Baryogenesis via leptogenesis in presence of cosmic strings

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Abstract

We study the effect on leptogenesis due to $B - L$ cosmic strings of a $U(1)_{B-L}$ extension of the Standard Model. The disappearance of closed loops of $B-L$ cosmic strings can produce heavy right handed neutrinos, N_R's, whose CP-asymmetric decay in out-of-thermal equilibrium condition can give rise to a net lepton (L) asymmetry which is then converted, due to sphaleron transitions, to a Baryon (B) asymmetry. This is studied by using the relevant Boltzmann equations and including the effects of both thermal and string generated non-thermal N_R's. We explore the parameter region spanned by the effective light neutrino mass parameter $\tilde{m}_1$, the mass M_1 of the lightest of the heavy right-handed neutrinos (or equivalently the Yukawa coupling h_1) and the scale of $B - L$ symmetry breaking, η_{B-L}, and show that there exist ranges of values of these parameters, in particular with $\eta_{B-L} > 10^{11}$ GeV and $h_1 \gtrsim 0.01$, for which the cosmic string generated non-thermal N_R's can give the dominant contribution to, and indeed produce, the observed Baryon Asymmetry of the Universe when the purely thermal leptogenesis mechanism is not sufficient. We also discuss how, depending on the values of η_{B-L}, $\tilde{m}_1$ and h_1, our results lead to upper bounds on $\sin \delta$, where δ is the the CP violating phase that determines the CP asymmetry in the decay of the heavy right handed neutrino responsible for generating the L-asymmetry.

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I. INTRODUCTION

Present low energy neutrino oscillation data [1, 2, 3] are elegantly explained by the neutrino oscillation hypothesis with very small masses ($\leq$ 1 eV) of the light neutrinos. Neutrinos can have either Dirac or Majorana masses. Small Majorana masses of the light neutrinos, however, can be generated in a natural way through the seesaw mechanism [4] without any fine tuning. This can be achieved by introducing right handed neutrinos, N_R's, into the electroweak model which are neutral under the known gauge symmetries. The Majorana masses of these N_R's are free parameters of the model and are expected to be either at TeV scale [5] or at a higher scale [6, 7]. This indicates the existence of new physics beyond Standard Model (SM) at some predictable high energy scale.

The heavy right handed Majorana neutrinos are also an essential ingredient in currently one of the most favored scenarios of origin of the observed baryon asymmetry of the Universe (BAU), namely, the "baryogenesis via leptogenesis" scenario [8, 9, 10, 11]. Majorana mass of the neutrino violates lepton number (L) and thus provides a natural mechanism of generating a lepton asymmetry in the Universe. Specifically, leptogenesis can occur via the L-violating, CP-asymmetric, out-of-equilibrium [12] decay of the N_R's into SM leptons and Higgs. The resulting L-asymmetry is then partially converted to a baryon (B)-asymmetry via the non perturbative $B + L$ violating (but $B - L$ conserving) electroweak sphaleron transitions [13]. The attractive aspect of the leptogenesis mechanism is the link it implies between the physics of the heavy right handed neutrino sector and the experimental data on light neutrino flavor oscillation, thus making the scenario subject to experimental tests. Indeed, the magnitude of the L (and thus B) asymmetry produced depends on, among other parameters, the masses of the heavy neutrinos, which in turn are related to the light neutrino masses via the seesaw mechanism. The mass-square differences amongst the three light neutrino species inferred from the results of neutrino experiments, therefore, place stringent constraints on the leptogenesis hypothesis.

A natural way to implement the leptogenesis scenario is to extend the SM to a gauge group which includes $B - L$ as a gauge charge. The heavy neutrino masses are then determined by the scale of spontaneous breaking of this gauge symmetry. Further, with $B - L$ a conserved gauge charge and $B + L$ anomalous, we can start with a net $B = L = 0$ at a sufficiently early stage in the Universe. The observed B asymmetry must then be generated only after

the phase transition breaking the $B-L$ gauge symmetry.

It is well known that phase transitions associated with spontaneous breaking of gauge symmetries in the early Universe can, depending on the structure of the symmetry group and its breaking pattern, lead to formation of cosmic topological defects [14, 15] of various types. In particular, the simplest choice for the $B-L$ gauge symmetry group being a $U(1)_{B-L}$, the phase transition associated with spontaneous breaking of this $U(1)_{B-L}$ in the early Universe would, under very general conditions, lead to formation of *cosmic strings* [14, 15] carrying quantized $B-L$ magnetic flux. These "$B-L$" cosmic strings can be a *non-thermal* source of N_R's whose decay can give an extra contribution to the L and thereby B asymmetry in addition to that from the decay of N_R's of purely thermal origin ("thermal" leptogenesis). This can happen in the following two ways:

First, since the Higgs field defining the $B-L$ cosmic string is the same Higgs that also gives mass to the N_R through Yukawa coupling, the N_R's can be trapped inside the $B-L$ cosmic strings as fermionic zero modes [16]. Existence of zero energy solutions of fermions coupled to a Higgs field that is in a topological vortex string configuration is well-known [17, 18], and has been studied in a variety of models allowing cosmic strings [19, 20, 21, 22, 23]. As closed loops of $B-L$ cosmic strings oscillate, they lose energy due to emission of gravitational radiation and shrink in size. Eventually, when the size of the loop becomes of the order of the width of the string, the string loop disappears into massive particles among which will be the N_R's which were trapped inside the string as zero modes. Each closed loop would be expected to release at least one N_R when it finally disappears, and the decay of these N_R's would then give a contribution to the BAU through the leptogenesis route [16].

Second, collapsing, decaying or repeatedly self-intersecting closed loops of cosmic strings would in general produce heavy gauge and Higgs bosons of the underlying spontaneously broken gauge theory. In the context of cosmic strings in Grand Unified Theories (GUTs), the CP asymmetric B-violating decay of the heavy gauge and Higgs bosons released from cosmic string loops would produce a net B asymmetry [24, 25, 26, 27]. The sphaleron transitions would of course erase the B asymmetry *unless a net $B-L$ was generated*. If the strings under consideration are the $B-L$ cosmic strings, which can be formed at an intermediate stage of symmetry breaking in a GUT model based on $SO(10)$, for example, then the heavy gauge and Higgs bosons released from the decaying or collapsing $B-L$ cosmic string loops can themselves decay to N_R's since the N_R's have Yukawa and gauge

couplings to the (string-forming) Higgs and gauge boson, respectively. The decay of these Higgs and gauge boson generated N_R's can produce a net $B - L$ and thus contribute to the BAU through the leptogenesis route irrespective of the existence of zero modes of N_R's on cosmic strings.

In a previous work [28], we made a general analytical estimate of the contribution of the non-thermal N_R's produced by $B - L$ cosmic string loops to BAU. It was shown there that, in order for the resulting B asymmetry not to exceed the measured BAU inferred from the WMAP results [29], the mass M_1 of lightest right handed Majorana neutrino had to satisfy the constraint

$$M_1 \lesssim 2.4 \times 10^{12} \left(\eta_{B-L}/10^{13}\,\mathrm{GeV} \right)^{1/2} \,\mathrm{GeV}, \tag{1}$$

where η_{B-L} is the $U(1)_{B-L}$ symmetry breaking scale. In the above mentioned analytical study we had (a) taken the CP asymmetry parameter ϵ_1 to have its maximum value (see below), (b) not taken into account the contribution to BAU from the decay of the already existing thermal N_R's, and (c) neglected all wash-out effects (see below) on the final $B - L$ asymmetry. Indeed, our aim there was to make a simple analytical estimate of the possible maximum contribution of the cosmic string generated non-thermal N_R's to the measured BAU. It is, of course, clear that a complete analysis of leptogenesis in presence of cosmic strings can only be done by solving the full Boltzmann equations [9, 11] that include the non-thermal N_R's of cosmic string origin as well as the already existing thermal N_R's and take into account all the relevant interaction processes including the wash out effects. This is the study taken up in this paper.

