Fermion mass hierarchy in a multiple warped braneworld model

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Abstract

A geometric understanding of the origin of mass hierarchy among the standard model fermions have been addressed in a multiple warped spacetime. We show that the requirement of 10^{16} order warping between the Planck and the visible branes without creating any significant hierarchy among the moduli results into splitting of fermion masses in the standard model. Values of the various parameters of the extra dimensional model and the corresponding fermion masses are determined.

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1 Introduction

The standard model (SM) of elementary particles, despite it's spectacular success in explaining Physics upto scale close to TeV, suffers from the well known gauge hierarchy problem, where the Higgs boson mass receives quantum loop corrections of the order of Planck scale. To restrict the mass of the Higgs boson within electroweak scale we have to fine tune these quantum corrections at each loop by adjusting the counter terms in the SM. Resolution to this problem has been explored in the context of supersymmetric and extra dimensional models.

Another problem encountered within the framework of SM is related to the masses of the fermions. The known fermions in the SM have masses ranging from 1 eV to 172 GeV. The reason why the fermions have different and hierarchical masses is one of the challenging problems in particle physics. Various approaches to solve the fermion mass hierarchy problem have been discussed over a wide range of works [1]. In this work we propose yet another possible solution to this problem in a multiply warped geometry model which has already been shown to offer a resolution of the gauge hierarchy problem in the context of Higgs mass.

Higher dimensional models offer a new explanation for stabilizing the Higgs boson mass around the electroweak scale. Among these, the model proposed by Randall and Sundrum (RS) has a warped geometry with one extra spatial dimension [2]. The extra spatial dimension in this model supports two 3-branes and the warping along this dimension can generate exponential mass scale suppression $\sim 10^{16}$ between these two 3-branes. Hence, one of the 3-branes can be interpreted as Planck-brane and the other one to be TeV-brane. By confining the Higgs boson on the TeV-brane, we generate TeV scale mass for the Higgs field. Subsequently this model was generalized to accommodate more than one extra warped dimension and it was qualitatively indicated that the requirements of the resolution of gauge hierarchy problem as well as little hierarchical moduli give rise to the possibility of resolving the fermion mass hierarchy problem[3]. Here, we demonstrate explicitly that in a multiply warped model, the fermion mass hierarchy problem can indeed be addressed.

In the multiply warped braneworld models [3], the number of 3-branes increases with the number of extra dimensions. Half of these 3-branes have energy scale clustered around TeV scale while the scale of the remaining 3-branes would be around Planck scale. For example, in a 6-dimensional doubly warped model, there will we four 3-branes, out of which two will have scale around TeV and the remaining two would be around the Planck

scale. By computing the fermion Yukawa couplings on different 3-branes clustered around TeV scale, we show that the respective masses of fermions will split. Based on this, we argue that in a multiply warped braneworld model, the fermion mass splitting between these 3-branes can offer an explanation of the observed fermion mass hierarchy of the SM. More specifically, we take the 6-dimensional warped model as our case study and we show that by adjusting the parameters of the model to appropriate values, the fermion mass splitting on the two TeV-branes can be generated. By extending this analysis to more than six dimensions, we show the possibility of explaining all fermion masses of the SM.

The paper is organized as follows. In the next section we give a brief overview of multiply warped braneworld models. In Sec. 3 we analyze the fermion mass splitting on two different 3-branes in the 6-dimensional warped model. We generalize this to higher dimensional multiply warped braneworld models, and argue that the different fermion masses in the SM can be explained from the mass splittings that are generated in these models. We conclude in Sec. 4.

2   Multiply warped braneworld model

In this section we give brief description of six and seven dimensional warped spacetime models [3], which are extended from the 5-dimensional RS model. Further generalization of these models beyond seven dimensions can be worked out in a similar fashion [3].

The six dimensional warped model has the usual four spacetime and two extra spatial dimensions. The two extra spatial dimensions are successively compactified on S_1/Z_2 orbifold. The manifold of the six dimensional spacetime is $[M^{1,3} \times S^1/Z_2] \times S^1/Z_2$. The metric ansatz consistent with this manifold is

$$ds^2 = b^2(z)[a^2(y)\eta_{\mu\nu}dx^\mu dx^\nu + R_y^2 dy^2] + r_z^2 dz^2, \tag{1}$$

where x^μ, $\mu = 0, \cdots, 3$, are the four non-compact spacetime coordinates, y and z are the angular coordinates of the two extra dimensions with the moduli R_y and r_z, respectively. $a(y)$ and $b(z)$ are some functions of the extra coordinates. Here, $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$. Since the y and z coordinates are orbifolded on S^1/Z_2, with the orbifold fixed points $y = 0, \pi$ and $z = 0, \pi$, there are four 4-branes at these fixed points. Intersection of any of the two 4-branes give one 3-brane. Hence, in this model there would be four different 3-branes located at $(y, z) = (\pi, \pi), (\pi, 0), (0, \pi)$ and $(0, 0)$. We take one of these 3-branes

as our Tev scale universe. The total bulk-brane action of this model has a form [3]

$$
\begin{aligned}
S &= S_6 + S_5, \quad S_6 = \int d^4x\,dy\,dz\sqrt{-g_6}(R_6 - \Lambda), \\
S_5 &= \int d^4x\,dy\,dz[V_1\delta(y) + V_2\delta(y - \pi)] + \int d^4x\,dy\,dz[V_3\delta(z) + V_4\delta(z - \pi)] \quad (2)
\end{aligned}
$$

