Bounds on neutrino masses from leptogenesis in type-II see-saw models

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Abstract

The presence of the triplet Δ_L in left-right symmetric theories leads to type-II see-saw mechanism for the neutrino masses. In these models, assuming a normal mass hierarchy for the heavy Majorana neutrinos, we derive a lower bound on the mass of the lightest of heavy Majorana neutrino from the leptogenesis constraint. From this bound we establish a consistent picture for the hierarchy of heavy Majorana neutrinos in a class of left right symmetric models in which we identify the neutrino Dirac mass matrix with that of Fritzsch type charged lepton mass matrix. It is shown that these values are compatible with the current neutrino oscillation data.

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I. INTRODUCTION

A plausible explanation of the observed baryon (B) asymmetry of the Universe is that it arose from a lepton (L) asymmetry [1, 2, 3, 4]. The conversion of the L-asymmetry to the B-asymmetry then occurs via the high temperature behavior of the $B + L$ anomaly of the Standard Model (SM) [5]. This is an appealing route for several reasons. The extremely small neutrino masses suggested by the solar [6] and atmospheric [7] neutrino anomalies and the KamLAND experiment [8], point to the possibility of Majorana masses for the neutrinos generated by the see-saw mechanism [9]. This suggests the existence of new physics at a predictable high energy scale. Since the Majorana mass terms violate lepton number they can generate L-asymmetry.

Early proposals along these lines relied on out-of-equilibrium decay of the heavy Majorana neutrinos to generate the L-asymmetry. In the simplest scenario a right-handed neutrino per generation is added to the SM [1, 2, 4]. They are coupled to left-handed neutrinos via Dirac mass matrix (m_D) which is assumed to be similar to charged lepton mass matrix [9]. Since the right handed neutrino is a singlet under SM gauge group a Majorana mass term (M_R) for it can be added to the Lagrangian. Diagonalisation of neutrino mass matrix leads to two Majorana neutrino states per generation: a light neutrino state (mass $\sim m_D^2/M_R$) which is almost left handed and a heavy neutrino state (mass $\sim M_R$) which is almost right handed. This is called type-I see-saw mechanism in which the left handed fields do not have Majorana mass terms in the Lagrangian.

It is desirable to consider neutrino masses in the context of grand unified theories ($GUTs$). The gauge groups of most of the GUTs contain the left-right symmetry group $SU(2)_L \times SU(2)_R$ as a subgroup [10]. In such models Majorana masses, M_L, for left handed neutrinos occur in general, through the vacuum expectation value (VEV) of the triplet Δ_L [11, 12, 13, 14, 15]. In these models also there is a light and a heavy neutrino state per generation. The heavy neutrino state has mass $\sim M_R$ but the light neutrino mass is $\sim (M_L - m_D^2/M_R)$. The presence of new scalars and their couplings, which give rise to M_L, can contain adequate CP-violation to accommodate L-asymmetry. The two contributions to the light neutrino mass, m_D^2/M_R and M_L are called type-I and type-II terms respectively.

An additional grace of left-right symmetric models [16] is that $B-L$ is a gauge symmetry in contrast to type-I models where $B - L$ conservation is ad-hoc. Because of $B - L$ is a gauge charge of this model, no primordial $B-L$ can exist. Further, the rapid violation of the $B+L$ conservation by the anomaly due to the high temperature sphaleron fields erases any $B+L$ generated earlier. Thus the lepton asymmetry must be produced entirely during or after the $B-L$ symmetry breaking phase transition.

Several authors [17, 18, 19, 20] have recently dealt with the bound on the mass scale of lightest right handed neutrino (M_1) in type-I see-saw models from the leptogenesis constraint. With the assumption of hierarchical mass spectrum of the heavy Majorana neutrinos, a common outcome was that $M_1 \geq 10^9$ GeV. With the same assumption, a bound on M_1 was obtained in type-II see-saw models in [21], which is an order of magnitude less than that in type-I case.

In this paper we revisit the lower bound on the mass scale of lightest heavy Majorana neutrino [17] due to its CP-violating decay to SM particles in generic type-II see-saw models. Recently it is claimed that the large atmospheric neutrino mixing can be achieved naturally in case of renormalisable $SO(10)$ theories if the type-II term dominates [22, 23]. Therefore, it is interesting to extend our formalism to these models and derive an upper bound on the CP-asymmetry. In both cases it is shown that the mass scale of lightest right handed neutrino satisfies the constraint, $M_1 \geq 2.5 \times 10^8$GeV in order to produce the present baryon asymmetry of the Universe. On the other hand leptogenesis in models, where the type-II term was included in the neutrino mass matrix, was considered in the literature [24, 25]. However, the contribution to the CP-violating parameter, ϵ_1, due to the triplet Δ_L in the loop, are not taken into account.