The main results obtained in the present paper can be summarized as follows: First, we confirm the result, obtained earlier in our analytical study [28], that $B - L$ cosmic string loops can give significant contribution to BAU only for $\eta_{B-L} \gtrsim 10^{11}\,\mathrm{GeV}$. Second, the numerical solution of the relevant Boltzmann equations in the present paper has enabled us to track the dynamical evolution of the contribution of the cosmic string generated non-thermal N_R's to the final BAU. Specifically, we find that for sufficiently large values of η_{B-L} and for appropriate ranges of values of the other relevant parameters $\tilde{m}_1$ and h_1 as detailed in sec. IV D, the effect of the cosmic string generated non-thermal N_R's is to produce a late-time increase of the final value of BAU as compared to its value in absence of cosmic strings. This can be understood from the fact that while the thermal abundance of N_R's decreases exponentially with decreasing temperature, the density of cosmic string generated

N_R's has a power law dependence (on temperature) inherited from the scaling behavior of the evolution of cosmic strings, leading to a domination of the string generated N_R's over the thermal N_R's at late times for sufficiently large values of η_{B-L}. In such situations, we are required to place an *upper* bound on the magnitude of $\sin\delta$, where δ is the CP violating phase that determines the CP asymmetry in the heavy neutrino sector, in order not to overproduce the BAU.

The rest of our paper is organized as follows. In section II, we briefly review the standard thermal baryogenesis via leptogenesis scenario, and discuss the required lower bounds on the mass of the lightest right handed neutrino and the $B - L$ symmetry breaking scale. In section III we introduce closed loops of $B - L$ cosmic strings as non-thermal sources of N_R's and write down the injection rates of these N_R's due to the two main processes of disappearance of cosmic string loops. Section IV is devoted to setting up and then solving the relevant Boltzmann equations for the evolution of the $B - L$ asymmetry, including the non-thermal N_R's of cosmic string origin in addition to the usual thermal ones. The effects of the cosmic string generated non-thermal N_R's on the evolution of the $B - L$ asymmetry are discussed. Finally, section V summarizes our main results.

II.   A BRIEF REVIEW OF BARYOGENESIS VIA LEPTOGENESIS

A.   The general framework

In the model we consider, the lepton asymmetry arises through the decay of right handed Majorana neutrinos to the SM leptons (ℓ) and Higgs (ϕ) via the Yukawa coupling

$$\mathcal{L}_Y = f_{ij}\bar{\ell}_i\phi N_{R_j} + \text{h.c.}\,, \tag{2}$$

where f_{ij} is the Yukawa coupling matrix, and $i, j = 1, 2, 3$ for three flavors.

We use the basis in which the mass matrix of the heavy Majorana neutrinos, M, is diagonal, where the Majorana neutrinos are defined by $N_j = \frac{1}{\sqrt{2}}(N_{R_j} + N_{R_j}^c)$. However, in this basis the Dirac mass matrix m_D ($= fv$, v being the vacuum expectation value of the SM Higgs) of the neutrinos is not diagonal. The canonical see-saw mechanism then gives the corresponding light neutrino mass matrix

$$m_\nu = -m_D M^{-1} m_D^T. \tag{3}$$

In this basis the light neutrino mass matrix m_ν can be diagonalized by the lepton mixing matrix [30] U_L to give the three light neutrino masses,

$$\mathrm{diag}(m_1, m_2, m_3) = U_L^T m_\nu U_L. \tag{4}$$

The best fit values of the mass-square differences and mixing angles of the light neutrinos from a global three neutrino flavors oscillation analysis are [31]

$$\theta_\odot \equiv \theta_1 \simeq 34°, \quad \theta_{atm} \equiv \theta_2 = 45°, \quad \theta_3 \leq 13°, \tag{5}$$

and

$$\Delta m_\odot^2 \equiv |m_2^2 - m_1^2| \simeq 7.1 \times 10^{-5}\,\mathrm{eV}^2\,, \tag{6}$$

$$\Delta m_{atm}^2 \equiv |m_3^2 - m_1^2| \simeq 2.6 \times 10^{-3}\,\mathrm{eV}^2\,. \tag{7}$$

In the absence of data about the overall scale of neutrino masses, we shall assume throughout this paper a normal hierarchy, $m_1 \ll m_2 \ll m_3$, for the light neutrino masses. With this assumption equations (6) and (7) give

$$m_2 \simeq \sqrt{\Delta m_\odot^2} = 0.008\,\mathrm{eV}\,,$$
$$m_3 \simeq \sqrt{\Delta m_{atm}^2} = 0.05\,\mathrm{eV}\,. \tag{8}$$

We will use the above value of m_3 to obtain the upper bound on CP-asymmetry in subsection II B.

B.   Decay of heavy Majorana neutrino and CP-asymmetry

The CP-asymmetry parameter in the decay of N_i is defined as

$$\epsilon_i \equiv \frac{\Gamma(N_i \to \bar{\ell}\phi) - \Gamma(N_i \to \ell\phi^\dagger)}{\Gamma(N_i \to \bar{\ell}\phi) + \Gamma(N_i \to \ell\phi^\dagger)}\,. \tag{9}$$

We assume a normal mass hierarchy in the heavy Majorana neutrino sector, $M_1 < M_2 < M_3$, and further assume that the final lepton asymmetry is produced mainly by the decays of the lightest right handed Majorana neutrino, N_1. The latter is justified [9, 11] because any asymmetry produced by the decay of N_2 and N_3 is erased by the lepton number violating interactions mediated by N_1. At an epoch of temperature $T > M_1$, all the lepton number violating processes mediated by N_1 are in thermal equilibrium. As the Universe expands and

cools to $T \lesssim M_1$, the L-violating scatterings mediated by N_1 freeze out, thus providing the out-of-equilibrium situation [12] necessary for the survival of any net L-asymmetry generated by the decay of the N_1's. The final L-asymmetry is, therefore, given essentially by the product of the equilibrium number density of the N_1's at $T \sim M_1$ and the CP-asymmetry parameter ϵ_1. The latter is given by [6, 7]

$$|\epsilon_1| = \frac{3}{16\pi v^2} M_1 m_3 \sin \delta \,, \tag{10}$$

where $v \simeq 174 \, \text{GeV}$ is the electroweak symmetry breaking scale and δ is the CP violating phase. Using the best fit value of m_3 from equation (8) the value of ϵ_1 can be written as

$$|\epsilon_1| \leq 9.86 \times 10^{-8} \left(\frac{M_1}{10^9 \, \text{GeV}} \right) \left(\frac{m_3}{0.05 \, \text{eV}} \right) \,. \tag{11}$$

C. Thermal leptogenesis and bound on M_1

The L-asymmetry, created by the decays of N_1, is partially converted to a B-asymmetry by the nonperturbative sphaleron transitions which violate $B + L$ but preserve $B - L$. Assuming that sphaleron transitions are ineffective at temperatures below the electroweak transition temperature (T_{EW}), the B-asymmetry is related to L-asymmetry by the relation [32]

$$B = \frac{p}{p-1} L \simeq -0.55 L \,, \tag{12}$$

where we have taken $p = 28/79$ appropriate for the particle content in SM. The net baryon asymmetry of the Universe, defined as $Y_B \equiv (n_B/s)$, can thus be written as

$$Y_B \simeq 0.55 \epsilon_1 Y_{N_1}|_{T \simeq M_1} \, d \,, \tag{13}$$

where the factor 0.55 in front comes from equation (12),

$$Y_{N_1} \equiv \frac{n_{N_1}}{s} \,, \tag{14}$$

n_{N_1} being the number density of N_1, and d is the dilution factor due to wash-out effects. In the above equations s stands for the entropy density and is given by

$$s = \frac{2\pi^2}{45} g_* T^3 \simeq 43.86 (g_*/100) T^3 \,, \tag{15}$$

where g_* is the total number of relativistic degrees of freedom contributing to entropy of the Universe.

