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Published in IET Communications
Received on 10th April 2013
Revised on 5th July 2013
Accepted on 30th July 2013
doi: 10.1049/iet-com.2013.0296

ISSN 1751-8628

Wireless local area network service providers' price competition in presence of heterogeneous user demand

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Abstract: Consider wireless local area network (WLAN) service providers (SPs) operating in an overlapping service area. The SPs compete with each other to attract users. The price charged is utilised by the SPs as a tool to maximise revenue, resulting in a price competition between the WLAN SPs. The users are assumed to be selfish, trying to maximise their individual utility. They have varied sensitivity towards quality of service experienced and the price charged. In such a scenario, the user demand distribution is the one that achieves Wardrop equilibrium. Approximate analytical expressions are obtained for the best response of SPs to each other's price. Existence of a Nash equilibrium (NE) between the competing SPs is proved and the price vector at which the NE occurs is obtained. It is found that, while in one extreme monopoly leads to very high revenue for WLAN SPs with minimal consumer surplus, in the other extreme unregulated duopoly/oligopoly leads to high consumer surplus at the cost of minimal revenue generation for the competing SPs. Thus, price regulation is proposed in the WLAN market for equitable distribution of the surplus among the SPs and the users.

1 Introduction

There has been a significant increase in the number of wireless local area network (WLAN) users across the globe. To capitalise on the ever increasing demand, more and more WLAN service providers (SPs) are entering the market. In this highly competitive market, several SPs may coexist with overlapping coverage areas. Catering to a common group of users, SPs price their service to attract maximum number of users and optimise their revenue. This results in a price competition [1] among the WLAN SPs. The price charged and quality of service (QoS) experienced at the SPs affect the users' choice of SP. Thus, pricing among the WLAN SPs needs to be studied as it impacts both the SPs' revenues and the overall WLAN usage and demand patterns.

The impact of price competition among SPs in different kinds of communication networks is of great interest to the policy makers, network regulators, users and the SPs themselves. Price competition in congested networks, in the presence of convex latency functions, has been analysed and bounds on price of anarchy (PoA) have been obtained in [2]. In [3], a pricing game for heterogeneous wireless access networks has been studied, suggesting game theoretic solutions to revenue sharing based on an N-person coalition game. Duopoly price competition among WLAN SPs for homogeneous users with packet loss rate (PLR) as the negative externality [1] has been studied in [4]. In [5], a novel game-theoretic approach for pricing and user network selection with performance-cost ratio maximisation in the presence of heterogeneous wireless networks is proposed. Convergence of price competitions through regulation in price jumps is discussed in [6]. A PoA-based study of opportunistic sensing in cognitive radio networks (CRNs) is presented in [7]. An adaptive competitive second-price pay-to-bid sealed auction game based approach for spectrum sharing in CRN is proposed in [8]. PoA-based efficiency analysis of WLAN SPs' duopoly price competition, in presence of four types of users, is investigated in [9].

In a real-life scenario, multiple WLAN SPs may coexist, at places like airports, universities and convention centres, with overlapping coverage areas. Existing analyses for price competition of WLAN SPs, like [4, 9], provide results for only two SPs in presence of homogeneous users and limited number of user types, respectively. An important metric for QoS at WLAN SPs is PLR [10]. PLR is a concave latency function [11] of the total number of users connected at an access point (AP). Hence, results obtained for convex latency functions like [2] cannot be directly used for PLR and utility calculations, as in [4]. Thus, a detailed study of price competition of multiple WLAN SPs with overlapping coverage areas and in the presence of diverse heterogeneous user demand with/without any price regulation is required. This is the motivation of this work.

The primary contributions of this paper are as follows. Firstly, the issue of multiple SPs operating in an overlapping WLAN coverage area being in a state of price competition is investigated. Secondly, for the case of non-atomic users, with diverse sensitivities to price and QoS, approximate analytical expressions for the user demand distribution in Wardrop equilibrium (WE) [12] and the best response (BR) of SPs to each other's price are obtained. A proof of existence of a Nash equilibrium (NE) [13] between the competing SPs and the price vector at which the NE occurs is derived for the duopoly scenario. This is a major contribution as compared with [9], because with an explicit NE price vector analytical results are presented for the SPs' revenues, consumer surplus and the PoA. Unlike the work in [9], the framework presented in this paper is shown to be generalisable to a scenario having more than two SPs and the impact of SPs' competition on their revenues is also investigated. Furthermore, numerical results are presented to show a need for regulation in WLAN SPs' market.

