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An Analysis of Stepped Trapezoidal Shaped Microcantilever beams for MEMS based Devices

An Analysis of Stepped Trapezoidal Shaped Microcantilever beams for MEMS based Devices

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Abstract

Microcantilever beam has been most widely used mechanical element in the design and fabrication of MEMS/NEMS based sensors and actuators. In this work, we have proposed a new design of microcantilever beam based on stepped trapezoidal shaped microcantilever. Single, double, triple and quadruple stepped trapezoidal shaped microcantilever beams along with the conventional rectangular shaped microcantilever beam were analysed experimentally, numerically and analytically. The microcantilever beams were fabricated from silicon dioxide material using wet bulk micromachining in 25 wt% TMAH. Length, width and thickness of the microcantilever beams are fixed as 200, 40 and 0.96 µm, respectively. Laser vibrometer is utilized to measure the resonance frequency and Q-factor of the microcantilever beams in vacuum as well as in ambient conditions. Furthermore, a finite element analysis software, ANSYS is employed to numerically analyse the resonance frequency, maximum deflection, and torsional end rotation of all designs of microcantilever beam. Analytical and numerical resonance frequencies are found to be in good agreement with experimental resonance frequencies. In stepped trapezoidal shaped microcantilever beams with increasing number of steps Q-factor, maximum deflection and torsional end rotation were improved, whereas the resonance frequency was slightly reduced. Nevertheless, the resonance frequency is higher than the basic rectangular shaped microcantilever beam. The observed quality factor, maximum deflection and torsional end rotation for quadruple stepped trapezoidal shaped microcantilever are 38%, 41% and 52%, respectively, higher than those of conventional rectangular shaped microcantilever beam. Furthermore, for the applied concentrated mass of 1 picogram on cantilever surface, a greater shift in frequency is obtained for all designs of stepped trapezoidal shaped microcantilever beams compared to conventional rectangular microcantilever beam.

Keywords: Microcantilever; Stepped beams; Bending and Torsion Motion.

1. Introduction

From past few years the use of microelectromechanical systems (MEMS) based sensors in various fields has been increasing tremendously because of the advantages such as low cost of manufacturing, high sensitivity, quick response, low power consumption, etc., offered by miniaturized sensors over the conventional sensors. One of the key constituents of a MEMS based sensor is the mechanical platform. Among many mechanical platforms, microcantilever has become most popular because of its simple design, ease of fabrication, low cost, higher sensitivity, etc. In addition, arrays of microcantilever beams can easily be fabricated. Thus, microcantilever-based sensors have been potentially employed in various fields [1-7]. Recently, the study of single cantilever or an array of microcantilever beams for chemical and biological sensors has been growing rapidly [8-12]. Therefore, as the scope of applicability of microcantilevers is broadening with time, it necessities intensified research on microcantilever beams. In the field of sensors, sensitivity is one of the key factors on which research has been mainly focused. Shape, geometry and resonating mode of the cantilever plays key role in the sensitivity of cantilever sensors as these parameters greatly influence the resonance frequency, quality factor and deflection of the cantilever which predominantly influence the sensitivity of sensors [13-27]. So far, numerous researchers have reported different shape and dimension of microcantilever beams as well as proposed higher resonating modes of vibration for improving sensitivity of microcantilever based sensors. Ansari *et al.* [14] investigated the deflection, resonance frequency and stress characteristics of rectangular, triangular and step profile microcantilevers numerically and analytically, and they found that triangular and step profile cantilevers have better frequency and deflection characteristics than the rectangular one. In another work, Ansari *et al.* [15] studied the deflection and vibration characteristics of rectangular and trapezoidal profile microcantilevers using ANSYS. They investigated three models for each profile and found that paddled trapezoidal profile microcantilever have better sensitivity. Subramanian *et al.* [16] proposed an improvement in the shape of the V-shaped microcantilever, they studied the variation of resonance frequency of V-shaped microcantilever as a function of changes in the microcantilever profile. Liu *et al.* [17] investigated the resonance frequency of microcantilevers with different shapes using analytical and finite element methods. Further, they also investigated the absorbed gas induced frequency shift of microcantilevers with various geometries and profiles of microcantilever. Hawari *et al.* [18] analysed six different designs of microcantilever using ANSYS, they investigated the effect of cantilever design on its fundamental resonance frequency, maximum deflection and equivalent (Von-Mises) stress. Lim *et al.* [19] investigated the influence of length, thickness, shape and material of microcantilever on the sensitivity of microcantilever biosensors. Furthermore, in order to investigate the effect of resonance mode on microcantilever mass-sensing performance, Xia and Li [20] theoretically, numerically and experimentally investigated the air drag damping effect on quality factor of first, second flexural and torsional modes for four different kinds of microcantilevers. Jin *et al.* [21] proposed piezoresistive microcantilever sensor which operates in second flexure mode for improved mass sensing performance in air. Furthermore, an optimized electromagnetic excitation method is proposed and developed to further improve the resolution [21]. Also, Li and Lee [22] presented review on integrated microcantilever devices for ultra-sensitive applications as well as for in-situ micro/nanoscale surface analysis and manipulation. The integrated cantilever devices are realized by integrating a microcantilever with self-sensing and self-actuating elements, specific sensing materials and functionalized nano-tips for transverse [22] as well as torsional motion [23]. To understand the effect of microcantilever shape, Singh *et al.* [24, 25] theoretically studied the influence of linearly and nonlinearly varying beam width of an unstepped (non-uniform) microcantilever on pull-in voltage, resonance frequency, and mass-sensitivity. Sahoo and Pandey [26] also utilized these variation in proposing a wide band piezoelectric energy harvester. Recently, A. Ashok *et al.* [27] performed numerical and experimental studies of the influence of mechanical coupled non-uniform beams on frequency tuning and amplitude tuning. Therefore, motivated by the literature on the influence of shape of micromachined cantilever beams for MEMS based sensors, we have proposed a new design of microcantilever, namely, stepped trapezoidal shaped microcantilever beams, for improving the performance of microcantilever based sensors compared to the basic rectangular shaped microcantilever based sensors. Resonance frequency, Q-factor, maximum deflection, frequency shift and torsional end rotation of both stepped trapezoidal and basic rectangular shaped microcantilever beams were analysed and compared.