The present-day observed value of the baryon-to-photon ratio $\eta \equiv (n_B - n_{\bar{B}})/n_\gamma$ inferred from the WMAP data is [29]

$$\eta_0^{\text{WMAP}} \simeq 7.0\, Y_{B,0} \tag{16}$$

$$= \left(6.1^{+0.3}_{-0.2}\right) \times 10^{-10} . \tag{17}$$

Using equation (11) in equation (13) and comparing with (17) we get

$$M_1 \geq 10^9\,\text{GeV} \times \left(\frac{1.6 \times 10^{-3}}{Y_{N_1}|_{T \simeq M_1} d}\right) \left(\frac{0.05 eV}{m_3}\right) . \tag{18}$$

Assuming that N_1's are initially (i.e., at $T \gg M_1$) in thermal equilibrium and they remain so till $T \simeq M_1$, one has $Y_{N_1}|_{T \simeq M_1} \approx 2.3 \times 10^{-3} \times (100/g_*)$. Equation (18) then indicates that the $B-L$ symmetry breaking scale must satisfy the constraint $\eta_{B-L} \gtrsim O(10^9)\,\text{GeV}$ for successful thermal leptogenesis, unless we allow the Majorana neutrino Yukawa coupling of the lightest right handed neutrino, h_1, to be greater than unity.

III. $B-L$ COSMIC STRINGS AS SOURCES OF NON-THERMAL HEAVY NEUTRINOS

A. $B-L$ cosmic strings and zero modes of N_R

The possibility of $B-L$ cosmic strings in various extensions of the SM were studied in [33, 34]. For simplicity, we consider in the present paper a model based on the gauge group $SU(2)_{\text{L}} \otimes U(1)_{\text{Y}} \otimes U(1)_{\text{Y}'}$, where Y' is a linear combination of Y and $B-L$ [35]. We then follow the symmetry breaking scheme

$$SU(2)_L \otimes U(1)_Y \otimes U(1)_{Y'} \xrightarrow{\langle \chi \rangle = \eta_{B-L}} SU(2)_L \otimes U(1)_Y$$
$$\xrightarrow{\langle \phi \rangle = v} U(1)_{EM}, \tag{19}$$

where χ is the Higgs boson required to break the $U(1)_{Y'}$ gauge symmetry and $\phi^T = (\phi^+, \phi^0)$ is the SM Higgs.

As the Universe cools below the critical temperature, T_{B-L}, of the $B-L$ symmetry breaking phase transition, the Higgs field χ develops a vacuum expectation value $\langle \chi \rangle = \eta_{B-L}$. The same Higgs field also forms strings. The mass per unit length of the string, μ, is given by [15] $\mu \sim \eta_{B-L}^2 \sim T_{B-L}^2$. The exact value of μ depends on the values of the parameters

of the model, in particular the relevant gauge and Higgs boson masses, and can differ from η_{B-L}^2 by up to an order of magnitude depending on the model [36]. Measurements of CMB anisotropies have been used to place upper bounds on the fundamental cosmic string parameter μ in a variety of different cosmic string models; see, for example, [37]. A recent analysis [38] of data on CMB anisotropies and large scale structure together puts the bound $G\mu < 3.4(5) \times 10^{-7}$ at 68 (95)% c.l. This most likely rules out cosmic string formation at a typical GUT scale $\sim 10^{16}\,\text{GeV}$. However, lighter cosmic strings arising from symmetry breaking at lower scales, for example, the $B - L$ cosmic strings discussed in this paper, are not ruled out.

The right handed neutrinos acquire Majorana masses from coupling to the same Higgs field χ,

$$-\mathcal{L}_{\chi-N_R} = \frac{1}{2}[ih_{ii}\chi\overline{N_{R_i}}N_{R_i}^c + \text{h.c}]\,, \tag{20}$$

where h is the Yukawa coupling matrix, and $N_R^c = i\sigma^2 N_R^*$ defines the Dirac charge conjugation operation. The equation of motion of N_R in the background of χ forming the string admits $|n|$ normalizable zero-modes in the winding number sector n [16, 17, 18, 22]. On a straight string these modes are massless. However, on a generic string they are expected to acquire effective masses proportional to the inverse radius of the string curvature. As soon as this mass becomes comparable to the mass of the free neutrinos in the bulk medium, these neutrinos can be emitted from the string. In any case, when the string loop shrinks and finally decays it emits various massive particles: the gauge bosons, the heavy Higgs (χ) and the massive right handed Majorana neutrinos (N_i).

B. N_R's from closed loops of $B - L$ cosmic strings

A key physical process that governs the evolution of cosmic string networks in an expanding Universe is the formation of sub-horizon sized closed loops which are pinched off from the network whenever a string segment curves over into a loop and intersects itself. It is this process that allows the string network to reach a scaling regime, in which the energy density of the string network scales as a fixed fraction of the radiation or matter energy density in the Universe; see, e.g., Ref. [15] for a text book discussion of evolution of cosmic strings in the Universe.

After their formation, the closed loops eventually disappear through either of the following

two processes (see Refs. [28, 39] for the relevant details):

1. *Slow death*

Closed loops born in non-selfintersecting configurations oscillate freely with oscillation time period $L/2$ (L being the length of the loop). In doing so they slowly lose energy due to emission of gravitational radiation, and thus shrink in size. Eventually, when the radius of the loop becomes of the order of the width ($\sim \eta_{B-L}^{-1}$) of the string — which happens over a time scale large compared to L (hence "slow") — the resulting "tiny" loop loses topological stability and decays into elementary particle quanta including the gauge- and Higgs bosons associated with the underlying broken symmetry as well the heavy neutrinos N_i's coupled to the gauge- and Higgs bosons. While we expect the final number N_N of the heavy neutrinos released per tiny loop to be of order one, it is difficult to be more precise, and we keep this number as an undetermined parameter. Assuming that the energy of a loop going into N_R's is predominantly in the form of the lightest of the heavy neutrinos, N_1's, the number of heavy Majorana neutrinos released from the closed loops disappearing through this "slow death" (SD) process per unit time per unit volume at any time t (in the radiation dominated epoch) can be written as [28]

$$\left(\frac{dn_{N_1}}{dt}\right)_{\mathrm{SD}} = N_N f_{\mathrm{SD}}\frac{1}{x^2}\left(\Gamma G\mu\right)^{-1}\frac{(K+1)^{3/2}}{K}t^{-4}\,,\tag{21}$$

where f_{SD} is the fraction of newly born loops which die through the SD process, $x \sim 0.5$ is a numerical factor that characterizes the scaling configuration of the string network, $\Gamma \sim 100$ is a numerical factor that determines the life time of a loop due to gravitational radiation emission, and $K \sim O(1)$ is a numerical factor that determines the average length of the closed loops at their birth. It is generally argued [15] that $f_{\mathrm{SD}} \simeq 1$.

Using (14), the above rate then gives the injection rate of massive Majorana neutrinos from cosmic string closed loops disappearing through the SD process per comoving volume as

$$\left(\frac{dY_{N_1}^{\mathrm{st}}}{dz}\right)_{\mathrm{SD}} = \frac{1.57 \times 10^{-17}}{z^4}N_N\left(\frac{M_1}{\eta_{B-L}}\right)^2\left(\frac{M_1}{\mathrm{GeV}}\right)\,,\tag{22}$$

where $z = M_1/T$ is the dimensionless variable with respect to which the evolution of the various quantities is studied. In the above we have used the following numerical values for the various constants: $\Gamma = 100$, $x = 0.5$, $g_* = 100$ and $K = 1$.