Here, $V_{1,2}$ and $V_{3,4}$ are brane tensions of the branes located at $y = 0, \pi$ and $z = 0, \pi$, respectively. Λ is the cosmological constant in 6-dimensions.

After solving Einstein's equations with orbifolded boundary conditions the solution to the warp functions of the metric of Eq. (1) has a form [3]

$$
\begin{aligned}
a(y) &= \exp(-c|y|), \quad b(z) = \frac{\cosh(kz)}{\cosh(k\pi)}, \\
c &\equiv \frac{R_y k}{r_z \cosh(k\pi)}, \quad k \equiv r_z\sqrt{\frac{-\Lambda}{10M_P^4}}.
\end{aligned}
\quad (3)
$$

Here, M_P is the Planck scale.

The functional form of $a(y)$ in Eq. (3) is similar to the exponential warping in the 5-dimensional RS model [2]. In addition the function $b(z)$ gives warping along z direction. It can be noticed that for the 3-brane located at $y = 0, z = \pi$ we have no warping and hence it is called Planck brane while for the 3-brane located at $y = \pi, z = 0$ the warping is maximum and is called standard model or TeV brane. However, from the relation for c in Eq. (3), in order not to have large hierarchy between R_y and r_z, one of c or k must be large and other is small. This indicates large warping in one direction and small along the other. It is now easy to show that when Eq. (3) is considered together with the maximum warping condition at the visible brane i.e. $a(\pi)b(0) = 10^{-16}$ then the minimum hierarchy between the two moduli can be achieved for $c \sim 11$ and $k \sim 0.4$. Now, with this choice of values for c and k, the 3-branes located at $(y, z) = (\pi, 0)$ and (π, π) get exponential suppression $\sim 10^{-16}$ with respect to the 3-branes located at $(y, z) = (0, 0)$ and $(0, \pi)$. However due to small warping along z-direction, the mass scales on the 3-branes at $(y, z) = (0, 0)$ and $(0, \pi)$ would of the order of Planck scale while the mass scales on the 3-branes at $(y, z) = (\pi, 0)$ and (π, π) would be suppressed close to TeV scale. So in the six dimensional warped model, the requirement of absence of any large hierarchy between the two moduli along with both the moduli implies that the scales of two 3-branes are clustered around TeV scale and that of the other two are at around Planck scale. The moduli of such six dimensional models can be stabilized to their desired values following the mechanism suggested by Goldberger and Wise where a bulk scalar field is introduced as a stabilizing field with appropriate vacuum expectation values at the boundaries [4]

We now describe the seven dimensional model which has three extra spatial dimensions. Each of the extra dimension is orbifolded by Z_2 symmetry. The manifold under consideration of this model is $[\{M^{1,3} \times S^1/Z_2\} \times S^1/Z_2] \times S^1/Z_2$. Denoting the three extra dimensions by angular coordinates y, z and w, the bulk-brane action in this model can be written as

$$
\begin{aligned}
S &= S_7 + S_6, \quad S_7 = \int d^4x dy dz dw \sqrt{-g_7}(R_7 - \Lambda_7), \\
S_6 &= \int d^4x dy dz dw [V_1\delta(w) + V_2\delta(w - \pi)] + \int d^4x dy dz dw [V_3\delta(z) + V_4\delta(z - \pi)] \\
&\quad + \int d^4x dy dz dw [V_5\delta(y) + V_6\delta(y - \pi)].
\end{aligned}
\tag{4}
$$

Here, Λ_7 is cosmological constant in 7-dimensions. $V_{1,\cdots,6}$ are brane tensions of 5-branes located at $w = 0, \pi$, $z = 0, \pi$ and $y = 0, \pi$, respectively. The metric solution of the above action which satisfies Einstein's equations and satisfies the orbifolded boundary condition is [3]

$$
\begin{aligned}
ds^2 &= \frac{\cosh^2(lw)}{\cosh^2(l\pi)} \left\{ \frac{\cosh^2(kz)}{\cosh^2(k\pi)} \left[\exp(-2cy)\eta_{\mu\nu}dx^\mu dx^\nu + R_y^2 dy^2 \right] + r_z^2 dz^2 \right\} + R_w^2 dw^2 \\
l^2 &\equiv \frac{\Lambda_7 R_w^2}{15}, \quad k \equiv \frac{l r_z}{R_w \cosh(l\pi)}, \\
c &\equiv \frac{l R_y}{R_w \cosh(k\pi)\cosh(l\pi)} = \frac{k R_y}{r_z \cosh(k\pi)}.
\end{aligned}
\tag{5}
$$

Here, R_w is the moduli corresponding to the w direction. Looking at the above solution of metric, we have exponential suppression along the y direction, whereas, the suppression along z and w are given by cosine-hyperbolic functions. Once again a consistent set of values for the dimensionless quantities would be: $c \sim 10$, $k \sim 0.1$ and $l \sim 0.1$, so that there will not be large hierarchies among the moduli R_y, r_z and R_w.