The rest of the paper is organised as follows. The system model is presented in Section 2. In Section 3, user demand distribution in WE is studied. The important issue of multiple SPs being in a state of price competition is also examined in Section 3. Section 4 analyses the price competition and proves the existence of NE. Section 5 contains analysis of SPs' revenues and consumer surplus. Simulation results are compared with the obtained analytical results in Section 6. Section 7 gives some concluding remarks.

2 System model

Consider multiple SPs operating in an IEEE 802.11 service area as shown in Fig. 1. Each SP controls a distinct AP; hence, the terms SP and AP are used interchangeably. Let the set of APs be denoted by M, where $M = \{1, 2, \ldots, M_t\}$. Distributed coordination function (DCF) is the most commonly used multiple access technique in WLAN operations. Hence, we consider the case when each AP $m \in M$ uses DCF. If multiple APs start operating on the same set of frequencies, packet collision occurs, resulting in poor QoS for the users. Thus, even competing SPs, operating in close vicinity of each other, will operate on disjoint sets of frequencies. Each user connected to any AP $m \in M$ is charged a corresponding price p_m for using the SPs services for a time slot T. In this paper, we consider the case when a continuum of users exists (i.e. non-atomic or infinitesimal users). Users are considered to be heterogeneous, in the sense that they value the same level of QoS and price charged differently. Let the 'set of user types' be denoted by N, where $N = \{1, 2, \ldots, N_t\}$.

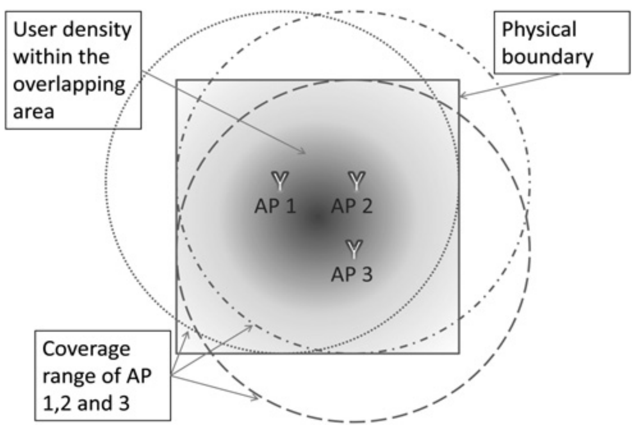

Fig. 1 *Service area of WLAN SPs*

We consider the user utility u_{mn} for any user of type n, $n \in N$, at AP m, $m \in M$, to be a convex strictly decreasing function of p_m and a concave strictly increasing function of q_m, the QoS experienced at SP m. Thus, u_{mn} is given by

$$u_{mn} = u_{mn}(q_m,\, p_m) \tag{1}$$

As in [14], this represents the law of diminishing marginal utility because of price and QoS. Let x_m be the total number of users connected to AP m and x_{mn} denote the 'user demand distribution', where x_{mn} represents the total number of type n users connected to AP m. Note that the QoS for WLAN users is a strictly decreasing convex function of x_m [4, 9]. The individual user or consumer surplus is the utility obtained by a type n user at AP m [15]. Thus, using (1), the aggregate consumer surplus cs of all users in the system is given by

$$cs \triangleq \sum_{m \in M} \sum_{n \in N} u_{mn} x_{mn} \tag{2}$$

Users connect to an AP, until the resultant user utility is non-negative. The 'revenue or utility' u_m of an SP $m \in M$ is a function of $(p_1, \ldots, p_{M_t})$, and is given by

$$u_m\left(p_1, \ldots, p_{M_t}\right) = p_m x_m \tag{3}$$

Under equilibrium conditions of user mobility and call arrival/completion rate [16], on an average the total number of active users in the WLAN service area is considered to be finite. Let x_t be the maximum possible number of active users in the system. Thus, we have the constraint