Rest of our paper is organised as follows. In section II, we derive an upper bound on the CP-asymmetry in type-II see-saw models assuming that type-I and type-II terms are similar in magnitude. In section III, we discuss the upper bound on the mass scale of lightest right handed neutrino from the leptogenesis constraint. In light of current neutrino oscillation data in section IV a consistent picture for the heavy Majorana neutrino mass hierarchies are obtained in a class of left-right symmetric models in which we identify the neutrino Dirac mass matrix with that of charged lepton mass matrix [9]. Further we choose this matrix to be of Fritzsch type [26]. Finally in Section V, we put our summary and conclusions.

II. UPPER BOUND ON CP-ASYMMETRY IN TYPE-II MODELS

Before proceeding with our analysis, we review the breaking scheme of $SO(10)$ grand unified theory through the left-right symmetric path [13, 15]. This breaking can be accomplished by using a $\{126\}$ of $SO(10)$ as an intermediate. Under $SU(2)_L \otimes SU(2)_R \otimes SU(4)_C (= SU(2)_L \otimes SU(2)_R \otimes SU(3)_C \otimes U(1)_{B-L})$ its decomposition can be written as

$$\{126\} = \Delta_L(3,1,10) + \Delta_R(1,3,10) + \Phi(2,2,15) + \sigma(1,1,6), \tag{1}$$

where $\sigma(1,1,6)$ is a singlet under $SU(2)_L \otimes SU(2)_R$ and has no role in neutrino mass generation. Since 126 of $SO(10)$ contains a pair of triplets Δ_L and Δ_R and the bidoublet Φ, left-right symmetry can be preserved at the intermediate level. As the right handed triplet $\Delta_R(1,3,10)$ gets a VEV v_R, left-right symmetry is broken to the SM symmetry. The scalars in both the triplets Δ_L and Δ_R acquire masses of the order v_R. At a lower scale, Φ gets a VEV v breaking the SM symmetry to $U(1)_{em}$. This induces a small VEV v_L for the neutral component of the triplet Δ_L [13, 14, 27]. The three VEVs are related by $v_L = \gamma v^2/v_R$, where γ is a model dependent parameter which depends on the quartic couplings of the Higgs and can be as small as 10^{-4}.

After the final symmetry breaking, the effective neutrino mass matrix is

$$M_\nu = \bar{\nu}_{Li} \tilde{m}_{D_{ij}} \nu'_{Rj} + \frac{1}{2}\left[v_L \bar{\nu}_{Li} f_{ij} \nu^c_{Lj} + v_R \bar{\nu'}^c_{Ri} f_{ij} \nu'_{Rj}\right] + H.C. \tag{2}$$

Because of $L \leftrightarrow R$ symmetry, the same symmetric Yukawa matrix f_{ij} gives rise to Majorana masses for both left and right handed fields. The Dirac mass matrix is given by $\tilde{m}_D = \tilde{h} v$, where $\tilde{h}$ is the Yukawa matrix for neutrino Dirac masses.

The Majorana mass matrix for the right handed neutrinos can be diagonalized by making the following rotation on ν'_R

$$\nu_R = U_R^\dagger \nu'_R. \tag{3}$$

In this basis, we have

$$U_R^T f U_R = f_{dia}, \tag{4}$$

$$h = \tilde{h} U_R. \tag{5}$$

In this rotated basis we get the mass matrix for the neutrinos

$$\begin{pmatrix} f v_L & m_D \\ m_D^T & M_R \end{pmatrix}, \tag{6}$$

where $M_R = f_{dia} v_R$ and $m_D = hv$. Diagonalising the mass matrix (6) into 3×3 blocks we get the mass matrix for the light neutrinos to be

$$\begin{aligned} m_\nu &= f v_L - \frac{v^2}{v_R} h f_{dia}^{-1} h^T \\ &= m_\nu^{II} + m_\nu^{I}. \end{aligned} \tag{7}$$

Note that in contrast to the present case in type-I models, m_ν^{II} is absent. Diagonalization of the above light neutrino mass matrix m_ν, through lepton flavour mixing PMNS matrix U_L, gives us three light Majorana neutrinos. Its eigenvalues are

$$U_L^\dagger m_\nu U_L^* = dia(m_1, m_2, m_3) \equiv D_m, \tag{8}$$

where the masses are real.