In the seven dimensional model, the 5-branes located at $w = 0, \pi$, $z = 0, \pi$ and $y = 0, \pi$ intersect and give twelve different 4-branes which lie at $(z, w) = (0, 0), (0, \pi), (\pi, 0), (\pi, \pi)$, $(z, y) = (0, 0), (0, \pi), (\pi, 0), (\pi, \pi)$ and $(y, w) = (0, 0), (0, \pi), (\pi, 0), (\pi, \pi)$. Intersection of 4-branes results in 3-branes, and in this model there are eight different possibilities, which lie at $(y, z, w) = (0, 0, 0), (0, 0, \pi), (0, \pi, 0), (\pi, 0, 0), (\pi, \pi, 0), (\pi, 0, \pi), (0, \pi, \pi)$ and (π, π, π). For the consistent set of values of c, k and l, which are described in the previous paragraph, the 3-branes located at $(0, 0, 0), (0, 0, \pi), (0, \pi, 0)$ and $(0, \pi, \pi)$ have no suppression, hence, these 3-branes can be identified as Planck scale branes. Whereas, the 3-branes located at $(\pi, 0, 0), (\pi, 0, \pi), (\pi, \pi, 0)$ and (π, π, π) have exponential suppression $\sim 10^{16}$ to TeV scale masses, and hence they may be identified as TeV scale branes. So in the seven dimensional

model, four 3-branes are clustered around TeV scale and the remaining four are clustered around Planck scale.

The six and seven dimensional braneworld models, which are described here, can easily be generalized to higher dimensions. A notable feature in all these models is that half of the total number of 3-branes would be clustered around TeV scale and the remaining 3-branes would be clustered around Planck scale. As explained in Sec. 1, we exploit this feature and try to see how much splitting between fermion masses can be obtained geometrically. In the next section we describe the calculation of fermion mass splitting in the six dimensional model. We can generalize this calculation to higher dimensions, and we argue that in a multiply braneworld model the hierarchical mass pattern of SM fermions can be explained.

3 Fermion mass splitting in a multiply warped braneworld model

As explained previously, we take 6-dimensional warped model as our case study and calculate fermion mass splitting between two different 3-branes. A viable way of generating fermion masses is through Yukawa couplings. We generalize the approach taken in [5], for generating fermion masses in multiply braneworld models. We assume that both the Higgs boson and left-handed doublet quark fields of SM are confined to the usual 4-dimensional world. We allow the right-handed fields of the SM to be in the bulk of 6-dimensions. Consider the Yukawa couplings for left-handed doublets with its right-handed counter-parts. Since right-handed fermions are allowed in the bulk, we can estimate the gauge invariant Yukawa couplings on both the 3-branes at $(y, z) = (\pi, 0), (\pi, \pi)$, which are given below.

$$
\begin{aligned}
S_{\text{vis}} &= \int d^4x \sqrt{-g_{\text{vis}}} \hat{Y}_6 \bar{L}_{0L}(x) H_0(x) \psi_R(x, \pi, 0) + \text{h.c.}, \\
S_{\text{vis}'} &= \int d^4x \sqrt{-g_{\text{vis}'}} \hat{Y}_6' \bar{L}_{0L}(x) H_0(x) \psi_R(x, \pi, \pi) + \text{h.c.}.
\end{aligned}
\tag{6}
$$

Here, $\hat{Y}_6, \hat{Y}_6'$ are Yukawa couplings in 6-dimensions in the two branes. Note that their dimensions are M^{-1}, where $M \sim M_P$. L_{0L} indicates left-handed SU(2) doublet of either quark or lepton field of the SM, and ψ_R is its right-handed counter-part. H_0 is the Higgs doublet. $g_{\text{vis}} = \det(g_{\mu\nu}^6(\pi, 0))$ and $g_{\text{vis}'} = \det(g_{\mu\nu}^6(\pi, \pi))$, where $g_{M,N}^6$ is the metric function defined in Eq. (1). Since the fermions are 8-component spinors in 6-dimensions, we have to choose the representation for Higgs field H_0 accordingly in the above equation. Also

since ψ_R is a bulk field, it decomposes into Kaluza-Klein (KK) fields in the 4-dimensional world. After the Higgs acquires vacuum expectation value, the Yukawa couplings in above equation give mixing masses between the left-handed fermion and the KK tower of right-handed fields. The zero mode of the KK tower is identified as the SM field. Nevertheless, to be consistent we have to include all possible mixings with the KK fermions, and after diagonalizing it we can identify the lowest eigenvalue to be the SM fermion mass. In fact, this is the procedure employed in [5], and here we generalize this to 6-dimensions. For this we have to study the KK excitations of a bulk fermion field, which is done in [6] for a 6-dimensional model. However, the nonzero KK masses and their wave functions are not computed in [6]. In the next subsection we calculate them which will be necessary for the computation of SM fermion masses.