2. Experimental Procedure

Czochralski (Cz) grown 4-inch diameter P-type boron doped (resistivity 8-12 Ωcm) {100} oriented single side polished silicon wafers were used for present experiments. The step by step fabrication process for the fabrication of silicon dioxide (SiO_2) hanging structures was explained elsewhere [27]. SiO_2 of about 1 μm thickness grown using thermal oxidation process is used as structural and masking material in the micromachining process. The oxidized wafers were patterned using UV photolithography (Midas Mask Aligner, MDA 400 M) followed by silicon dioxide etching in buffered hydrofluoric acid (BHF) solution. The patterned wafers were diced into small chips of size 2×2 cm^2. Chips are properly cleaned in piranha (H_2SO_4:H_2O_2:: 1:1) bath followed by rinsing with deionized (DI) water. This step is followed by removal of chemical oxide in 1% hydrofluoric acid (HF) solution and thorough rinse in DI water. Etching is carried out in 25 wt% TMAH (Tetramethylammonium hydroxide) (99.999%, Alfa Aesar) at 74.5 ±0.5 ˚C to fabricate overhanging SiO_2 microcantilevers. After etching process the samples were rinsed with DI water.

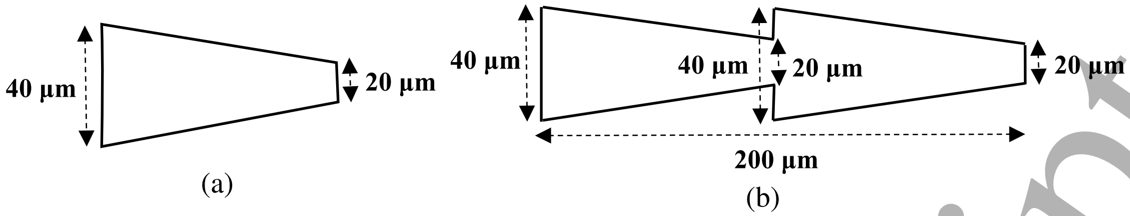


Figure 1. Schematic of (a) basic building block (trapezoid) of stepped trapezoidal shaped microcantilever beams (b) Single stepped trapezoidal shaped microcantilever beam.

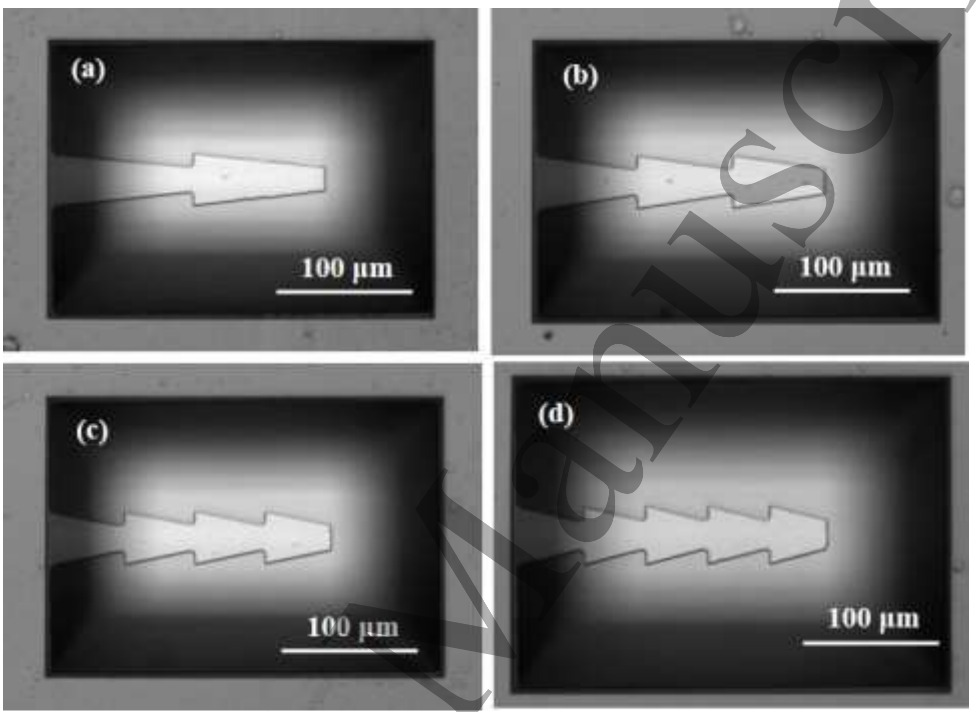


Figure 2. Optical micrographs of (a) single (b) double (c) triple and (d) quadruple stepped trapezoidal shaped microcantilever beams.

The length and width of the proposed and rectangular shaped microcantilevers are fixed as 200 μm and 40 μm, respectively. Figures 1(a) and (b) illustrate the schematic of trapezoid (basic building block of stepped trapezoidal profile microcantilever beams) and single stepped trapezoidal shaped microcantilever, respectively. As shown in Fig. 1, trapezoid width at the opening and ending are fixed as 40 μm and 20 μm, respectively, total length of all stepped trapezoidal shaped microcantilevers is fixed as 200 μm. In a given stepped trapezoidal shaped microcantilever, length of each trapezoid is same, however, its length depends on number of steps in the stepped trapezoidal shaped microcantilever beam. Figure 2 illustrates the optical micrographs of single, double, triple and quadruple stepped trapezoidal shaped microcantilever beams. Laser Scanning Vibrometer (Polytec, OFV-5000 vibrometer) whose outline of experimental set-up is explained elsewhere [27] was employed to investigate the modal resonance frequencies and corresponding mode shapes of the microcantilever beams. A finite element analysis software, ANSYS was used for performing the numerical analyses of the proposed microcantilever beams. For the analysis, length, thickness and material properties were kept same for all the designs of microcantilever beams proposed in this work. The material, silicon dioxide properties used in the present work are 66.26 Gpa for Elastic modulus, 30.12 Gpa for shear modulus, 2200 kg/m^3 for density and 0.1 for Poisson's ratio. The procedure of computing Elastic modulus of silicon dioxide for the present study is same as we explained it in our previous work [27]. Shell 181 element with four nodes and six degrees of freedom at each node available in the software is selected for the present study. Furthermore, quadratic type mesh element is utilized in the meshing process and the mesh convergence is achieved above 2000 number of elements with percentage error of less than 1 % with respect to steady state results.

3. Results and Discussion

3.1 Modal Analysis

The resonance frequencies of stepped trapezoidal as well as rectangular shaped microcantilever beams were measured in vacuum (8×10^{-3} mbar) and ambient (1.01325 bar) conditions using laser vibrometer setup. Figure 3 shows the comparison of first transverse and torsional mode resonance frequencies for stepped trapezoidal and rectangular shaped microcantilever beams. From the graph it is evident that the transverse and torsional mode resonance frequencies of all designs of stepped trapezoidal shaped microcantilevers are greater than the conventional rectangular shaped microcantilever beam.