2. Quick death

Some fraction of closed loops may be born in configurations with waves of high harmonic number. Such string loops have been shown [40] to have a high probability of self-intersecting. In this case a loop of length L can break up into a debris of tiny loops of size η_{B-L}^{-1} (at which point they turn into the constituent massive particles) on a time-scale $\sim L$ (hence "quick"). Since gravitational energy loss occurs over a time scale much larger than L, these loops do not radiate any significant amount of energy in gravitational radiation, and thus almost the entire original energy of these loops would eventually come out in the form of massive particles.

Assuming again that each tiny loop of size $\sim \eta_{B-L}^{-1}$ yields a number N_N of heavy neutrinos, the injection rate of the massive Majorana neutrinos due to this "quick death" (QD) process is given by [28]

$$\left(\frac{dn_{N_1}}{dt}\right)_{\mathrm{QD}} = f_{\mathrm{QD}} N_N \frac{1}{x^2} \frac{\eta_{B-L}}{t^3} , \tag{23}$$

where f_{QD} is the fraction of loops that undergo QD.

Note that while each tiny loop yields the same number N_N of the heavy neutrinos irrespective of the SD or QD nature of the demise of the loop, the total injection rates of the heavy neutrinos in the two cases are different because of the different number of tiny loops that result from an initial big loop and the different time scales involved in the two cases.

Using (14) the above rate (23) can be rewritten as

$$\left(\frac{dY_{N_1}^{\mathrm{st}}}{dz}\right)_{\mathrm{QD}} \simeq \frac{1.36 \times 10^{-36}}{z^2} f_{\mathrm{QD}} N_N \left(\frac{\eta_{B-L}}{\mathrm{GeV}}\right) \left(\frac{M_1}{\mathrm{GeV}}\right) . \tag{24}$$

While the value of f_{QD} is not known, there are constraints [39, 41] on f_{QD} from the measured flux of the ultrahigh energy cosmic rays above $\sim 5 \times 10^{19}\,\mathrm{eV}$ and more stringently from the cosmic gamma ray background (CGRB) in the 10 MeV – 100 GeV energy region measured by the EGRET experiment [42]. The latter gives

$$f_{\mathrm{QD}} \eta_{16}^2 \lesssim 9.6 \times 10^{-6} , \tag{25}$$

where $\eta_{16} \equiv (\eta_{B-L}/10^{16}\,\mathrm{GeV})$. For GUT scale cosmic strings with $\eta_{16} = 1$, for example, the above constraint implies that $f_{\mathrm{QD}} \leq 10^{-5}$, so that most loops should be in non-selfintersecting configurations, consistent with our assumption of $f_{\mathrm{SD}} \sim 1$. Note, however, that f_{QD} is not constrained by (25) for cosmic strings formed at scales $\eta_{B-L} \lesssim 3.1 \times 10^{13}\,\mathrm{GeV}$.

IV. LEPTOGENESIS IN PRESENCE OF COSMIC STRINGS: TOWARDS A COMPLETE ANALYSIS

A. Analytical estimate

In our previous work [28] we made simple analytical estimates of the maximum possible contributions of cosmic string loops to leptogenesis. Neglecting the contribution of the thermal N_R's, allowing the maximal value of the CP-violation parameter given by equation (11) and demanding that the resulting value of the baryon-to-photon ratio not exceed the observed value given by equation (17), we derived upper bounds on the mass M_1 for the SD and QD processes of decay of cosmic string loops. This is shown in Figure 1 for $N_N = 10$ for illustration.
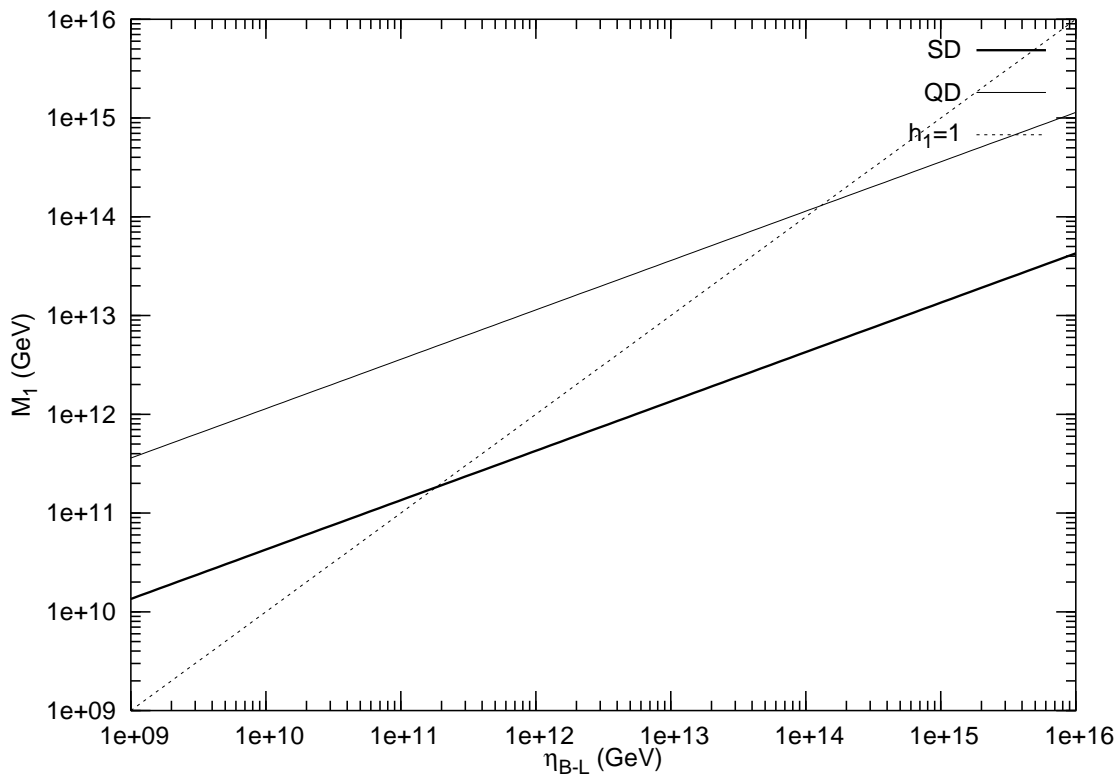

FIG. 1: Constraints on $h_1 = M_1/\eta_{B-L}$ from consideration of possible maximal contributions of $B - L$ cosmic string loops to B-asymmetry. Models lying on the thick (thin) solid line can in principle produce the observed BAU entirely due to the SD (QD) process of $B - L$ cosmic string loops. Models above the thick solid line are ruled out from consideration of overproduction of the B asymmetry. The $h_1 = 1$ line is also shown for comparison.

Figure 1 allows us to identify regions in the $\eta_{B-L} - M_1$ plane determining models for which cosmic string processes discussed above can play a significant role in the leptogenesis mechanism. Clearly, from Figure 1, cosmic string are relevant for leptogenesis only for $\eta_{B-L} \gtrsim 1.7 \times 10^{11}\,\mathrm{GeV}$; lower values of η_{B-L} are relevant only if we allow $h_1 \geq 1$. Note also that the above lower limit on η_{B-L} (for cosmic strings to be relevant for leptogenesis) is determined by the SD process; the QD process becomes relevant only at much higher values of η_{B-L}, namely, $\eta_{B-L}^{\mathrm{QD}} \gtrsim 1.2 \times 10^{14}\,\mathrm{GeV}$.