3.1   KK fermions

The kinetic part of the action for a bulk fermion field Ψ in 6-dimensions is [5, 7, 8]

$$S_f = \int d^4x dy dz \sqrt{-G} E_a^A \left[\frac{i}{2} \left(\bar{\Psi}\Gamma^a \partial_A \Psi - \partial_A \bar{\Psi}\Gamma^a \Psi \right) + \frac{\omega_{bcA}}{8} \bar{\Psi}\{\Gamma^a, \sigma^{bc}\}\Psi \right] \qquad (7)$$

Here, the capital letter A denotes index in the curved space and the lower case letters a, b, c denote indices in the tangent space. ω_{bcA} is spin connection and E_a^A is inverse vielbein. Here, we do not write the bare mass of bulk fermion since the SM fermions acquire mass through the Higgs mechanism via Eq. (6). The spin connection term in the above equation give zero contribution due to the diagonal form of the metric in Eq. (1). The order of the Dirac matrices Γ^a in 6-dimensions would be 8×8, and they can be taken as [9]:

$$\Gamma^\mu = \gamma^\mu \otimes \sigma^0, \quad \Gamma^4 = i\gamma_5 \otimes \sigma^1, \quad \Gamma^5 = i\gamma_5 \otimes \sigma^2 \qquad (8)$$

Here, γ^μ are the Dirac matrices in 4-dimensions and $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$. σ^i, $i = 1, 2, 3$, are the Pauli matrices and σ^0 is the 2×2 unit matrix. The chirality in 6-dimensions is defined by the matrix $\bar{\Gamma} = \Gamma^0\Gamma^1\Gamma^2\Gamma^3\Gamma^4\Gamma^5$ such that $\bar{\Gamma}\Psi_\pm = \pm\Psi_\pm$. The chiral fermions in 6-dimensions have both left- and right-handed chirality of 4-dimensions, which can be projected by the operators $P_{L,R} = (1 \mp i\Gamma^0\Gamma^1\Gamma^2\Gamma^3)/2$. Since, fermions have chirality in six dimensions, we can write $\psi = \psi_+ + \psi_-$. Each field carrying either plus or minus chirality can further be decomposed into left- ($\psi_{\pm L}$) and right-chiral ($\psi_{\pm R}$) fields in four

dimensions. These fields can be expressed in KK expansion as [6]

$$\Psi_{+L,-R}(x^\mu, y, z) = \frac{1}{\sqrt{R_y r_z}} \sum_{j,k} \psi^{(j,k)}_{+L,-R}(x^\mu) f^{(j,k)}_{+L,-R}(y,z) \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix},$$

$$\Psi_{-L,+R}(x^\mu, y, z) = \frac{1}{\sqrt{R_y r_z}} \sum_{j,k} \psi^{(j,k)}_{-L,+R}(x^\mu) f^{(j,k)}_{-L,+R}(y,z) \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \qquad (9)$$

Here, fields of the form $\psi^{(j,k)}(x^\mu)$ are the KK fields in 4-dimensions and the KK wave functions should satisfy the following normalization condition and eigenvalue equations [6].

$$\int dy dz b^4(z) a^3(y) \left(f^{(j,k)}_{+R,+L,-R,-L}(y,z) \right)^* f^{(j',k')}_{+R,+L,-R,-L}(y,z) = \delta^{j,j'} \delta^{k,k'}, \qquad (10)$$

$$\begin{aligned}
(i\mathcal{D}_y + \mathcal{D}_z) f^{(j,k)}_{+R}(y,z) &= -M_{j,k} f^{(j,k)}_{+L}(y,z), \\
(-i\mathcal{D}_y + \mathcal{D}_z) f^{(j,k)}_{+L}(y,z) &= -M_{j,k} f^{(j,k)}_{+R}(y,z), \\
(i\mathcal{D}_y + \mathcal{D}_z) f^{(j,k)}_{-L}(y,z) &= M_{j,k} f^{(j,k)}_{-R}(y,z), \\
(-i\mathcal{D}_y + \mathcal{D}_z) f^{(j,k)}_{-R}(y,z) &= M_{j,k} f^{(j,k)}_{-L}(y,z), \qquad (11)
\end{aligned}$$