$$\sum_{m \in M} \sum_{n \in N} x_{mn} \le x_t \tag{4}$$

3 User demand distribution

In the presence of competing SPs, users will connect to the AP offering maximum utility (1). Users will continue to connect until either all the users are connected or the utility on all the APs becomes non-positive. No user will connect to an AP offering negative utility. Users will continue to switch from an AP with lower utility to an AP with higher utility until such transition results in no further increase in utility. Given non-atomic user demand, all the above scenarios can be characterised by the WE [12]. Thus, at WE, any user type $n \in N$ will have non-zero x_{mn} at an AP m only if the utility is equal to the maximum possible utility from any of the other APs. Otherwise, the users of type n will connect to the other APs offering higher utility, resulting in x_{mn} equal to zero. Hence, WE can be mathematically represented as

$$\begin{aligned} & x_{mn}\left(u_{mn} - \max_{m \in M}\{u_{mn}\}\right) = 0 \\ & \text{s.t.} \quad u_{mn} \ge 0 \quad \forall m \in M,\ n \in N \end{aligned} \tag{5}$$

We consider the case when all the SPs use the latest IEEE 802.11 APs (i.e. IEEE 802.11e or beyond). The beacon frames of such APs contain the load information at the AP. By deciphering the beacon frames, at any point of time the

IET Commun., 2013, Vol. 7, Iss. 18, pp. 2113–2121
doi: 10.1049/iet-com.2013.0296

SPs know the total number of active users connected to all the SPs, denoted by x_0. Thus, we have

$$\sum_{m \in M} x_m = x_0, \quad x_0 \in [0, x_t] \tag{6}$$

Note that the WE defined by (5) exists irrespective of the prices charged by the various SPs. Thus, the multiple SPs operating in the vicinity of each other in the WLAN service area can be competing in a non-segmented market (oligopoly), colluding (monopoly) or operating in a perfectly segmented market (disjoint operation akin to independent monopolies). Since the focus of this work is oligopoly (i.e. more than one SP) price competition, we next present a proposition on existence of equilibrium characterising user type essential for the multiple SPs to be in a state of price competition with each other.

Proposition 1: M_t SPs, operating in each other's vicinity, are in a state of M_t^{th} order oligopoly competition if there exists at least one user type $l \in N$, such that

$$u_{1,l} = u_{m,l} \forall \quad m \in M \tag{7}$$

We further define such user type l as the 'equilibrium characterising user type'.

Proof: Given the prices charged by the SPs, let us represent the user demand as a matrix $\boldsymbol{X}^{\mathrm{WE}}$ given by

$$\boldsymbol{X}^{\mathrm{WE}} = \begin{bmatrix} x_{11} & \cdots & x_{1N_t} \\ \vdots & \vdots & \vdots \\ x_{M_t 1} & \cdots & x_{M_t N_t} \end{bmatrix} \tag{8}$$

Without loss of any generality, we have the elements in $\boldsymbol{X}^{\mathrm{WE}}$, such that, the rows representing SPs are sorted in increasing order of the price charged and the columns representing user types are sorted in increasing order of the utility obtained at the maximum price and the minimum QoS. Thus, we have the set of price charged by the SPs such that $p_1 \leq \ldots \leq p_{M_t}$. Similarly, the user types are such that $u_{M_t 1} \leq \cdots \leq u_{M_t N_t}$. Based on the prices and the user types, several scenarios are possible for the elements in $\boldsymbol{X}^{\mathrm{WE}}$.

In the case of monopoly or a perfectly segmented market, each SP will be catering to a disjoint set of users. Hence, we will have a sparse matrix $\boldsymbol{X}^{\mathrm{WE}}$, such that at most one entry in any column is non-zero. More than one non-zero element in a column implies that users of the same type are connecting to more than one SP, resulting in a price competition. Thus, for any two SPs to be in a state of competition against each other, they should have non-zero number of users of at least one common user type in (8) to compete over. Using (5), non-zero number of users of any type at both the SPs implies the utility for such type of users is equal at both the SPs. Now consider the case of all the M_t SPs competing with each other. To be in a state of competition, all the M_t SPs must compete over a common resource (in this case any user type). Hence, at least one column in (8) must have non-zero entries for all these SPs. From (5), this implies $u_{1l} = u_{ml} \forall m \in M$. The user type defined by this column is the 'equilibrium characterising user type'. This proves Proposition 1. □

Note that scenarios can exist such that SPs compete with each other in pairs but not with all the SPs. For example, consider the case of three SPs. It is possible for these SPs to compete over different user types in pairs. Thus, in such a case (7) will not hold. Nevertheless, two simultaneous duopolies over non-identical resources (in this case different user types) are not equivalent to an oligopoly (of order 3). Thus, as shown in Proposition 1, if SPs are in a state of M_t^{th} order oligopoly competition, (7) will hold.