We assume a normal mass hierarchy for heavy Majorana neutrinos. In this scenario while the heavier neutrinos, N_2 and N_3, decay yet the lightest of heavy Majorana neutrinos is still in thermal equilibrium. Any asymmetry produced by the decay of N_2 and N_3 will be washed out by the lepton number violating interactions mediated by N_1. Therefore, the final lepton asymmetry is given only by the CP-violating decay of N_1 to standard model leptons (l) and Higgs (ϕ). The CP-asymmetry in this scenario is given by

$$\epsilon_1 = \epsilon_1^I + \epsilon_1^{II}, \tag{9}$$

where the contribution to ϵ_1^I comes from the interference of tree level, self-energy correction and the one loop radiative correction diagrams involving the heavier Majorana neutrinos N_2 and N_3. This contribution is the same as in type-I models [17, 18] and is given by

$$\epsilon_1^I = \frac{3M_1}{16\pi v^2} \frac{\sum_{i,j} Im\left[(h^\dagger)_{1i}(m_\nu^I)_{ij}(h^*)_{j1}\right]}{(h^\dagger h)_{11}}. \tag{10}$$

On the other hand the contribution to ϵ_1^{II} in equation (9) comes from the interference of tree level diagram and the one loop radiative correction diagram involving the triplet Δ_L. It is given by [21, 27]

$$\epsilon_1^{II} = \frac{3M_1}{16\pi v^2} \frac{\sum_{i,j} Im\left[(h^\dagger)_{1i}(m_\nu^{II})_{ij}(h^*)_{j1}\right]}{(h^\dagger h)_{11}}. \tag{11}$$

Substituting (10) and (11) in equation (9) we get the total CP-asymmetry

$$\epsilon_1 = \frac{3M_1}{16\pi v^2} \frac{Im(h^\dagger m_\nu h^*)_{11}}{(h^\dagger h)_{11}}. \tag{12}$$

Using (8) in the above equation (12) we get

$$\begin{aligned}\epsilon_1 &= \frac{3M_1}{16\pi v^2}\frac{Im(h^\dagger U_L D_m U_L^T h^*)_{11}}{(h^\dagger h)_{11}} \\ &= \frac{3M_1}{16\pi v^2}\frac{\sum_i m_i Im(U_L^T h^*)_{i1}^2}{\sum_i |(U_L^T h^*)_{i1}|^2}.\end{aligned} \tag{13}$$

With an assumption of normal mass hierarchy for the light Majorana neutrinos the maximum value of CP-asymmetry (13) can be given by

$$\epsilon_{1,max} = \frac{3M_1}{16\pi v^2} m_3. \tag{14}$$

Note that the above upper bound (14) for ϵ_1 as a function of M_1 and m_3 was first obtained for the case of type-I see-saw models [17]. However, the same relation holds in the case of type-II see-saw models also [21] *independent of the relative magnitudes of* m_ν^I *and* m_ν^{II}.

III. ESTIMATION OF BARYON ASYMMETRY

A net $B-L$ asymmetry is generated when left-right symmetry breaks. A part of this $B-L$ asymmetry then gets converted to B-asymmetry by the high temperature sphaleron transitions. However these sphaleron fields conserve $B-L$. Therefore, the produced $B-L$ will not be washed out, rather they will keep on changing it to B-asymmetry. In a comoving volume a net B-asymmetry is given by

$$Y_B = \frac{n_B}{s} = \frac{28}{79}\epsilon_1 Y_{N_1}\delta, \tag{15}$$

where the factor $\frac{28}{79}$ in front [28] is the fraction of $B-L$ asymmetry that gets converted to B-asymmetry. Further Y_{N_1} is density of lightest right handed neutrino in a comoving volume given by $Y_{N_1} = n_{N_1}/s$, where $s = (2\pi^2/45)g_* T^3$ is the entropy density of the Universe at any epoch of temperature T. Finally δ is the wash out factor at a temperature just below the mass scale of N_1. The value of Y_{N_1} depends on the source of N_1. For example, the value of Y_{N_1} estimated from topological defects [29] can be different from thermal scenario [4, 18]. In the present case we will restrict ourselves to thermal scenario only.