where the differential operators: $\mathcal{D}_y = \frac{i}{2R_y}(4\partial_y a + 2a\partial_y)$, $\mathcal{D}_z = \frac{i}{2r_z}a(5\partial_z b + 2b\partial_z)$ and $M_{j,k}$ is the mass of the KK fermions $\psi^{(j,k)}$. In [6] the KK wave function for zero mode has been solved, which is identified with $M_{0,0} = 0$. By writing $f^{(0,0)}_{+R}(y,z) = f_y(y)f_z(z)$, we get the solution for f_y and f_z as

$$f_y(y) = \exp\left(\frac{1}{2}(c_1 + 4c)y \right), \quad f_z(z) = \frac{\exp\left(\frac{-ic_1 r_z}{kR_y} \tan^{-1}(\tanh(kz/2)) \cosh(k\pi) \right)}{\cosh^{5/2}(kz)} \qquad (12)$$

Here, c_1 is a separation constant which is defined in [6], and can be worked out from the normalization of wave function $f^{(0,0)}_{+R}(y,z)$, using Eq. (10). The nonzero KK masses and their wave functions are necessary to calculate the mixing masses of Eq. (6), which will be worked out below.

The eigenvalue equations in Eq. (11) can be transformed into second order differential equations as

$$\begin{aligned}
(-i\mathcal{D}_y + \mathcal{D}_z)(i\mathcal{D}_y + \mathcal{D}_z) f^{(j,k)}_{+R,-L} &= M^2_{j,k} f^{(j,k)}_{+R,-L}, \\
(i\mathcal{D}_y + \mathcal{D}_z)(-i\mathcal{D}_y + \mathcal{D}_z) f^{(j,k)}_{-R,+L} &= M^2_{j,k} f^{(j,k)}_{-R,+L}. \qquad (13)
\end{aligned}$$

Here we will show how the nonzero eigenvalues $M_{j,k}$ can be found for the wave function $f^{(j,k)}_{+R}$. The wave functions of other chiral modes can be analogously worked out. By writing

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$f_{+R}^{(j,k)} = \chi^{(j,k)}(y)\xi^{(j,k)}(z)$, the above second order differential equation can be recast into

$$\frac{1}{(2R_y)^2}\frac{[24c^2 - 20c\partial_y + 4\partial_y^2]\chi^{(j,k)}}{\chi^{(j,k)}} + \frac{M_{j,k}^2}{a^2}$$
$$+\frac{1}{(2r_z)^2}\frac{(5\partial_z b + 2b\partial_z)^2\xi^{(j,k)}}{\xi^{(j,k)}} + \frac{2ic}{2R_y 2r_z}\frac{(5\partial_z b + 2b\partial_z)\xi^{(j,k)}}{\xi^{(j,k)}} = 0, \qquad (14)$$

where the first and second lines of the above equation depends on functions of y and z, respectively. Hence, we can choose

$$\frac{1}{(2R_y)^2}\frac{[24c^2 - 20c\partial_y + 4\partial_y^2]\chi^{(j,k)}}{\chi^{(j,k)}} + \frac{M_{j,k}^2}{a^2} = m_\chi^2,$$
$$\frac{1}{(2r_z)^2}\frac{(5\partial_z b + 2b\partial_z)^2\xi^{(j,k)}}{\xi^{(j,k)}} + \frac{2ic}{2R_y 2r_z}\frac{(5\partial_z b + 2b\partial_z)\xi^{(j,k)}}{\xi^{(j,k)}} = -m_\chi^2, \qquad (15)$$

where m_χ^2 is a separation constant of mass-square dimensions. By making the transformations

$$\chi^{(j,k)} = e^{\frac{5}{2}cy}\tilde{\chi}^{(j,k)}, \quad Z_p = \frac{M_{j,k}}{k'}\cosh(k\pi)e^{cy}, \quad k' = k/r_z, \qquad (16)$$

the first of Eq. (15) can be put into a Bessel's equation as

$$Z_p^2\frac{\partial^2\tilde{\chi}^{(j,k)}}{\partial Z_p^2} + Z_p\frac{\partial\tilde{\chi}^{(j,k)}}{\partial Z_p} + (Z_p^2 - \nu^2)\tilde{\chi}^{(j,k)} = 0, \qquad (17)$$

where

$$\nu^2 = \frac{25}{4} + \left(m_\chi^2 - \frac{6c^2}{R_y^2}\right)\frac{\cosh^2(k\pi)}{k'^2}. \qquad (18)$$