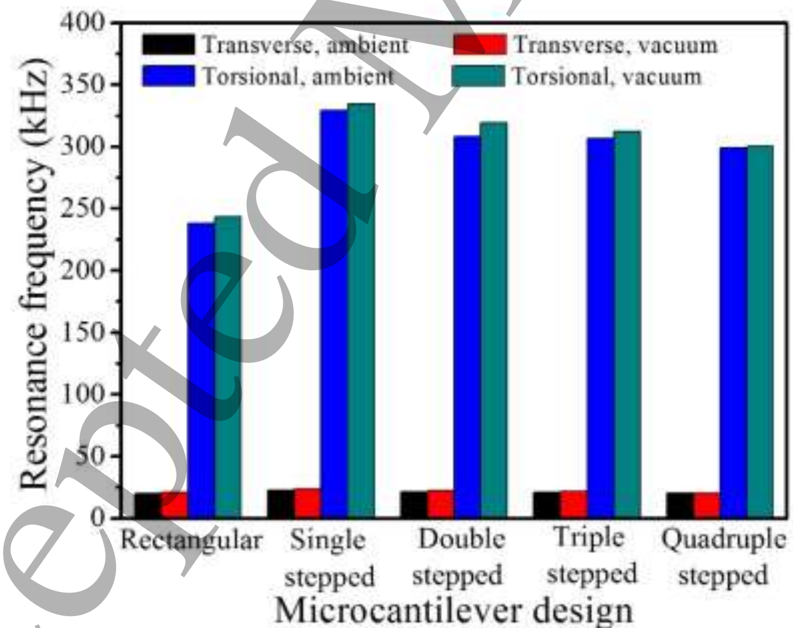


Figure 3. Dependence of first transverse and torsional mode resonance frequencies on microcantilever design.

It is well known that the resonance frequency of a cantilever is directly proportional to the square root of spring constant and inversely proportional to square root of mass of the cantilever [15]. Therefore, in this work, increase in resonance frequency of stepped trapezoidal shaped microcantilevers compared to rectangular shaped microcantilever may be due to marginal decrease in the mass or increase in effective stiffness of stepped trapezoidal shaped microcantilevers resulting from the shape change [13, 15, 18]. Further, it can also be observed from the graph that resonance frequency decreases as number of steps in the stepped trapezoidal shaped microcantilevers were increased. This could be due to relative reduction in stiffness (spring constant) and increase in effective mass of the stepped trapezoidal shaped microcantilevers with increasing number of steps [13-15, 18].

As stated before, to numerically analyse the resonance frequencies of microcantilevers finite element software, ANSYS is utilized. Table 1 shows the comparison of experimental and numerical resonance frequencies of first transverse and torsional modes for stepped trapezoidal and rectangular shaped microcantilevers. From the table it can be noticed that the error percentage is as small as 0.1% to maximum of 8% which manifests that experimental and numerical resonance frequencies are in good agreement.

Table 1: Comparison of experimental and numerical resonance frequencies of first transverse and torsional modes for rectangular and stepped trapezoidal shaped microcantilever beams

Cantilever type	Resonance frequency (kHz)			
	Transverse mode		Torsional mode	
	Experimental / Numerical	Error (%)	Experimental / Numerical	Error (%)
Rectangular beam	21.27 / 21.30	0.1	243.18 / 231.23	4.9
Un-stepped	- / 26.13	-	- / 394.67	-
Single stepped	23.64 / 23.38	1	334.60 / 311.72	6.8
Double stepped	22.44 / 22.34	0.4	319.45 / 296.09	7.3
Triple stepped	21.98 / 21.77	0.9	312.31 / 287.47	7.9
Quadruple stepped	20.65 / 21.37	3.4	297.59 / 281.62	5.3

Also, we have performed analytical study to determine fundamental mode resonance frequency of stepped trapezoidal as well as rectangular shaped microcantilever beams based on Rayleigh method as described by Purohit *et al.* [28] for monolithic cantilever beams. Table 2 presents the comparison of experimental and analytically measured fundamental resonance frequencies for stepped trapezoidal as well as rectangular shaped microcantilever beams. It can be noticed from the table that analytically computed resonance frequencies follow the similar trend followed by experimentally measured resonance frequencies. Error percentage of less than 1% to maximum of 11% between the experimental and analytically evaluated resonance frequencies indicates that the experimental and analytical results are in good agreement.

Table 2: Comparison of experimental and analytical resonance frequency of first transverse mode for rectangular and stepped trapezoidal shaped microcantilever beams

Cantilever type	Resonance frequency of fundamental mode (kHz) Experimental / Analytical	Error (%)
Rectangular	21.27 / 21.36	0.42
Un-stepped	- / 26.43	-
Single stepped	23.64 / 22.83	3.42
Double stepped	22.44 / 20.94	6.68
Triple stepped	21.98 / 19.51	11.23
Quadruple stepped	20.65 / 18.32	11.28

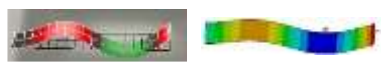
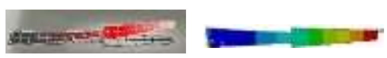
Furthermore, ANSYS workbench is employed to investigate the shift in fundamental frequency of microcantilever designs by applying a point mass of 1 picogram on cantilever's top surface. The point mass is positioned at the centre of beam, 10 μm from the free end of cantilever. The material properties and dimensions of microcantilever designs used were same as those used in ANSYS APDL software. The frequencies obtained in workbench were maximum of 8% less compared to the frequencies found using APDL software. Table 3 shows the comparison of fundamental resonance frequency of proposed and rectangular profile microcantilever beams corresponding to without and with adsorbed mass of 1 picogram. As it is evident from Table 3 that for the fixed weight of applied mass, shift in frequency for proposed stepped trapezoidal shaped microcantilever beams is higher than that of basic rectangular shaped microcantilever, which manifests that the proposed stepped designs would be more sensitive than the basic rectangular profile microcantilever. The increase in frequency shift of stepped trapezoidal shaped microcantilever beams compared to rectangular profile microcantilever beam is attributed to higher stiffness or reduced effective mass of stepped trapezoidal shaped microcantilever beams compared to the rectangular profile microcantilever beam [20]. In the present work, shift in frequency for un-stepped and single stepped microcantilever beams is improved by 94% & 57%, respectively, compared to shift in frequency of rectangular profile microcantilever beam.

Table 3. Frequency shift comparison for rectangular and stepped trapezoidal shaped microcantilever beams, mass attached is 1 picogram

Cantilever design	Frequency w/o mass F_1 (Hz)	Frequency with mass F_2 (Hz)	Frequency shift Δf (Hz)
Rectangular	19831	19350	481
Un-stepped	24278	23343	935
Single stepped	21681	20925	756
Double stepped	20685	19985	700
Triple stepped	20131	19460	671
Quadruple stepped	19752	19099	653

Mode shapes are also important along with resonance frequencies in designing the sensors. Hence, experimental and numerical resonance frequencies and corresponding vibrational mode shapes of transverse and torsional modes up to mode three for the rectangular and stepped trapezoidal shaped microcantilevers were presented in the following tables. Tables 4(a) and 4(b) illustrates the experimental and numerical resonance frequencies and corresponding vibrational mode shapes of transverse and torsional modes up to mode three for the rectangular and stepped trapezoidal shaped microcantilevers. The colour scales illustrated at the bottom of the tables represent the magnitude of vibration amplitude of vibrating cantilever relative to the stationary cantilever (black = 0, green = negative (min.), red = positive (max.) for experimental and blue = 0, red = maximum for numerical).