In this context, it may be mentioned here that, following our previous work, a recent work [43] has found that in the case of degenerate neutrinos (as opposed to hierarchical neutrinos assumed in our work), the $B-L$ cosmic strings become relevant for leptogenesis at much higher values of η_{B-L} compared to those in the case of hierarchical neutrinos, e.g., $\eta_{B-L} \gtrsim 3.3 \times 10^{15}\,\mathrm{GeV}$ for the SD process and $\eta_{B-L} > 10^{16}\,\mathrm{GeV}$ for the QD process. As already mentioned, we assume hierarchical neutrino masses in the present paper.

We now proceed towards a complete analysis of leptogenesis in presence of cosmic strings by first setting up and then solving the relevant Boltzmann equations that include the non-thermal N_R's produced from the decaying cosmic string loops as well as the already existing thermal N_R's and also include all the relevant wash-out effects.

B. Boltzmann Equations

The total rate of change of the abundance of N_1's can be written as

$$\frac{dY_{N_1}}{dz} = \left(\frac{dY_{N_1}}{dz}\right)_{D,S} + \left(\frac{dY_{N_1}}{dz}\right)_{\mathrm{injection}} . \tag{26}$$

The first term on the right hand side of equation (26) is given by the usual Boltzmann equation [9, 11]

$$\left(\frac{dY_{N_1}}{dz}\right)_{D,S} = -(D + S)\left(Y_{N_1} - Y_{N_1}^{\mathrm{eq}}\right) , \tag{27}$$

where D and S constitute the decay and $\Delta L = 1$ lepton number violating scatterings, respectively, which reduce the number density of N_1, and $Y_{N_1}^{\mathrm{eq}}$ is the abundance of N_1 in thermal equilibrium.

The second term on the right hand side of equation (26) gives the rate of injection of

N_1's due to disappearance of $B-L$ cosmic string loops:

$$\left(\frac{dY_{N_1}}{dz}\right)_{\text{injection}} = \left(\frac{dY_{N_1}^{\text{st}}}{dz}\right)_{\text{SD}} + \left(\frac{dY_{N_1}^{\text{st}}}{dz}\right)_{\text{QD}}, \tag{28}$$

where the two terms on the right hand side are given by the equations (22) and (24), respectively.

The two terms in equation (26) compete with each other. While the first term reduces the density of N_1's, the second term tries to increase it through continuous injection of N_1's from the disappearance of cosmic string loops. The CP-violating decays of the N_1's produce a net $B-L$ asymmetry. This can be calculated by solving the Boltzmann equation

$$\frac{dY_{B-L}}{dz} = -\epsilon_1 D \left(Y_{N_1} - Y_{N_1}^{\text{eq}}\right) - W Y_{B-L}. \tag{29}$$

The first term in equation (29) involving the decay term D produces an asymmetry while a part of it gets erased by the wash out terms represented by W which includes the processes of inverse decay as well as the $\Delta L = 1$ and $\Delta L = 2$ lepton number violating scatterings. In a recent work [44] it has been claimed that thermal corrections to the above processes as well as processes involving gauge bosons are important for the final $B-L$ asymmetry. However, this is under debate [45], and we have not included these corrections in the present paper.

In equations (27) and (29), $D = \Gamma_D/Hz$, where

$$\Gamma_D = \frac{1}{16\pi v^2}\tilde{m}_1 M_1^2 \tag{30}$$

is the decay rate of N_1, with

$$\tilde{m}_1 \equiv \frac{(m_D^\dagger m_D)_{11}}{M_1} \tag{31}$$

the *effective neutrino mass* parameter [11], and H is the Hubble expansion parameter. Also, $S = \Gamma_S/Hz$, where Γ_S is the rate of $\Delta L = 1$ lepton number violating scatterings and $W = \Gamma_W/Hz$, where Γ_W is the rate of wash out effects involving the $\Delta L = 1$ and $\Delta L = 2$ lepton number violating processes and inverse decay. The various Γs are related to the scattering densities [9] γ's as

$$\Gamma_i^X(z) = \frac{\gamma_i(z)}{n_X^{\text{eq}}}. \tag{32}$$

The dependence of the scattering rates involved in $\Delta L = 1$ lepton number violating processes on $\tilde{m}_1$ and M_1 are similar to that of the decay rate Γ_D. As the Universe expands these Γ's

compete with the Hubble expansion parameter. In a comoving volume we have (with same notations as in [9])

$$\left(\frac{\gamma_D}{sH(M_1)}\right), \left(\frac{\gamma_{\phi,s}^{N_1}}{sH(M_1)}\right), \left(\frac{\gamma_{\phi,t}^{N_1}}{sH(M_1)}\right) \propto k_1 \tilde{m}_1. \tag{33}$$

On the other hand the γ's for the $\Delta L = 2$ lepton number violating processes depend on $\tilde{m}_1$ and M_1 as

$$\left(\frac{\gamma_{N_1}^l}{sH(M_1)}\right), \left(\frac{\gamma_{N_1,t}^l}{sH(M_1)}\right) \propto k_2 \tilde{m}_1^2 M_1. \tag{34}$$

In the above equations (33), (34), k_i, $i = 1, 2, 3$ are dimensionful constants determined from other parameters; see [9] for details.

C. Constraint on the effective neutrino mass ($\tilde{m}_1$)

In order to satisfy the out of equilibrium condition the decay rate of N_1 has to be less than the Hubble expansion parameter. This imposes a constraint on the effective neutrino mass parameter $\tilde{m}_1$ as follows. At an epoch $T < M_1$,

$$\frac{\Gamma_D}{H} \equiv \frac{\tilde{m}_1}{m_*} = K < 1, \tag{35}$$

where m_*, the *cosmological neutrino mass parameter*, is given by [46]

$$m_* \equiv 4\pi g_*^{1/2} \frac{G_N^{1/2}}{\sqrt{2}G_F} = 6.4 \times 10^{-4}\,\text{eV}, \tag{36}$$

where g_* is the effective number of relativistic degrees of freedom. We see that a net $B - L$ asymmetry can be generated dynamically provided $\tilde{m}_1 < m_*$ at an epoch $T < M_1$. This constraint on $\tilde{m}_1$ can be realized in a model as follows.

We assume a charge-neutral lepton symmetry for which we take the texture of the Dirac mass of the neutrino of the form [47]

$$m_D = \begin{pmatrix} 0 & \sqrt{m_e m_\mu} & 0 \\ \sqrt{m_e m_\mu} & m_\mu & \sqrt{m_e m_\tau} \\ 0 & \sqrt{m_e m_\tau} & m_\tau \end{pmatrix}. \tag{37}$$

Using (18) and (37) in equation (31) we get the constraint to be

$$\tilde{m}_1 \leq 5.25 \times 10^{-6}\,\text{eV}, \tag{38}$$

which is in consonance with the requirement that $\tilde{m}_1 < m_*$ at any epoch $T < M_1$.

D. Solving the Boltzmann equations

At an epoch $T \gg M_1$ the lepton number violating processes are sufficiently fast as to set the $B - L$ asymmetry to zero. As the temperature falls and becomes comparable to M_1, a net $B - L$ asymmetry is generated through the CP-violating decay of N_1. The resulting asymmetry can be obtained by solving the Boltzmann equations. We solve equations (26) and (29) numerically with the following initial conditions

$$Y_{N1}^{\text{in}} = Y_{N_1}^{\text{eq}} \ \text{and} \ Y_{B-L}^{\text{in}} = 0. \tag{39}$$

Using the first initial condition we solve equation (26) for Y_{N_1}, and the corresponding $B - L$ asymmetry Y_{B-L} is obtained from equation (29) by using the second initial condition of equation (39).