Now, the solution to the wave function $\chi^{(j,k)}(y)$ can be written as

$$\chi^{(j,k)}(y) = \frac{e^{\frac{5}{2}cy}}{N}[J_\nu(Z_p) + bY_\nu(Z_p)], \qquad (19)$$

where N and b are some constants. N can be determined from the normalization condition of Eq. (10). b and the KK masses $M_{j,k}$ can be determined by applying the following boundary conditions $\partial_y\chi^{(j,k)}(y)|_{y=0,\pi} = 0$. The boundary condition at $y = 0$ gives us that the constant b is negligibly small and the condition at $y = \pi$ gives the following relation

$$5J_\nu(\tilde{y}) + \tilde{y}[J_{\nu-1}(\tilde{y}) - J_{\nu+1}(\tilde{y})] = 0, \qquad (20)$$

where $\tilde{y} = \frac{M_{j,k}}{k'}\cosh(k\pi)e^{c\pi}$. The above equation determines the nonzero mass eigenvalues $M_{j,k}$ of the KK modes for a specific value of ν. The index ν depends on the separation constant m_χ^2. This value can be determined by solving the equation along the z, which is the second of Eq. (15).

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The second of Eq. (15) can be expressed as

$$(5\partial_z b + 2b\partial_z)[(5\partial_z b + 2b\partial_z)\xi^{(j,k)} + k_1\xi^{(j,k)}] = k_2\xi^{(j,k)} \tag{21}$$

where $k_1 = 2icr_z/R_y$ and $k_2 = -4r_z^2 m_\chi^2$. The solution to the above equation can be found from the linear equation $(5\partial_z b + 2b\partial_z)\xi^{(j,k)} = k_3\xi^{(j,k)}$, where k_3 satisfies the quadratic equation $k_3(k_3 + k_1) = k_2$. After doing this, the general solution to the second of Eq. (15) can be written as

$$
\begin{aligned}
\xi^{(j,k)}(z) &= a_1\xi^+(z) + a_2\xi^-(z), \\
\xi^\pm(z) &= \frac{\exp(k_3^\pm \tan^{-1}[\tanh(\frac{kz}{2})]\cosh(k\pi)/k)}{\cosh^{5/2}(kz)}, \\
k_3^\pm &= \frac{icr_z}{R_y}[-1 \pm \sqrt{1 + \frac{4R_y^2 m_\chi^2}{c^2}}],
\end{aligned}
\tag{22}
$$

where $a_{1,2}$ are constants, and one of them can be determined by the boundary condition $\partial_z\xi^{(j,k)}(z)|_{z=0,\pi} = 0$. The condition at $z = 0$ gives us $a_1 = -a_2 k_3^-/k_3^+$. And the condition at $z = \pi$ gives the following relation

$$k_3^-(k_3^+ - 5k\tanh(k\pi))\xi^+(\pi) - k_3^+(k_3^- - 5k\tanh(k\pi))\xi^-(\pi) = 0. \tag{23}$$

The undetermined constant a_2 can be found from the normalization condition of Eq. (10). The left-hand side of Eq. (23) has a sinusoidal behavior, and zeros of this function determine the nonzero values of m_χ^2, which should be put in Eq. (20) to find the nonzero KK masses $M_{j,k}$ of fermions.

3.2 Fermion mass splitting

After calculating the KK masses and their wave functions we now return to Eq. (6), and calculate the SM fermion masses on both the 3-branes at $(y, z) = (\pi, 0), (\pi, \pi)$. To make the kinetic term canonical we need to do rescaling of the SM doublet L_{0L} and the Higgs field H_0, since we have confined them in the 4-dimensional world. Imagine that our universe is identified as $(y, z) = (\pi, 0)$, then the kinetic energy terms of a fermion ψ_0 and scalar ϕ_0 on this brane are

$$S = \int d^4x\sqrt{-g_{\text{vis}}}\left[g_{\text{vis}}^{\mu\nu}\partial\phi_0^*\partial_\nu\phi_0 + E_a^\mu(\pi, 0)\bar{\psi}_0\Gamma^a\partial_\mu\psi_0 + \cdots\right] \tag{24}$$

The inverse vielbein is given by $E_a^\mu(\pi, 0) = \delta_a^\mu e^{c\pi}\cosh(k\pi)$ and we have $g_{\text{vis}}^{\mu\nu} = e^{2c\pi}\cosh^2(k\pi)\eta^{\mu\nu}$. Substituting them in the above equation, we get

$$S = \int d^4x\left[\frac{e^{-2c\pi}}{\cosh^2(k\pi)}\eta^{\mu\nu}\partial\phi_0^*\partial_\nu\phi_0 + \frac{e^{-3c\pi}}{\cosh^3(k\pi)}\bar{\psi}_0\Gamma^\mu\partial_\mu\psi_0 + \cdots\right] \tag{25}$$

Hence, to get the canonical kinetic terms for scalar and fermion fields in 4-dimension, we have to do the following rescalings

$$\phi_0 = e^{c\pi}\cosh(k\pi)\phi, \quad \psi_0 = e^{\frac{3}{2}c\pi}\cosh^{3/2}(k\pi)\psi, \tag{26}$$

where ϕ and ψ are renormalized scalar and fermion fields. Recall Sec. 2 where we pointed out that $c \sim 10$ and $k \sim 0.1$ give a consistent set of values. For this set of values $e^{c\pi}\cosh(k\pi) \sim 10^{16}$, which naturally gives $\langle\phi\rangle \sim 100$ GeV. This is in similar lines of what it is shown in the 5-dimensional RS model [5].