Let the equilibrium characterising user type be l. In the presence of multiple user types, users that obtain maximum utility even with poor QoS and high prices are the equilibrium characterising user types. The reason for such behaviour of users comes from (1). Even for poor QoS and high prices, such users can obtain some utility from an SP. Thus, they are the last ones to switch from one SP to another. Hence, in presence of competing SPs they define the equilibrium of the price competition. The same is further explored through simulations in the numerical results section. In the absence of such a user type, a pure strategy NE may still exist between the SPs, but the SPs will not be strictly competing over price. Consider the case when all the SPs have complete information about the users' utility and each SP has complete information about the other SPs' user demand. Further, given that an equilibrium characterising user type exists, (7) represents a set of $M_t - 1$ equations in $x_m \forall m \in M$. Thus, jointly solving (6) and (7), the SPs can obtain the exact user demand distribution.

4 Price competition analysis

In this section, using the user demand distribution proposed in Section 3, we analyse the price competition for competing WLAN SPs for the scenario when all the SPs have complete information about every users' utility functions. In such a case, given that an equilibrium characterising user type exists, the SPs' price competition can be modelled as a non-cooperative game. The SP m, $m \in M$, represents the players. For any player $m \in M$, the price choice p_m represents its strategy. Let $\boldsymbol{p}_{-m}$ represent the price vector of the other APs besides AP m. Thus, the action profile of the players or SPs is denoted by the price vector $\boldsymbol{p} = \left(p_1, \ldots, p_{M_t}\right)$. The individual utility of any SP m, as given in (3), is $u_m\left(p_1, \ldots, p_{M_t}\right) = p_m x_m$. The NE in this price competition game is defined as follows [13].

Definition 1: A price vector $\boldsymbol{p}^{\mathrm{NE}} = \left(p_1^{\mathrm{NE}}, \ldots, p_{M_t}^{\mathrm{NE}}\right)$ corresponds to an NE if

$$\begin{gathered} u_m\left(p_m^{\mathrm{NE}}, \boldsymbol{p}_{-m}^{\mathrm{NE}}\right) \geq u_m\left(p_m, \boldsymbol{p}_{-m}^{\mathrm{NE}}\right) \\ \forall m \in M \quad \text{and} \quad p_m, \boldsymbol{p}_{-m} \geq 0 \end{gathered} \tag{9}$$

Equation (9) clearly implies that at NE each AP m chooses the price p_m which is the BR of the AP, in response to the price $\boldsymbol{p}_{-m}$ of the other APs. Next, we present a proposition on the existence of an NE in the oligopoly scenario.

Proposition 2: Given M_t competing SPs, an NE always exists if the user demand at any SP is a concave function of the price charged by the other SPs.

Proof: From (3), the utility of any SP is $p_m x_m$. Given that x_m is a concave function of $p_k \forall k \in \mathrm{M} - \{m\}$, $p_m x_m$ is a concave function of $p_k \forall k \in M - \{m\}$. The individual SP's utility is continuous in p_m and $\boldsymbol{p}_{-m}$. The strategy space p_m is convex, compact and non-empty for each m. Therefore as in [17], at least one NE always exists. □

For illustration, we analyse the case of two SPs that is a duopoly scenario. As in [4, 9], let the user utility function of the equilibrium characterising user type n be

$$u_{mn} = \alpha_n q_m - \beta_n p_m, \tag{10}$$

where p_m is the price charged by SP m, q_m is the QoS experienced at SP m, α_n represents the 'QoS sensitivity' and β_n denotes the 'price sensitivity' of user type n. The QoS is discussed in detail in the next paragraph. The parameters α_n and β_n make it possible to model the heterogeneity in user utility. A user can have a real time/non-real time, low/high bandwidth demand based on the type of call like data, voice, email or video stream. Thus, users will have varying urgency to communicate. Similarly, they will have diverse ability to pay for the same QoS. Several such scenarios can easily be modelled by varying α_n and β_n, accordingly. Consider the user density distribution over the plane of various QoS and price sensitivity as depicted in Fig. 2. In a real-life scenario, a large number of users will want to have a good QoS at minimum possible price. However, WLANs are contention-based networks which are available in some places even for free. Thus, user density will be greater for users with low sensitivity to QoS and high sensitivity to price. Hence, in Fig. 2 the user density ϕ is more for users with low sensitivity to QoS and high sensitivity to price (i.e. the lower right corner of Fig. 2), as compared with the user density of users that are more sensitive to QoS and price (as observed in the upper right corner of Fig. 2).

