta_1 = \theta_\odot \simeq 34^\circ, \quad \theta_2 = \theta_{atm} = 45^\circ, \quad \theta_3 \leq 10^\circ, \tag{58}$$

and

$$\begin{aligned} \Delta m_\odot^2 &= m_2^2 - m_1^2 \ \simeq \ m_2^2 = 7.1 \times 10^{-5} \text{ eV}^2 \\ \Delta m_{atm}^2 &= m_3^2 - m_1^2 \ \simeq \ m_3^2 = 2.6 \times 10^{-3} \text{ eV}^2. \end{aligned} \tag{59}$$

Using equation (19) we rewrite the f-matrix

$$f = (\frac{eV}{v_L}) \left[(m_\nu/eV) + \frac{4}{(1.054165)^2} \frac{1}{(M_1/\text{GeV})} \begin{pmatrix} \frac{a^2}{\alpha_A} & 0 & \frac{ab}{\alpha_A} \\ 0 & a^2 + \frac{b^2}{\alpha_B} & \frac{bc}{\alpha_B} \\ \frac{ab}{\alpha_A} & \frac{bc}{\alpha_B} & \frac{b^2}{\alpha_A} + \frac{c^2}{\alpha_B} \end{pmatrix} \right], \tag{60}$$

where the neutrino mass matrix m_ν is given by equation(31). The constrained eigenvalues α_A and α_B are given by equation (56).

In the following, we choose M_1 to be larger than the bound given by type-II phase choice (54) and m_1 such that $m_1^2 << \Delta_{sol}$. For such m_1 and M_1, we choose suitable α_A and α_B that are compatible with the low energy neutrino oscillation data. In particular here we choose $m_1 = 1.0 \times 10^{-3} eV$, $M_1 = 1.0 \times 10^8$ GeV, $\alpha_A = 17$, $\alpha_B = 170$ and $\theta_3 = 6°$. Then we get

$$f_{dia} = \frac{2.16 \times 10^{-3} eV}{v_L} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 17.3 & 0 \\ 0 & 0 & 169.7 \end{pmatrix}. \tag{61}$$

Thus, for the above values of m_1 and M_1, the assumed hierarchy of right-handed neutrino masses is consistent with global low energy neutrino data. Comparing equation (61) with (45) we get

$$\frac{M_1}{v_R} = \frac{2.16 \times 10^{-3} eV}{v_L}. \tag{62}$$

This implies that $v_R = O(10^{10})$ GeV for $v_L = 0.1$ eV. These values of v_L and v_R are compatible with genuine see-saw $v_L v_R = \gamma v^2$ for a small value of $\gamma \simeq O(10^{-4})$ [26]. On the other hand if we choose the parameters $m_1 = 1.0 \times 10^{-3}$ eV, $M_1 = 1.0 \times 10^9$ GeV, $\alpha_A = 17$, $\alpha_B = 65$ and $\theta_3 = 6°$ we get

$$f_{dia} = \frac{1.6 \times 10^{-3} eV}{v_L} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 16.76 & 0 \\ 0 & 0 & 64.68 \end{pmatrix}. \tag{63}$$

Once again we have consistency between the assumed hierarchy of right-handed neutrino masses and global low energy neutrino data. Again comparing equation (63) with (45) we get

$$\frac{M_1}{v_R} = \frac{1.6 \times 10^{-3} eV}{v_L}. \tag{64}$$

Thus for $v_L = 0.1$ eV one can get $v_R = O(10^{11}$ GeV). Again these values are compatible with see-saw for $\gamma \simeq O(10^{-3})$.

Here we demonstrated the consistency of our choice of the matrix f with neutrino data for two different choices of α_A and α_B. For other choices of these parameters, to be consistent with $1 << \alpha_A << \alpha_B$, one can choose appropriate values of $m_1 \leq 10^{-3}$ eV and $M_1 \geq 10^8$ GeV in equation (60) which will reproduce the correct eigenvalues of the matrix f.

VI. CONCLUSION

In this work we derived an upper bound on the CP-violating parameter ϵ_1 in left-right symmetric models by assuming the case of spontaneous CP-violation. Further we assumed a normal mass hierarchy among the heavy Majorana neutrinos. A class of left-right symmetric models are then considered in which we assume neutrino Dirac masses are of the Fritzsch type. We found that keeping the Majorana phase in type-II mass term of the light neutrinos, gives rise to a lower bound on the lightest right handed neutrino mass, whereas keeping the phase in type-I mass term, gives rise to bounds on the mass ratios M_2/M_1 and M_3/M_1. We further checked that these bounds are consistent with the present neutrino oscillation data.

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