Table 4(a): Experimental and numerical resonance frequencies and corresponding mode shapes of first three transverse modes for rectangular and stepped trapezoid microcantilever beams

Cantilever type	Resonance frequency (kHz) Experimental / Numerical	Mode shape Experimental / Numerical
Rectangular Mode 1	21.27 / 21.3	
Mode 2	132.37 / 133.52	
Mode 3	370.32 / 373.98	
Single Stepped Mode 1	23.64 / 23.38	
Mode 2	141.96 / 142.04	
Mode 3	389.20 / 392.94	

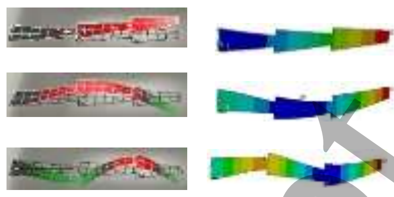
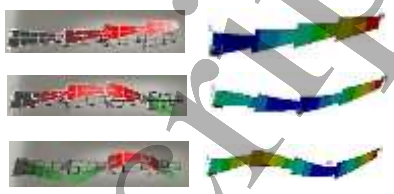
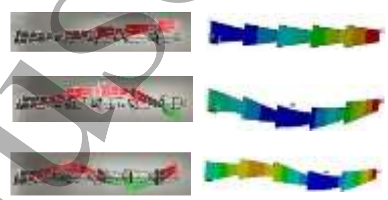
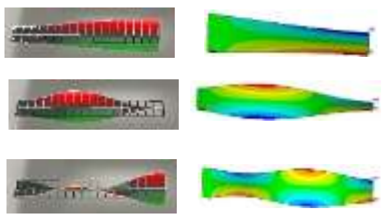
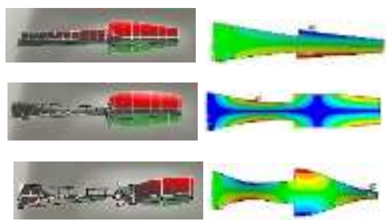
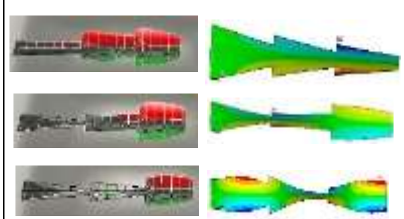
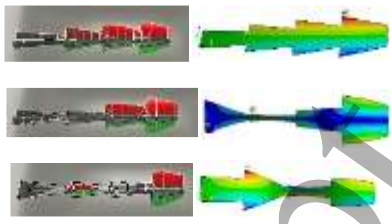
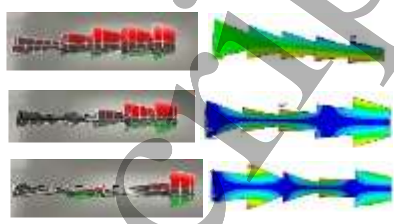
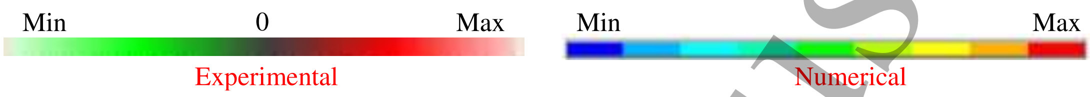
Double Stepped Mode 1 Mode 2 Mode 3	22.44 / 22.34 138.06 / 138.92 376.40 / 379.05	
Triple Stepped Mode 1 Mode 2 Mode 3	21.98 / 21.77 136.36 / 135.87 377.56 / 377.51	
Quadruple Stepped Mode 1 Mode 2 Mode 3	20.65 / 21.37 127.80 / 133.60 353.34 / 372.52	

Table 4(b): Experimental and numerical resonance frequencies and corresponding mode shapes of first three torsional modes for rectangular and stepped trapezoidal microcantilever beams

Cantilever type	Resonance frequency (kHz) Experimental / Numerical	Mode shape Experimental / Numerical
Rectangular Mode 1 Mode 2 Mode 3	243.20 / 231.23 722.98 / 703.67 825.09 / 1212.9	
Single Stepped Mode 1 Mode 2 Mode 3	334.60 / 311.72 691.23 / 1102.3 1182.26 / 1506.1	
Double Stepped Mode 1 Mode 2 Mode 3	319.45 / 296.09 608.73 / 894.71 964.37 / 1810.1	

Triple Stepped Mode 1 Mode 2 Mode 3	312.31 / 287.47 566.23 / 868.95 949.49 / 1462.5	
Quadruple Stepped Mode 1 Mode 2 Mode 3	297.59 / 281.62 536.23 / 851.85 907.96 / 1440.0	



3.2 Quality Factor

Quality factor (Q or Q-factor) which determines the sensing resolution of a cantilever sensor is one of the key paramters in the field of sensors. In order to investigate the influence of number of steps on the quality factor of stepped trapezoidal shaped microcantilevers and also to compare them with Q-factor of rectangular shaped microcantilever, Q-factor corresponding to first transverse and torsional modes for the proposed and rectangular shaped microcantilevers were evaluated in ambient (1.01325 bar) as well as in vacuum (8×10^{-3} mbar) conditions. In the present study, the Q-factor values were calculated from frequency spectrums based on half width method.

Figure 4 presents the Q-factor values corresponding to first transverse and torsional modes for rectangular as well as stepped trapezoidal shaped microcantilevers measured in vacuum as well as in ambient conditions. After performing measurements on more than three beams of same kind, a range of Q-factor values were presented corresponding to each mode in Fig.4. From the figure, one can observe that the quality factor of stepped trapezoidal shaped microcatilevers is higher than the reactangular shaped microcantilever.