In the case of WLAN users, the QoS is characterised by the data rate, delay and the congestion experienced. Given a constant non-overlapping bandwidth for the WLAN SPs, QoS is directly related to the congestion experienced at an AP. From [9, 10], PLR is an appropriate measure of congestion in a WLAN environment. As in [4, 9], we consider the case when SPs advertise their QoS in the form of the PLR. Thus, users have prior information of the expected PLR while selecting any SP. The QoS q_m at an AP m is given by

$$q_m(x_m) = 1 - \mathrm{PLR}_m(x_m) \tag{11}$$

where $\mathrm{PLR}_m(x_m)$ denotes the PLR at an AP m because of x_m users connected to it. Note that PLR being a strictly increasing concave function of the number of connected users [11], the quantity $q_m(x_m)$ is a strictly decreasing convex function, capturing the essence of the law of diminishing marginal utility (i.e. as more and more users gain access to the SP, the overall rate of gain in utility keeps decreasing). The PLR is further expressed as a function of the packet collision rate P_m^{c} and the minimal packet transmission error rate P_m^{t} supported by AP m as

$$\mathrm{PLR}_m(x_m) = 1 - \left(1 - P_m^{\mathrm{t}}\right)\left(1 - P_m^{\mathrm{c}}\right) \tag{12}$$

Note that P_m^{t} depends on the wireless link between users and the SP. Since a user can move to a satisfactory location to ensure a minimal acceptable P_m^{t} based on the QoS requirements, we consider that P_m^{t} at any SP is constant for the users at that SP. Given that the SPs want to maximise individual profits, as in [4, 9], we consider the case of maximum saturation throughput for the users. Thus, an approximation of the packet collision rate P_m^{c} can be written as [11]

$$P_m^{\mathrm{c}} = 1 - \left(1 - \frac{1}{x_m K}\right)^{x_m - 1} \tag{13}$$

where $K = \sqrt{T_{\mathrm{c}}/2}$ and T_{c} is the average time the channel is sensed busy by a user during a collision. For a given physical and media access control layer mechanism, K is a constant [11]. Using (12), (13) becomes

$$\begin{aligned}\mathrm{PLR}_m(x_m) &= 1 - \left(1 - P_m^{\mathrm{t}}\right)\left(1 - \frac{1}{x_m K}\right)^{(x_m - 1)} \\ &= 1 - \left(1 - P_m^{\mathrm{t}}\right)\mathrm{e}^{(x_m - 1)\ln\left(1 - \left(1/x_m K\right)\right)} \\ &\simeq 1 - \left(1 - P_m^{\mathrm{t}}\right)\mathrm{e}^{(x_m - 1)\left(-\frac{1}{x_m K} - \left(1/\left(2(x_m K)^2\right)\right)\right)} \\ &\simeq 1 - b_m\left(1 + \frac{a}{x_m}\right)\end{aligned} \tag{14}$$

where

$$b_m = \left(1 - P_m^{\mathrm{t}}\right)\mathrm{e}^{-(1/K)}, \quad a = \frac{1}{K} - \frac{1}{2K^2} \tag{15}$$

For values of $x_m \geq 4$, and values of K, P_m^t meeting the QoS requirements of WLAN in [10], the approximated PLR in (14) lies within 0.7% of the actual value. Note that b_m in (15) is directly related to the minimal packet error transmission rate P_m^{t} supported by SP m. P_m^{t} is a function of the coverage radius r_m, via the maximum power transmitted by the AP m [11]. In most countries, the maximum permissible transmission power for WLAN is bounded by law. Given a realistic user spatial density distribution, coverage area is always proportional to the number of connected users. Thus, it is reasonable to assume