In the usual thermal scenario the $B - L$ asymmetry depends on $\tilde{m}_1$ and M_1. In presence of cosmic strings there is in addition an explicit dependence on η_{B-L} since the injection rate of N_R's from the cosmic string loops explicitly depends on it. To illustrate the effect of the cosmic string generated non-thermal N_R's on the evolution of the $B - L$ asymmetry, we display in Figures 2 and 3 the results obtained by numerically solving the Boltzmann equations described above for some specific values of the various relevant parameters. It can be seen from Figure 2(a), for example, that in the absence of cosmic strings the $B - L$ asymmetry approaches the final value at $T \simeq 0.1 M_1$ when all the wash out processes fall out of equilibrium. In contrast, in the presence of cosmic strings, for sufficiently large values of M_1, the $B - L$ asymmetry continues to build up until the injection rate of N_R's from cosmic string loops becomes insignificant. For $h_1 = 1$ this happens around $T \simeq M_1/600$ (see Figure 2(b)), which is far lower than in the purely thermal case. As a result of this, in presence of cosmic strings, for a fixed value of the symmetry breaking scale $\eta_{B-L} = 10^{13}\,\text{GeV}$, for example, the final $B-L$ asymmetry is enhanced by three orders of magnitude for the effective neutrino mass $\tilde{m}_1 = 10^{-4}\,\text{eV}$ (Fig. 2(b)) and by two orders of magnitude for $\tilde{m}_1 = 10^{-5}\,\text{eV}$ (Fig. 3(b)). For $\eta_{B-L} = 10^{13}\,\text{GeV}$, the effect of cosmic string essentially disappears for $h_1 < 0.01$

The above results, in particular the dependence on the value of $\tilde{m}_1$, can be understood as follows: First let us consider the purely thermal leptogenesis case, i.e., in absence of cosmic strings. Below a certain temperature when the wash out processes fall out of equilibrium the asymmetry produced by the decay of N_1's does not get wiped out, and the produced
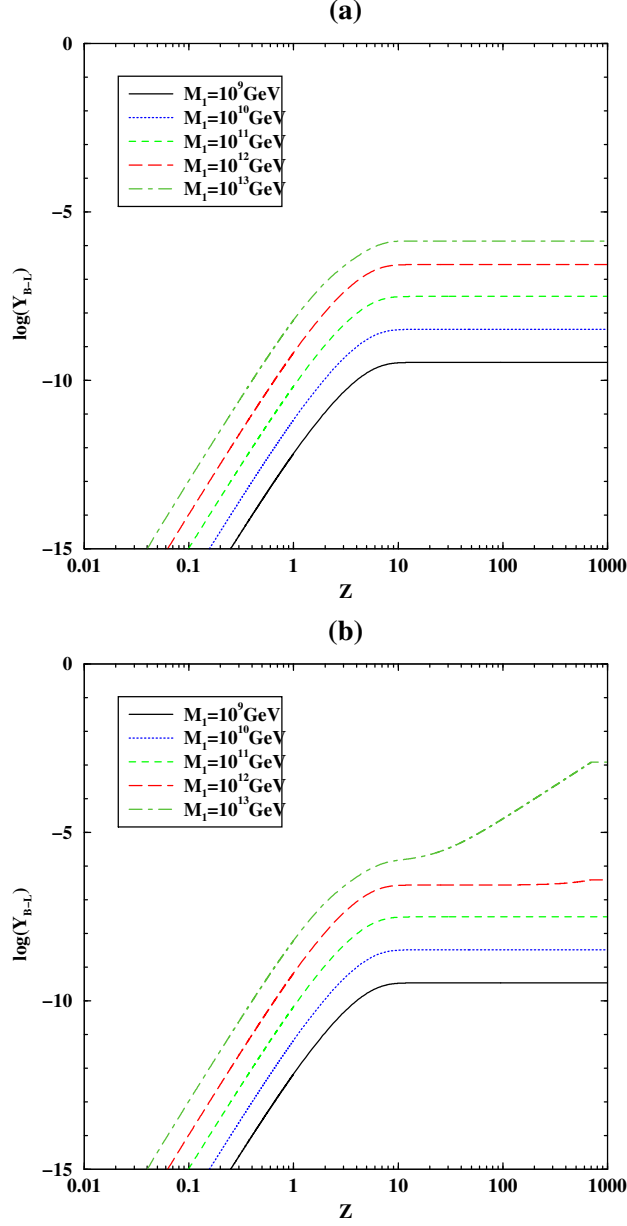
FIG. 2: Evolution of the $B - L$ asymmetry for $\tilde{m}_1 = 10^{-4}\,\text{eV}$, $\eta_{B-L} = 10^{13}\,\text{GeV}$ and different values of M_1 (a) in absence of cosmic strings, and (b) in presence of cosmic strings. The CP violation parameter ϵ_1 has been given its maximal value.

$B - L$ asymmetry remains as the final asymmetry. Since the decay rate of N_1's depends linearly on $\tilde{m}_1$ (see equation (33)), a larger value of $\tilde{m}_1$ implies that the condition of decay in out-of-equilibrium situation is satisfied at a later time when the abundance of thermal N_1's is smaller, thus yielding a smaller final value of Y_{B-L}. And as expected, this effect is larger for larger values of M_1. Now, in the presence of cosmic strings, for values of η_{B-L} for which
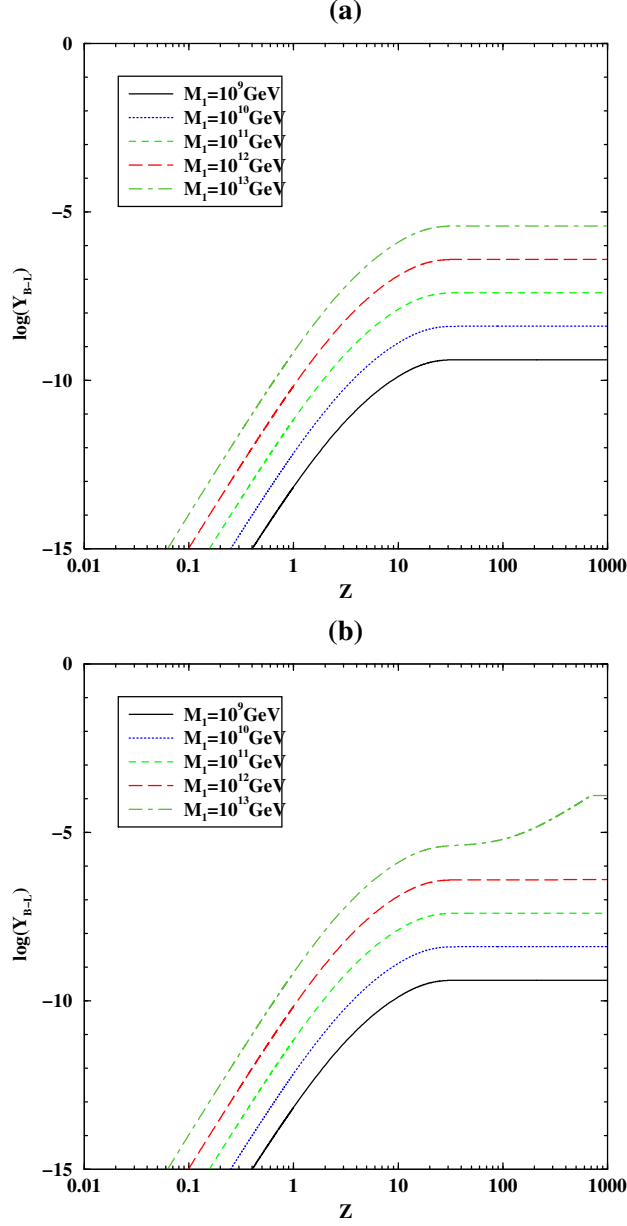
FIG. 3: Evolution of the $B - L$ asymmetry for $\tilde{m}_1 = 10^{-5}\,\mathrm{eV}$, $\eta_{B-L} = 10^{13}\,\mathrm{GeV}$ and different values of M_1 (a) in absence of cosmic strings, and (b) in presence of cosmic strings. The CP violation parameter ϵ_1 has been given its maximal value.