Recent observations from WMAP show that the matter-antimatter asymmetry in the present Universe measured in terms of (n_B/n_γ) is [30]

$$\left(\frac{n_B}{n_\gamma}\right)_0 \equiv \left(6.1^{+0.3}_{-0.2}\right) \times 10^{-10}, \tag{16}$$

where the subscript 0 presents the asymmetry today. Therefore, we recast equation (15) in terms of (n_B/n_γ) and is given by

$$\left(\frac{n_B}{n_\gamma}\right)_0 = 7(Y_B)_0$$
$$= 2.48\epsilon_1 Y_{N_1}\delta. \quad (17)$$

Substituting equation (14) in (17) we get

$$\left(\frac{n_B}{n_\gamma}\right)_0 \leq 2.48\left(\frac{3M_1}{16\pi v^2}\right) m_3 Y_{N_1}\delta. \quad (18)$$

The present neutrino oscillation data favours the bilarge neutrino mixing with the mass squared differences $\Delta m^2_{\rm atm} \equiv |m_3^2 - m_1^2| \approx 2.6 \times 10^{-3} eV^2$ and $\Delta m^2_{\rm sol} \equiv |m_2^2 - m_1^2| \approx 7.1\times10^{-5}eV^2$. Assuming a normal mass hierarchy ($m_3^2 \gg m_2^2 \gg m_1^2$) for the light Majorana neutrinos the above number gives $m_3 \simeq (\Delta m^2_{\rm atm})^{1/2} \simeq 0.05eV$. With this approximation we get from equation (18)

$$M_1 \geq 2.5\times10^8 GeV\left(\frac{10^{-2}}{Y_{N_1}\delta}\right)\left(\frac{0.05eV}{m_3}\right). \quad (19)$$

For the above lower limit on the mass scale of lightest right handed neutrino, $M_1 \geq 10^8$ GeV, we now proceed to find the plausible hierarchies (M_2/M_1) and (M_3/M_1) of the massive Majorana neutrinos in a model that are compatible with the current neutrino oscillation data. We check that the values we obtained are consistent with our assumptions.

IV. EXAMINING THE CONSISTENCY OF F-MATRIX EIGENVALUES

The solar and atmospheric evidences of neutrino oscillations are nicely accommodated in the minimal framework of the three-neutrino mixing, in which the three neutrino flavours ν_e, ν_μ, ν_τ are unitary linear combinations of three neutrino mass eigenstates ν_1, ν_2, ν_3 with masses m_1, m_2, m_3 respectively. The mixing among these three neutrinos determines the structure of the lepton mixing matrix [31] which can be parameterized as

$$U_L = \begin{pmatrix} c_1c_3 & s_1c_3 & s_3e^{i\delta} \\ -s_1c_2 - c_1s_2s_3e^{i\delta} & c_1c_2 - s_1s_2s_3e^{i\delta} & s_2c_3 \\ s_1s_2 - c_1c_2s_3 & -c_1s_2 - s_1c_2s_3e^{i\delta} & c_2c_3 \end{pmatrix} dia(1, e^{i\alpha}, e^{i(\beta+\delta)}), \quad (20)$$

where c_j and s_j stands for $cos\theta_j$ and $sin\theta_j$. The two physical phases α and β associated with the Majorana character of neutrinos are not relevant for neutrino oscillations [32] and will be set to zero here onwards. While the Majorana phases can be investigated in neutrinoless double beta decay experiments [33], the CKM-phase $\delta \in [-\pi, \pi]$ can be investigated in long base line neutrino oscillation experiments. For simplicity we set it to zero, since we are interested only in the magnitudes of elements of U_L. The best fit values of the neutrino masses and mixings from a global three neutrino flavors oscillation analysis are [34]

$$\theta_1 = \theta_\odot \simeq 34^\circ, \quad \theta_2 = \theta_{atm} = 45^\circ, \quad \theta_3 \leq 13^\circ, \tag{21}$$

and

$$\begin{aligned} \Delta m^2_\odot &= m_2^2 - m_1^2 \;\simeq\; m_2^2 = 7.1 \times 10^{-5} \text{ eV}^2 \\ \Delta m^2_{atm} &= m_3^2 - m_1^2 \;\simeq\; m_3^2 = 2.6 \times 10^{-3} \text{ eV}^2. \end{aligned} \tag{22}$$

From equation (7) and (8) we have

$$f = \frac{1}{v_L}\left[(U_L D_m U_L^T) + \frac{v^2}{v_R}(h f_{dia}^{-1} h^T)\right]. \tag{23}$$