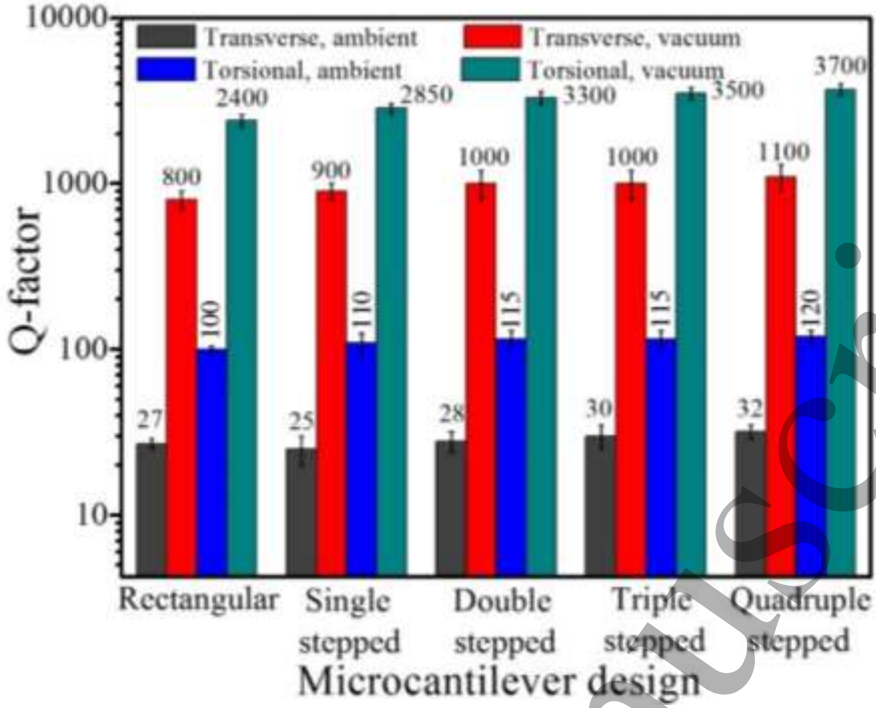


Figure 4. Variation in Q-factor of first transverse and torsional modes with respect to microcantilever design.

It can also be noticed that with increasing number of steps in the stepped trapezoidal shaped microcantilever, Q-factor is not altered significantly. However, it increases marginally with number of steps. In the present work, torsional mode of vibrations exhibits highest Q-factor values. It is well known that under vacuum condition, Q-factor of microcantilever beam is influenced not only by shape of the microcantilever but also by other factors such as support loss, thermoelastic damping and surface loss. Therefore, in the present work the difference in Q-factor with respect to microcantilever design is due to the influence of all aforementioned parameters. The first transverse and torsional mode's Q-factor of quadruple stepped trapezoidal profile microcantilever were improved by 38 and 54%, respectively, compared to those of rectangular profile microcantilever.

3.3 Deflection Analysis using ANSYS

In order to investigate the maximum deflection of stepped trapezoidal and rectangular shaped microcantilever beams, a fixed load of 100 μN is applied to top surface of cantilever at free end of the beam using ANSYS simulation software. Figure 5(a) shows the comparison of deflection produced in stepped trapezoidal and rectangular shaped microcantilevers due to applied load of 100 μN. From the graph it is clear that for applied fixed load, the deflection resulted in stepped trapezoidal shaped microcantilevers is greater compared to the deflection produced in rectangular shaped microcantilever. Further, it can also be noticed that as number of steps in the stepped trapezoidal shaped microcantilever increased, deflection slightly increases. The deflections found in quadruple and double stepped trapezoidal shaped microcantilever beams are 40 and 34%, respectively, higher than the deflection in rectangular profile microcantilever. The enhancement in the deflection of stepped trapezoidal shaped microcantilevers with increasing number of steps is possible mainly because of reduction in flexural stiffness of the microcantilever with increasing number of steps [13-15]. The increase in deflection of stepped trapezoidal shaped microcantilevers is manifested by decrease in the resonance frequency as observed in Fig. 3.

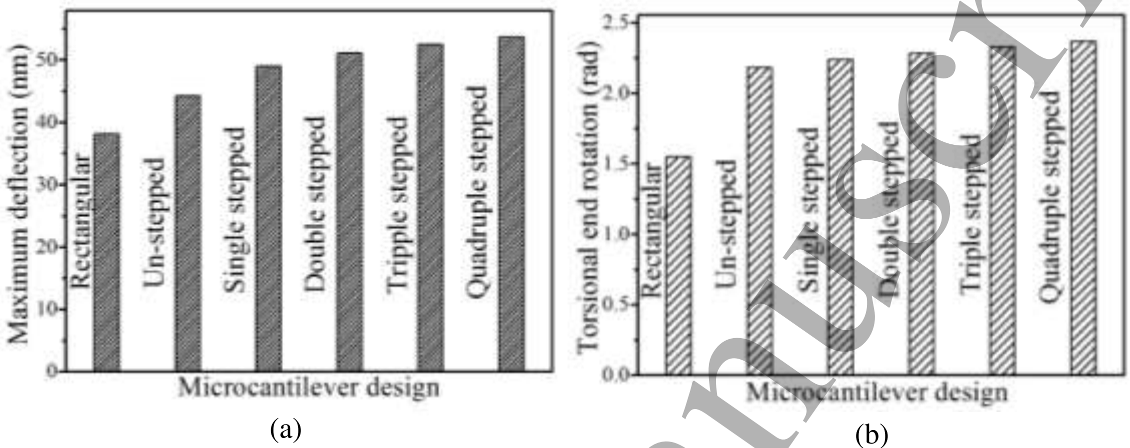


Figure 5. Comparison of (a) maximum deflection corresponding to applied load of 100 μN (b) torsional end rotation corresponding to applied of 100×10^{-6} μNm for stepped trapezoidal and rectangular shaped microcantilever beams.

Similar to maximum deflection, in order to investigate the torsional end rotation of microcantilever beams, 100×10^{-6} μNm torque is applied to cantilevers surface about the X-axis using ANSYS. Figure 5(b) illustrates the torsional end rotation of stepped trapezoidal and rectangular shaped microcantilevers corresponding to applied torque of 100×10^{-6} μNm. From the graph, it can be observed that the torsional end rotation of stepped trapezoidal microcantilevers increases linearly with number of steps in the stepped trapezoidal microcantilever. Maximum torsional end rotation of 2.369 rad is obtained for the quadruple stepped trapezoidal shaped microcantilever beam which is 52% greater than the torsional end rotation of rectangular shaped microcantilever. Thus, from this study it can be concluded that the deflection of stepped trapezoidal shaped microcantilevers can be further enhanced by increasing the number of steps in stepped microcantilever.