the contribution of the cosmic string generated N_1's to Y_{B-L} always remains insignificant compared to that due to the thermal N_1's, the dependence of the final value of Y_{B-L} on $\tilde{m}_1$ is essentially the same as in the absence of strings as explained above. However, for those values of η_{B-L} for which the string contributions to Y_{B-L} are significant compared to the thermal contribution, the dependence of the final value of Y_{B-L} on $\tilde{m}_1$ is opposite to that in

the absence of cosmic strings, i.e., the string contribution increases with increasing values of $\tilde{m}_1$. This is easy to understand: The string generated N_R's make dominant contribution to Y_{B-L}, if at all, only at late times ($T \ll M_1$) when the abundance of the thermal N_R's has fallen to insignificant levels. The decay of the string generated N_R's at such late times automatically satisfies the out-of-equilibrium condition. In this situation a larger value of $\tilde{m}_1$ simply implies a larger rate of decay of the N_R's leading to a quicker development of the string contribution to Y_{B-L}. This, together with the fact that the injection rate of the N_R's from cosmic string loops is higher at earlier times, leads to a higher final value of Y_{B-L}. Figures 2(b) and 3(b) plotted for $\tilde{m}_1 = 10^{-4}\,\mathrm{eV}$ and $\tilde{m}_1 = 10^{-5}\,\mathrm{eV}$ simultaneously bear out these expectations.

Note that in Figures 2 and 3 the CP asymmetry parameter ϵ_1 has been taken to have its maximum value, i.e., $\sin\delta = 1$ in equation (10). Clearly, for this value of ϵ_1 the produced B asymmetry for $\eta_{B-L} = 10^{13}\,\mathrm{GeV}$ and $M_1 \gtrsim 10^{10}\,\mathrm{GeV}$ (i.e., $h_1 \gtrsim 10^{-3}$) exceeds the observed value given by $Y_{B-L}^{\mathrm{obs}} \sim O(10^{-10})$ even for purely thermal leptogenesis. Including the effect of cosmic strings only makes the situation worse. This implies that we must place an upper bound on the CP asymmetry phase δ depending on the values of η_{B-L}, M_1 and $\tilde{m}_1$, in order not to overproduce the B asymmetry. At the same time, to produce the observed BAU for $\eta_{B-L} = 10^{13}\,\mathrm{GeV}$ and $h_1 = 1$, for example, we require $\sin\delta = \sin\delta_{\mathrm{reqd}} = O(10^{-7})$ and $\sin\delta_{\mathrm{reqd}} = O(10^{-6})$ for $\tilde{m}_1 = 10^{-4}\,\mathrm{eV}$ and $\tilde{m}_1 = 10^{-5}\,\mathrm{eV}$, respectively. These values of $\sin\delta$ diminish the purely thermal contributions, Y_{B-L}^{th}, to $O(10^{-13})$ and $O(10^{-12})$, respectively.

We have calculated the value of Y_{B-L} for a range of values of η_{B-L}, h_1 and $\tilde{m}_1$, both in the purely thermal case, Y_{B-L}^{th}, and with the effect of cosmic strings included, $Y_{B-L}^{\mathrm{th+st}}$. The results for $\eta_{B-L} = 10^{12}\,\mathrm{GeV}$ and $\eta_{B-L} = 10^{11}\,\mathrm{GeV}$ are summarized in Tables I and II, respectively, where we also indicate the order of magnitude of $\sin\delta_{\mathrm{reqd}}$, the value of $\sin\delta$ required to produce the observed B asymmetry, for each set of the chosen parameters.

From these Tables as well as from Figures 2 and 3 we see that there exist ranges of values of the parameters η_{B-L}, h_1, $\tilde{m}_1$ and δ for which the cosmic string generated N_R's make the dominant contribution to and indeed produce the observed BAU while the purely thermal leptogenesis mechanism is not sufficient.

h_1	$\tilde{m}_1 = 10^{-4}$ eV			$\tilde{m}_1 = 10^{-5}$ eV		
	Y_{B-L}^{th}	$Y_{B-L}^{\text{th+st}}$	$\log(\sin\delta_{\text{reqd}})$	Y_{B-L}^{th}	$Y_{B-L}^{\text{th+st}}$	$\log(\sin\delta_{\text{reqd}})$
1	2.72×10^{-7}	1.23×10^{-5}	-5	3.90×10^{-7}	1.59×10^{-6}	-4
0.1	3.10×10^{-8}	3.22×10^{-8}	-2	3.97×10^{-8}	3.98×10^{-8}	-2
0.01	3.25×10^{-9}	3.25×10^{-9}	-1	4.04×10^{-9}	4.04×10^{-9}	-1
0.001	3.39×10^{-10}	3.39×10^{-10}	0	4.11×10^{-10}	4.11×10^{-10}	0

TABLE I: The $B-L$ asymmetry in the purely thermal leptogenesis case (Y_{B-L}^{th}) and in presence of $B-L$ cosmic strings ($Y_{B-L}^{\text{th+st}}$) for $\eta_{B-L} = 10^{12}$ GeV and different values of the Yukawa coupling h_1 and the effective neutrino mass $\tilde{m}_1$. The order of magnitude of the value of $\sin\delta$ required to produce the observed BAU, $\sin\delta_{\text{reqd}}$, for each set of values of the parameters $\tilde{m}_1$ and h_1, is also given.

h_1	$\tilde{m}_1 = 10^{-4}$ eV			$\tilde{m}_1 = 10^{-5}$ eV		
	Y_{B-L}^{th}	$Y_{B-L}^{\text{th+st}}$	$\log(\sin\delta_{\text{reqd}})$	Y_{B-L}^{th}	$Y_{B-L}^{\text{th+st}}$	$\log(\sin\delta_{\text{reqd}})$
1	3.10×10^{-8}	1.51×10^{-7}	-3	3.97×10^{-8}	5.17×10^{-8}	-2
0.1	3.25×10^{-9}	3.27×10^{-9}	-1	4.04×10^{-8}	4.04×10^{-9}	-1
0.01	3.39×10^{-10}	3.39×10^{-10}	0	4.11×10^{-10}	4.11×10^{-10}	0

TABLE II: Same as Table I but for $\eta_{B-L} = 10^{11}$ GeV.

V. CONCLUSION

We have studied the effect of $B-L$ cosmic strings arising from the breaking of a $U(1)_{B-L}$ gauge symmetry, on the baryon asymmetry of the Universe. The disappearance of closed loops of $B-L$ cosmic strings can produce heavy right handed neutrinos, N_R's, whose CP-asymmetric decay in out-of-thermal equilibrium condition can give rise to a net lepton (L) asymmetry which is then converted, due to sphaleron transitions, to a Baryon (B) asymmetry. We have solved the relevant Boltzmann equations that include the effects of both thermal and string generated non-thermal N_R's. By exploring the parameter region

spanned by the effective light neutrino mass parameter $\tilde{m}_1$, the mass M_1 of the lightest of the heavy right-handed neutrinos (or equivalently the Yukawa coupling h_1) and the scale of the $B - L$ symmetry breaking, η_{B-L}, we found that there exist ranges of values of these parameters, in particular with $\eta_{B-L} > 10^{11}$ GeV and $h_1 \gtrsim 0.01$, for which the cosmic string generated non-thermal N_R's can give the dominant contribution to, and indeed produce, the observed Baryon Asymmetry of the Universe when the purely thermal leptogenesis mechanism is not sufficient. We have also discussed how, depending on the values of η_{B-L}, $\tilde{m}_1$ and h_1, our results lead to upper bounds on the CP violating phase δ that determines the relevant CP asymmetry in the decay of the heavy right handed neutrino responsible for generating the L-asymmetry.