The SU(2) doublet fermion of the SM can be put in either of the $+$ or the $-$ chirality of the 6-dimensional world. Without any loss of generality, we choose it to be in the $-$ chirality of 6-dimensional spinor. Then, after substituting the KK wave function expansion for Ψ_R, Eq. (9), and also doing the rescaling of the 4-dimensional fields L_{0L} and H_0, the first of Eq. (6) turns out to be

$$S_{\text{vis}} = \sum_{j,k} \int d^4x \frac{e^{-\frac{3}{2}c\pi}}{\cosh^{3/2}(k\pi)} f_{+R}^{(j,k)}(\pi,0) Y_6 \bar{L}_L H \psi_{+R}^{(j,k)}. \tag{27}$$

Here, $Y_6 = \hat{Y}_6/\sqrt{R_y r_z}$. Y_6 is the effective 4-dimensional Yukawa coupling which is an $\mathcal{O}(1)$ quantity. After the Higgs field acquires vacuum expectation value (vev), the left-handed SM fermion gets mixing masses with all the right-handed KK fields, which can be written as

$$[m_D]_{ij} = \frac{e^{-\frac{3}{2}c\pi}}{\cosh^{3/2}(k\pi)} f_{+R}^{(j,k)}(\pi,0) Y_6 v, \tag{28}$$

where v is the vev of Higgs field. In the previous subsection, we have seen that the fields $\psi_{+R}^{(j,k)}$ have Dirac masses with their left-handed counter parts. Hence, in the basis $\Psi_L = (\psi_L, \psi_{+L}^{(1,0)}, \psi_{+L}^{(0,1)}, \psi_{+L}^{(1,1)}, \cdots)$, $\Psi_R = (\psi_R, \psi_{+R}^{(1,0)}, \psi_{+R}^{(0,1)}, \psi_{+R}^{(1,1)}, \cdots)$, where ψ_L is a component of the SM doublet L_L and $\psi_R = \psi_{+R}^{(0,0)}$, we get the following form for mixing matrix $\bar{\Psi}_L \mathcal{M} \Psi_R + \text{h.c.}$,

$$\mathcal{M} = \begin{pmatrix} [m_D]_{0,0} & [m_D]_{1,0} & [m_D]_{0,1} & [m_D]_{1,1} & \cdots \\ 0 & M_{1,0} & 0 & 0 & \cdots \\ 0 & 0 & M_{0,1} & 0 & \cdots \\ 0 & 0 & 0 & M_{1,1} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{29}$$

We have numerically computed the nonzero KK masses $M_{i,j}$, whose values are coming in the multi-TeV scale. From the relations given in previous subsection, the KK masses

depend on r_z apart from k and c. For $r_z^{-1} \sim 10^{18}$ GeV, the lowest KK mass has been found to be around 6 TeV. Here we have fixed $k = 0.7$ and $c = 11.24$ where the justification of these values will be given later (see Table 1). Some of the modes like $M_{0,1}$, $M_{0,2}$, etc. may decouple away since their masses and wave functions are found out to be close to zero. We have also found that the off-diagonal values of $[m_D]_{i,j}$ are coming in the electroweak scale. As a result due to the high values of the KK masses, after diagonalizing the above matrix, we can see that the zeroth mode mass $[m_D]_{0,0}$ is negligibly affected. Hence, the SM fermion mass is approximately the same as the zeroth mode mass $[m_D]_{0,0}$. The same kind of feature can be seen if we repeat this calculation on the other 3-brane at $(y, z) = (\pi, \pi)$, i.e. on the second term of Eq. (6). Because of this, to calculate the fermion mass splitting between the two 3-branes, $(y, z) = (\pi, 0), (\pi, \pi)$, we work out the corresponding values of the zeroth mode.

From Eq. (28) we can notice that the following factor

$$F = \frac{e^{-\frac{3}{2}c\pi}}{\cosh^{3/2}(k\pi)} f_{+R}^{(0,0)}(\pi, 0) = \frac{e^{\frac{1}{2}(c_1+c)\pi}}{\cosh^{3/2}(k\pi)} \tag{30}$$

determines the zeroth mode mass on the 3-brane at $(y, z) = (\pi, 0)$ where $Y_6 v \sim 100$ GeV. In the above equation we have used $f_{+R}^{(0,0)}(y, z) = f_y(y)f_z(z)$ and also Eq. (12). The corresponding factor on the 3-brane at $(y, z) = (\pi, \pi)$ would be

$$F' = e^{-\frac{3}{2}c\pi} \cosh^{5/2}(k\pi)|f_{+R}^{(0,0)}(\pi, \pi)| = e^{\frac{1}{2}(c_1+c)\pi}. \tag{31}$$