3.4 Tuning of Stepped Trapezoidal Shaped Microcantilever beam for Coinciding Step Locations with Vibrational Nodes

The advantage of stepped microcantilever is that it can be modelled such that the step locations of beams can be made to coalescence the vibrational nodes of microcantilever. The number of step beams or step locations and the length of each step beam in stepped microcantilever depends on how many nodal points need to be coincided with step locations which in turn depends on the number of modes of interest. In this work, we have modelled a stepped trapezoidal shaped microcantilever corresponding to a microcantilever of length 200 μm and the number of modes of interest are up to four modes. In order to design a stepped microcantilever equivalent to a microcantilever of length 200 μm and to coincide all the nodal point encounters in up to fourth mode total seven step beams or six step locations are required. Figures 6(a)&(b) illustrates the scanning points and zero locations in mode 1 to 4 for rectangular as well as quadruple stepped trapezoidal shaped microcantilever beams produced using Laser vibrometer. The length of step beams were find out based on the idea of distance between vibrational nodes and fixed position of cantilever as illustrated in Figures 6(a)&(b).

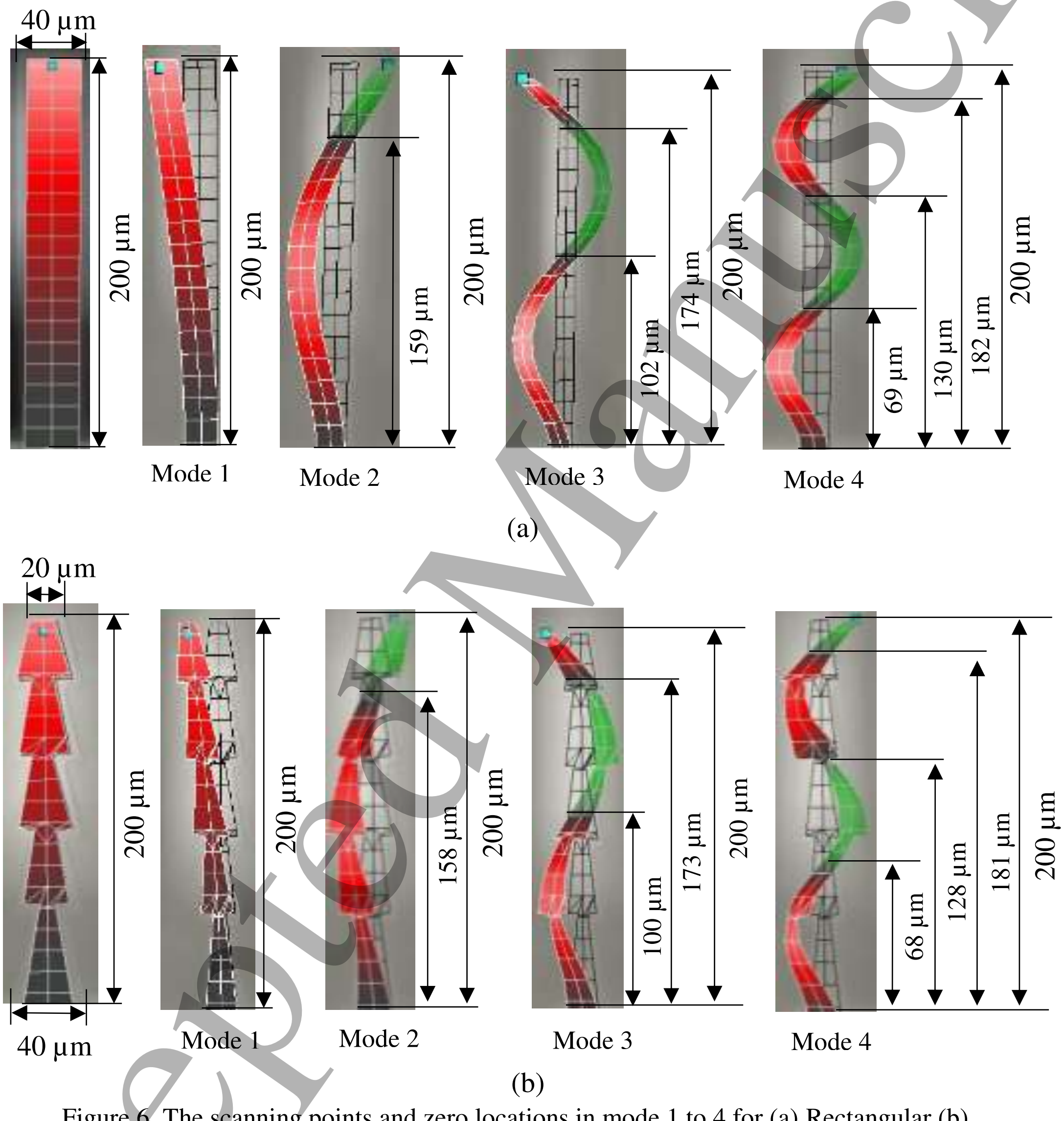


Figure 6. The scanning points and zero locations in mode 1 to 4 for (a) Rectangular (b) Quadruple stepped trapezoidal shaped microcantilever beams.

As we can observe from the figures, nodes distances were almost same for the rectangular as well as stepped trapezoidal shaped microcantilever beams. Therefore, using these nodal distances as length of step beams, stepped trapezoidal shaped microcantilever beam with seven step beams of different lengths is created to coalescence the vibrational nodes of microcantilever with step locations. Further, stepped trapezoidal shaped microcantilever comprising seven step beams of equal length (200 / 7 = 28.57 μm) is also created to compare the resonance frequencies and zero locations of vibrations with stepped trapezoidal shaped microcantilever comprising of unequal length step beams.

Figures 7(a)-(c) shows the comparison of resonance frequencies and corresponding mode shapes up to four modes for stepped trapezoidal shaped microcantilevers comprising of equal and unequal length step beams, respectively. From Fig. 7(a) it can be observed that the resonance frequencies of both type of stepped trapezoidal shaped microcantilevers are almost same, however, the resonance frequencies of stepped trapezoidal microcantilever comprising of unequal length step beams are slightly greater than the resonance frequencies of stepped trapezoidal microcantilever comprising of equal length step beams.