We now assume a hierarchical texture for the Majorana Yukawa coupling to be

$$f_{dia} = \frac{M_1}{v_R}\begin{pmatrix} 1 & 0 & 0 \\ 0 & \alpha_A & 0 \\ 0 & 0 & \alpha_B \end{pmatrix}, \tag{24}$$

where $1 \ll \alpha_A = (M_2/M_1) < \alpha_B = (M_3/M_1)$. We identify the neutrino Dirac Yukawa coupling h with that of charged leptons [9]. Further we assume it to be of Fritzsch type [26]

$$h = \frac{(m_\tau/v)}{1.054618}\begin{pmatrix} 0 & a & 0 \\ a & 0 & b \\ 0 & b & c \end{pmatrix}. \tag{25}$$

By choosing the values of a,b and c suitably one can get the hierarchy for charged leptons. In particular, the values

$$a = 0.004, \quad b = 0.24 \quad \text{and} \quad c = 1 \tag{26}$$

give the mass hierarchy of charged leptons. For this set of values the mass matrix h is normalized with respect to the τ-lepton mass. The values of a, b and c are rougly in geometric progression. They can be expressed in terms of the electro-weak gauge coupling $\alpha_w = \frac{g^2}{4\pi} =$

$\frac{\alpha}{sin^2\theta_w}$. In particular $a = 2.9\alpha_w^2$, $b = 6.5\alpha_w$ and $c = 1$. Here onwards we will use this set of values for the parameters of h.

Substituting (24) and (25) in equation (23) we get

$$f = (\frac{eV}{v_L}) \left[((U_L D_m U_L^T/eV) + \frac{4}{(1.054165)^2} \frac{1}{(M_1/\text{GeV})} \begin{pmatrix} \frac{a^2}{\alpha_A} & 0 & \frac{ab}{\alpha_A} \\ 0 & a^2 + \frac{b^2}{\alpha_B} & \frac{bc}{\alpha_B} \\ \frac{ab}{\alpha_A} & \frac{bc}{\alpha_B} & \frac{b^2}{\alpha_A} + \frac{c^2}{\alpha_B} \end{pmatrix} \right], \quad (27)$$

For the demonstration purpose we choose a typical value of $M_1 = 2.5 \times 10^8$ GeV. Now by suitably choosing the parameters $m_1 = 1.0 \times 10^{-4} eV$, $\alpha_A = 51$, $\alpha_B = 191$, $\theta_{13} = 10^\circ$ we get

$$f_{dia} = \frac{6.42 \times 10^{-4} eV}{v_L} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 50.98 & 0 \\ 0 & 0 & 190.47 \end{pmatrix}. \quad (28)$$

Thus, for the above values of m_1 and M_1, the assumed hierarchies of right-handed neutrino masses are consistent with global low energy neutrino data. Further we emhasize that the value of m_1 for which the consistency is obtained is compatible with our assumption that

$$m_1^2 \ll m_3^2 \equiv \Delta m_{atm}^2. \quad (29)$$

Comparing equation (28) with (24) one can get

$$\frac{M_1}{v_R} = \frac{6.42 \times 10^{-4} eV}{v_L}. \quad (30)$$

This implies that $v_R = O(10^{11})$ GeV for $v_L = 0.1$ eV. These values of v_L and v_R are compatible with genuine see-saw $v_L v_R = \gamma v^2$ for a small value of $\gamma \simeq O(10^{-3})$. Although in the literature frequently it is stated γ is to be order of unity, we see, however, here that it is required to be $O(10^{-3})$ for the consistency. See, for example, the recently proposed type-II see-saw models in which $\gamma = v_L v_R / v^2 \geq O(10^{-4})$ [35].

Here we demonstrated the consistency of our choice of the matrix f with the current neutrino data for the minimum value of M_1. For higher values of M_1, to be consistent with $1 << \alpha_A < \alpha_B$, one can choose appropriate values of $m_1 \leq 10^{-3}$ eV and $\theta_{13} \leq 13^\circ$ in equation (27) which will reproduce the correct eigenvalues of the matrix f.

V. CONCLUSION

In this work we proved the universality of the upper bound on the CP-violating parameter ϵ_1 in general type-II see-saw models. Irrespective of any assumption regarding the magnitude of type-I and type-II terms we found that same bound holds for all cases. Assuming a normal mass hierarchy for the massive Majorana neutrinos we demonstrated that in a thermal scenario the present baryon asymetry can be explained for all values of $M_1 \geq O(10^8)GeV$ in case of left-right symetric models. We also consider a class of left-right symmetric models in which we assume the neutrino Dirac masses are of the Fritzsch type. In such models we found that plausible hierarchies for the massive Majorana neutrinos can be obtained which are compatible with the current neutrino oscillation data.

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