The ratio of masses confined on 3-branes at $(y, z) = (\pi, 0), (\pi, \pi)$, is

$$R = \frac{F}{F'} = \frac{1}{\cosh^{3/2}(k\pi)} \tag{32}$$

This ratio R gives us the splitting between the fermions on these two 3-branes. The mass term of a fermion confined on $(y, z) = (\pi, 0)$ brane and the mass term of a fermion confined on the other TeV 3-brane at $(y, z) = (\pi, \pi)$ will have a relative suppression by a factor given by R. In Table 1, we give possible values of R in the 6-dimensional model. Here we have chosen c so that $e^{c\pi} \cosh(k\pi) = 10^{16}$, which yields the desired suppression from Planck to TeV scale in the 6-dimensional model [6]. Table 1 has been constructed for different choices for the two moduli in a six dimensional model where the Planck to TeV scale warping is achieved for every choice. Such a hierarchical pattern of masses can simultaneously be obtained by including larger number of extra dimensions and adjusting the different moduli according to Table 1. It has been explained in section 2 that for seven

k	c	$\frac{R_y}{r_z}$	R
0.7	11.24	73.3	$\frac{1}{9.75}$
1.2	10.75	194.34	$\frac{1}{101.1}$
1.7	10.24	628.9	$\frac{1}{1065.7}$
2.2	9.75	2223.7	$\frac{1}{11243.4}$
2.7	9.25	8268.9	$\sim \frac{1}{10^5}$
4.2	7.75	495757.0	$\sim \frac{1}{10^8}$
6.1	5.85	$\sim 10^8$	$\sim \frac{1}{10^{12}}$

Table 1: Various mass splitting ratios R for different values of k and c. $\frac{R_y}{r_z}$ gives the hierarchy between the two moduli.

or higher dimensional models the stack of 3-branes have energy scales clustered around TeV scale and Planck scale.

Here, using Table 1, we give few examples of how the fermion mass hierarchy can be addressed in multiply braneworld models. Without loss of generality we can localize top quark to the TeV-brane at $(y, z) = (\pi, 0)$, which has a mass of $\approx$172 GeV. Then the fermion mass at the other TeV-brane, i.e. at $(y, z) = (\pi, \pi)$, depends on the values of k and c. For $k = 0.7$ and $c = 11.24$, the fermion field localized at $(y, z) = (\pi, \pi)$ would have a mass of $\sim$10 GeV, which is a reasonable value to explain the bottom quark mass of $\approx$4.2 GeV. On the other hand, if we set $k = 1.2$ and $c = 10.75$, the fermion mass on the 3-brane at $(y, z) = (\pi, \pi)$ would be suppressed to $\sim$1 GeV, which is in the right ball park region to explain either the charm quark mass ($\approx$1.3 GeV) or the tau lepton mass ($\approx$1.77 GeV). Likewise, if we set $k = 6.1$ and $c = 5.85$, the suppression on the 3-brane at $(y, z) = (\pi, \pi)$ is such that we can even explain the neutrino mass scale of 1 eV. However, the values in Table 1 are generated for the case of six dimensional warped model, where we could only explain the masses of two fermion fields. By generalizing this to seven dimensions, we will have four 3-branes at around TeV scale and we also get additional freedom in the choice of the moduli R_w. Hence, in the seven dimensional warped model, adjusting the value of the other modulus according to Table 1 we can explain four fermion masses. These arguments can now easily be generalized in a braneworld model having more than seven dimensions where SM fermions of different masses are localized on different 3-branes of the stack of clustered 3-branes around Tev scale through the usual localization mechanism [10]. The hierarchy in standard model fermion masses thus can be explained.

4   Conclusions

In this work we have proposed a possible origin of the fermion mass hierarchy in the standard model through multiple warped geometry. The requirement of the resolution of gauge hierarchy problem restricts the energy scale of half of the 3-branes in such model around TeV while the remaining half are close to Planck scale. Using the mass generation mechanism of the standard model fermions proposed in [5], where the right handed fermions are allowed to propagate in the bulk and both the Higgs boson and left-handed fermion fields of SM are confined to the 3-branes, we have shown that we can adjust and tune various moduli to have different mass splittings once one mass parameter is determined by fixing the value of the only unknown parameter Y_{6D}. Assuming that the standard model fermions are localized at different 3-branes closely clustered around TeV the slight difference in warping results into different contributions from the right handed fermion wave function in the Yukawa coupling which in turn leads to different masses of the SM fermion. The hierarchical mass pattern of the standard model fermions thus can be generated by this mechanism. It is interesting to note from Table 1 that larger is the ratio between two moduli more is the splitting of fermion masses which are localized in those two branes. The multiple warped model thus proposes a geometric understanding of the splitting of the masses of standard model fermions.

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