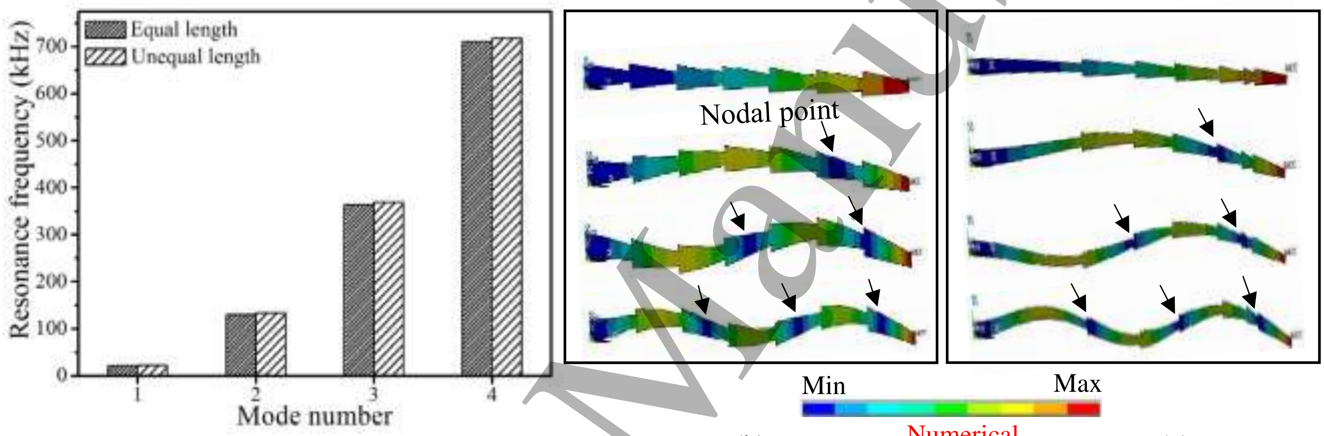


Figure 7. Comparison of bending mode (a) resonance frequencies, (b) and (c) mode shapes up to mode four for stepped trapezoidal shaped microcantilevers comprising of equal and unequal length step beams.

Figures 7(b) and (c) illustrates mode shapes up to fourth mode for stepped trapezoidal shaped microcantilever beams comprising of equal and unequal length step beams, respectively. In Fig. 7(b), it can be noticed that the length of each step beam is same which is 28.57 μm, step junctions are not coinciding the nodal points (shown by arrows) of the vibration mode shapes. Whereas, Fig. 7(c) shows the mode shapes corresponding to tuned stepped trapezoidal shaped microcantilever, in which length of step beams are adjusted in such a way that the step locations exactly coincide the nodal points of the vibrational modes which can be clearly seen in the figure. Hence, in this way one can utilize the stepped beams to coalescence the nodal points of vibrational modes without much effects on the modal frequencies which in turn can increase the deflection of stepped beams in higher modes.

Furthermore, variation in fundamental resonance frequency of stepped trapezoidal beam was investigated with respect to number of steps in the beam. Figure 8 shows variation in resonance frequency of fundamental bending mode with respect to number of steps in stepped trapezoidal shaped microcantilever beam, for comparison, resonance frequency of rectangular beam is also included in the figure. It can be observed from the figure that resonance frequency of an un-stepped trapezoidal beam is greater than the stepped trapezoidal and rectangular shaped microcantilever beams, initially with increasing number of steps resonance frequency decreases and by further increasing number of steps frequency reduces down to frequency of conventional rectangular beam.

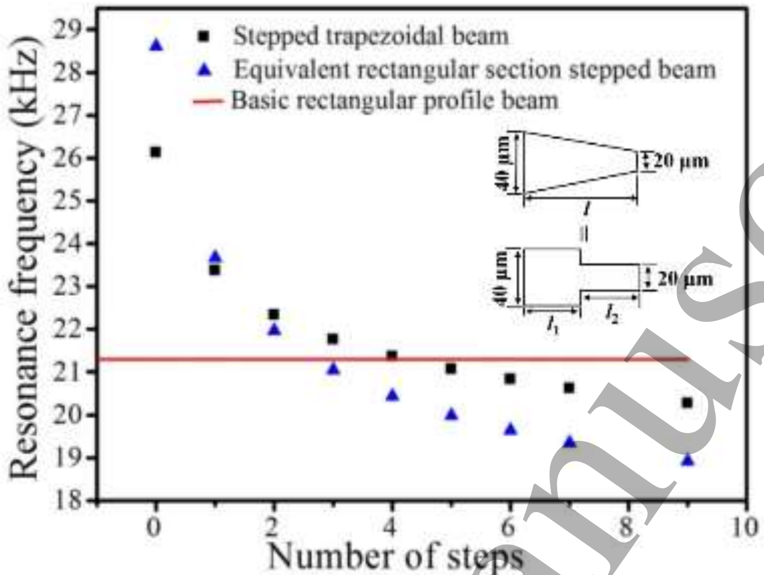


Figure 8. Variation in fundamental bending mode frequency of stepped trapezoidal shaped microcantilever beam with respect to number of steps.

In order to investigate the limiting value of frequency for stepped trapezoidal shaped microcantilever beam, we have created equivalent rectangular section stepped beams corresponding to stepped trapezoidal shaped beams. The schematic of equivalent rectangular section stepped beam corresponding to an un-stepped trapezoidal beam is shown in Figure 8. In a similar way, equivalent rectangular section stepped beams can be created corresponding to stepped trapezoidal shaped microcantilever beams. Then, we have investigated fundamental resonance frequency of bending mode for both stepped trapezoidal as well as their corresponding rectangular section stepped beams. In the present work, l_1 and l_2 are fixed as $l/2$, using this value we have obtained a percentage error of 8% for an un-stepped trapezoidal beam. Using same ratio of l_1 and l_2 for stepped trapezoidal beams of different step numbers, we have obtained maximum error percentage of 6%. However, error percentage can be reduced down to 1% by varying l_1 and l_2 ratios for stepped trapezoidal microcantilever beams having step beams of different tapering values. Thus, equivalent rectangular section stepped beams corresponding to different stepped trapezoidal microcantilever beams can be created. Hence, the limiting frequency value of stepped trapezoidal microcantilever beam can be determined using analytical formulas based on composite element model [29] or equivalent beam transformation technique [30].

4. Conclusions

In the present study, a new design of micromachined cantilever beam in the form of stepped trapezoidal shaped microcantilever is proposed. The proposed microcantilevers were fabricated and analysed based on their resonance frequency, Q-factor, maximum deflection, torsional end rotation and frequency shift, compared with conventional rectangular shaped microcantilever. Over the range of designs considered in the present work, resonance frequency of all proposed designs is comparable to or greater than the conventional rectangular shaped microcantilever, whereas quality factor, maximum deflection, frequency shift and torsional end rotation are higher than the rectangular shaped microcantilever beam. Analytical and numerical resonance frequencies of proposed and rectangular shaped microcantilever beams were found in good agreement with experimental resonance frequencies. With increasing number of steps in stepped trapezoidal shaped microcantilever beam, resonance frequency is slightly reduced, however, the Q-factor, maximum deflection and torsional end rotation are improved. Though all proposed designs exhibits improved characteristics compared to rectangular profile microcantilever, if sensitivity is primary concern then may be doubly stepped trapezoidal shaped microcantilever beam would be the best choice to enhance sensitivity and sensing resolution of microcantilever sensor compared to rectangular profile microcantilever since doubly stepped trapezoidal shaped microcantilever beam has up to 38% improvement in the Q-factor along with 46 % improvement in the frequency shift compared to conventional rectangular profile microcantilever. If maximum deflection and torsional end rotation are of primary importance then stepped microcantilever with greater number of steps must be best choice of microcantilever.