[1] S. Fukuda et al (Super-Kamiokande Collaboration), Phys. Rev. Lett. **86**, 5656 (2001).

[2] S.N. Ahmed et al (SNO collaboration), arXiv:nucl-ex/0309004; Q.R. Ahmed et al, Phys. Rev. Lett. **89**, 011301-011302 (2002).

[3] K. Eguchi et al (KamLAND collaboration), Phys. Rev. Lett. **90**, 021802 (2003).

[4] M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity*, ed. P. van Nieuwenhuizen and D. Freedman (North Holland 1979); T. Yanagida, in Proc. of *Workshop on unified theory and baryon number in the universe*, ed. O. Sawada and A. Sugamoto (KEK 1979); S. L. Glashow, *Cargese Lectures* (1979); R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980). For a text book discussion see, e.g., R.N. Mohapatra and P.B. Pal, *Massive Neutrinos in Physics and Astrophysics* (3rd edition) (World Scientific, Singapore, 2004).

[5] N. Sahu and U.A. Yajnik, Phys. Rev. D**71**, 2005 (023507) [arXiv:hep-ph/0410075], [arXiv:hep-ph/0509285].

[6] S. Davidson and A. Ibarra, Phys. Lett. B **535**, 25 (2002); W. Buchmuller, P. Di Bari and

M. Plumacher, Nucl. Phys. B **643**, 367 (2002).

[7] S. Antusch and S. F. King, Phys. Lett. B **597**, 199 (2004) [arXiv:hep-ph/0405093]; N. Sahu and S. Uma Sankar, Phys. Rev. D**71**, 2005 (013006) [arXiv:hep-ph/0406065]; Pei-hong Gu and Xiao-jun Bi, Phys. Rev. D**70**, 2004 (063511).

[8] M. Fukugita and T. Yanagida, Phys. Lett. B**174**, 45 (1986).

[9] M.A. Luty, Phys. Rev. D**45**, 455 (1992).

[10] R.N. Mohapatra and X. Zhang, Phys. Rev. D **46**, 5331 (1992).

[11] M. Plümacher, Z. Phy. C**74**, 549 (1997).

[12] A.D. Sakharov, JETP Lett. **5**, 24 (1967).

[13] V.A. Kuzmin, V.A. Rubakov and M.E. Shaposhnikov, Phys. Lett. B**155**, 36 (1985).

[14] T.W.B. Kibble, J. Phys. **A9**, 1387 (1976).

[15] A. Vilenkin and E.P.S. Shellard, *Cosmic strings and other topological defects* (Cambridge University Press, 1994).

[16] R. Jeannerot, Phys. Rev. Lett. **77**, 3292 (1996).

[17] R. Jackiw and P. Rossi, Nucl. Phys. B **190**, 681 (1981).

[18] E.J. Weinberg, Phys. Rev. D**24**, 2669 (1981).

[19] S. R. Das, Nucl. Phys. **B227**, 462 (1983).

[20] E. Witten, Nucl. Phys. **B249**, 557 (1985).

[21] A. Stern and U. A. Yajnik, Nucl. Phys. B **267**, 158 (1986).

[22] S.C. Davis, Int. J. Theor. Phys. **38**, 2889 (1999).

[23] G. Starkman, D. Stojkovic and T. Vachaspati, Phys. Rev. D**63**, 085011 (2001); Phys. Rev. D**65**, 065003 (2002).

[24] P. Bhattacharjee, T.W.B. Kibble and N. Turok, Phys. Lett. B**119**, 95 (1982).

[25] R.H. Brandenberger, A.C. Davis and M. Hindmarsh, Phys. Lett. B **263**, 239 (1991).

[26] H. Lew and A. Riotto, Phys. Rev. D **49**, 3837 (1994).

[27] P. Bhattacharjee, Phys. Rev. Lett. **81**, 260 (1998).

[28] P. Bhattacharjee, N. Sahu and U.A. Yajnik, Phys. Rev. D **70**, 083534, 2004 [arXiv: hep-ph/0406054].

[29] D.N. Spergel *et. al.*, Astrophys. J. Suppl. **148**, 175 (2003) [astro-ph/0302209].

[30] Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962).

[31] M.C. Gonzalez-Garcia and C. Pena-Garay, Phys. Rev. D**68**, 093003 (2003).

[32] J.A. Harvey and M.S. Turner, Phys. Rev. D**42**, 3344 (1990).

[33] R. Jeannerot and A.C. Davis, Phys. Rev. D **52**, 7220 (1995); R. Jeannerot Phys. Rev. D **53**, 5426 (1996); R. Jeannerot, J. Rocher, M. Sakellariadou, Phys. Rev. D **68** 103514 (2003).

[34] U. A. Yajnik, H. Widyan, D. Choudhari, S. Mahajan and A. Mukherjee, Phys. Rev. D **59**, 103508 (1999).

[35] W. Buchmuller, C. Greub and P. Minkwoski, Phys. Lett. B**267**, 395 (1991). See also ref. [11].

[36] C.T. Hill, H.M. Hodges, and M.S. Turner, Phys. Rev. Lett. **59**, 2493 (1987).

[37] R. Jeannerot and M. Postma, JCAP **0512**, 006 (2005); M. Landriau and E.P.S. Shellard, Phys. Rev. D **69**, 023003 (2004); N. Bevis, M. Hindmarsh and M. Kunz, Phys. Rev. D **70**, 043508 (2004); R. Durrer, M. Kunz and A. Melchiorri, Phys. Rept. **364**, 1 (2002); C. Contaldi, M. Hindmarsh and J. Magueijo, Phys. Rev. Lett. **82**, 679 (1999).

[38] M. Wyman, L. Pogosian and I. Wasserman, Phys. Rev. D **72**, 023513 (2005) [arXiv: astro-ph/0503364].

[39] P. Bhattacharjee and G. Sigl, Phys. Rept. 327: 109 - 247 (2000).

[40] X. A. Siemens and T.W.B. Kibble, Nucl. Phys. B **438**, 307 (1995).

[41] P. Bhattacharjee and N.C. Rana, Phys. Lett. B **246**, 365 (1990).

[42] P. Sreekumar et al, Astrophys. J. **494**, 523 (1998).

[43] P. H. Gu and H. Mao, Phys. Lett. B **619**, 226 (2005) [arXiv:hep-ph/0503126].

[44] G.F. Giudice, A. Notari, M. Raidal, A. Riotto and A. Strumia, Nucl. Phys. B **685**, 89 (2004) [arXiv:hep-ph/0310123].

[45] W. Buchmuller, P. Di Bari and M. Plumacher, [arXiv:hep-ph/0401240].

[46] W. Fischler, G.F. Guidice, R.G. Leigh, and S. Paban Phys. Lett. **B258** 45 (1991).

[47] D. Falcone and F. Tramontano, Phys. Rev. D **63**, 073007 (2001) [arXiv:hep-ph/0011053]; B. R. Desai and A. R. Vaucher, matrix, Phys. Rev. D **63**, 113001 (2001) [arXiv:hep-ph/0007233]; J. L. Chkareuli and C. D. Froggatt, Phys. Lett. B **450**, 158 (1999) [arXiv:hep-ph/9812499].