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References

1. Ndieyira J W *et al.* 2008 Nanomechanical detection of antibiotic-mucopeptide binding in a model for superbug drug resistance *Nat Nano.* **3** 691-6

2. Abu-Lail N I, Kaolek M, LaMattina B, Clak R L and Zauscher S 2006 Micro-cantilevers with end-grafed stimulus-responsive polymer brushes for actuation and sensing *Sensors and Actuators B: Chemical* **114** 371-8

3. Nugaeva N, Gfeller K Y, Backman N, Lang H P, Duggelin M and Hegner M 2005 Micromechanical cantilever array sensors for selective fungal immobilization and fast growth detection, *Biosensors and Bioelectronics* **21** 849-56

4. Calleja M, Nordstrom M, Alvarez M, Tamayo J, Lechuga L M, and Boisen A 2005 Highly sensitive polymer based cantilever sensors for DNA detection *Ultramicroscopy* **105** 215-22

5. Yue M, Lin H, Dedrick D E, Satyanaayana S, Majumda A, Bedekar A S, Jenkins J W, and Sundaram S 2004 A 2-D Microcantilever array for multiplexed biomolecular analysis *Journal of Microelectromechanical Systems* **13** 290-9

6. Wee K W, Kang G Y, Park J, Kang J Y, Yoon D S, Park J H and Kim T S 2005 Novel electrical detection of label-free disease maker proteins using piezoresistive self-sensing micro-cantilevers *Biosensors and Bioelectronics* **20** 1932-8

7. Kiselev G, Kudrinskii P, Yaminskii I and Vinogradova O 2008 Studying intermolecular processes in thin surface layers with microcantilever transducers. Formation of protein fibrils on a solid support *Protection of Metals* **44** 535-41

8. Kendry R. Mc *et al.* 2002 Multiple label-free bio detection and quantitative DNA, binding assays on a nanomechanical cantilever array *Proc. Natl. Acad. Sci.* **99** 9783-8

9. Nordstrom M, Keller S, Lillemose M, Johansson A, Dohn S, Haefliger D, Blagoi G, Havsteen-Jakobsen M, Boisen A 2008 SU-8 cantilevers for bio/chemical sensing; fabrication, characterisation and development of novel read-out methods *Sensors* **8** 1595-612

10. Zhang J *et al.* 2006 Rapid and label-free nanomechanical detection of biomarker, transcripts in human RNA *Nature Nanotechnol.* **1** 214-20

11. Suri C R, Kaur J, Gandhi S and Shekhawat G S 2008 Label-free ultra-sensitive detection of atrazine based on nanomechanics *Nanotechnol.* **19** 235502-600

12. Arntz Y, Seelig J D, Lang H P, Zhang J, Hunziker P, Ramseyer J P, Meyer E, Hegner M and Gerber C 2003 Label-free protein assay based on a nanomechanical cantilever array *Nanotechnol.* **14** 86-90

13. Zahid Ansari M and Cho C 2008 A study on increasing sensitivity of rectangular microcantilevers used in biosensors *Sensors* **8** 7530-44

14. Zahid Ansari M and Cho C 2009 Deflection, frequency, and stress characteristics of rectangular, triangular, and step profile microcantilevers for biosensors *Sensors* **9** 6046-57

15. Zahid Ansari M, Cho C, Kim J and Bang B 2009 Comparison between deflection and vibration characteristics of rectangular and trapezoidal profile microcantilevers *Sensors* **9** 2706-18

16. Subramanian S and Gupta N 2009 Improved V-shaped microcantilever width profile for sensing applications *J. Phys. D: Appl. Phys.* **42** 185501 (6pp)

17. Liu Y, Wang H, Qin H, Zhao W and Wang P 2017 Geometry and profile modification of microcantilevers for sensitivity enhancement in sensing applications *Sensors and Materials* **29(6)** 689-98

18. Hawari H F, Wahab Y, Azmi M T, Md. Shakaff A Y, Hashim U and Johari S 2014 Design and Analysis of various microcantilever shapes for MEMS based sensing *J. Phys.: Conf. Ser.* **495** 012045 (9pp)

19. Lim Y C, Kouzani A Z, Duan W and Kaynak A 2010 Effects of design parameters on sensitivity of microcantilever biosensors, in *Proc. IEEE Int. Conf. Complex Med. Eng.,* Gold Coast, Australia, July 13–15, 2010, 177–181

20. Xia X, Li X 2008 Resonance-mode effect on microcantilever mass-sensing performance in air *Rev. Sci. Instrum.***79** 074301(8 pp)

21. Jin D, Li X, Liu J, Zuo G, Wang Y, Liu M and Yu H 2006 High-mode resonant piezoresistive cantilever sensors for tens-femtogram resoluble mass sensing in air *J. Micromech. Microeng.* **16** 1017-1023

22. Li X, Lee D-W 2012 Integrated micro-cantilevers for ultra-resoluble sensing and probing *Meas. Sci. Technol.* **23** 022001(40 pp)

23. Jin D, Li X, Bao H, Zhang Z, Wang Y, Yu H and Zuo G 2007 Integrated cantilever sensors with a torsional resonance mode for ultraresoluble on-the-spot bio/chemical detection *Appl. Phys. Lett.* **90** 041901 (3 pp)

24. Singh S S, Pal P and Pandey A K 2015 Pull-in analysis of non-uniform microcantilever beams under large deflection *J. Appl. Phys.* **118** 204303 (11 pp)

25. Singh S S, Pal P and Pandey A K 2016 Mass sensitivity of non-uniform microcantilever beam *J. Vib. Acoust.* **138(6)** 064502 (7 pp)

26. Sahoo D K and Pandey A K 2018 Performance of non-uniform cantilever based piezoelectric energy harvester *ISSS J. Micro Smart Syst.* https://doi.org/10.1007/s41683-018-0018-2

27. Ashok A, Manoj Kumar P, Singh S S, Raju P, Pal P and Pandey A K 2018 Achieving Wideband Micromechanical System using Coupled Non-Uniform Beams Array *Sensors and Actuators A: Physical* **273** 12-18

28. Purohit B, Jain P C and Pandey A K 2016 Modal Analysis of Monolithic and Jointed type Cantilever Beams with Non-Uniform Section *Exp.Mech.***56 (6)** 1083-1094

29. Lu Z R, Huang M, Liu J K, Chen W H and Liao W Y 2009 Vibration analysis of multi-stepped beams with the composite element model *J. Sound and Vibration* **322(4-5)** 1070-1080

30. Moon S P and Hong S J 2008 Fundamental natural frequency analysis of stepped cantilever beams by equivalent beam transformation technique *Journal of the Computation Structural Engineering Institute of Korea* **21 (4)** 401-410