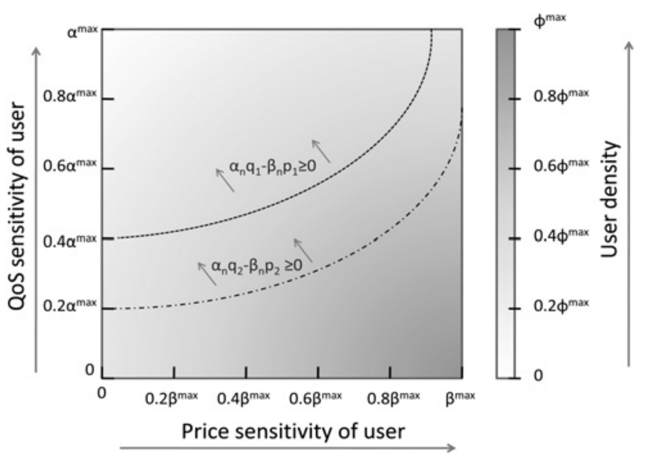

Fig. 2 *Price and QoS sensitivity of the users*

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doi: 10.1049/iet-com.2013.0296

that all the SPs will operate at the maximum permissible power. Hence, we consider the case when $b_1 = b_2 = \cdots = b_{M_t}$, that is

$$b_m = \left(1 - P_m^{\mathrm{t}}\right)\mathrm{e}^{-(1/K)} = b \quad \forall \quad m \in M \tag{16}$$

where b is a constant. Since $\mathrm{PLR}_1(x_1) = \mathrm{PLR}_2(x_2)$ as long as $x_1 = x_2$, the function $\mathrm{PLR}_m(x_m)$ is hereby referred to as PLR (x_m). From (14), $\mathrm{PLR}(x_m)$ is given by

$$\mathrm{PLR}(x_m) = 1 - b\left(1 + \frac{a}{x_m}\right) \tag{17}$$

Thus, using (10), (14) and (17), the net utility u_{ml} for a type l users at an AP m is given by

$$u_{ml} = \alpha_l b\left(1 + \frac{a}{x_m}\right) - \beta_l p_m \tag{18}$$

Given that the equilibrium characterising user type l experiences the same utility at all the APs, from (7), we obtain $M_t - 1$ linear equations in $\{1/x_m\}$, which are

$$\begin{aligned} \alpha_l b\left(1 + \frac{a}{x_1}\right) - \beta_l p_1 &= \alpha_l b\left(1 + \frac{a}{x_2}\right) - \beta_l p_2 \\ &\vdots \\ \alpha_l b\left(1 + \frac{a}{x_1}\right) - \beta_l p_1 &= \alpha_l b\left(1 + \frac{a}{x_{M_t}}\right) - \beta_l p_{M_t} \end{aligned} \tag{19}$$

From (6), we obtain

$$x_1 + \cdots + x_{M_t} = x_0 \tag{20}$$

'Consider the case of duopoly, which implies' $M_t = 2$. Jointly solving (19) and (20), we obtain the numbers of users at the APs for $M_t = 2$ as

$$\begin{aligned} &\text{for} \quad p_1 < p_2, \quad x_1 = \frac{x_0}{2} + \frac{1}{\gamma_l} + \sqrt{\frac{x_0^2}{4} + \frac{1}{\gamma_l^2}}, \\ &\qquad\qquad\qquad x_2 = \frac{x_0}{2} - \frac{1}{\gamma_l} - \sqrt{\frac{x_0^2}{4} + \frac{1}{\gamma_l^2}} \\ &\text{for} \quad p_1 > p_2, \quad x_1 = \frac{x_0}{2} + \frac{1}{\gamma_l} - \sqrt{\frac{x_0^2}{4} + \frac{1}{\gamma_l^2}}, \\ &\qquad\qquad\qquad x_2 = \frac{x_0}{2} - \frac{1}{\gamma_l} + \sqrt{\frac{x_0^2}{4} + \frac{1}{\gamma_l^2}} \\ &\text{for} \quad p_1 = p_2, \quad x_1 = x_2 = \frac{x_0}{2} \end{aligned} \tag{21}$$

where $\gamma_l = \beta_l(p_1 - p_2)/(\alpha_l ab)$.

Given p_2, the BR of AP 1, that is, p_1^{BR} is obtained using (3) and (21). Differentiating the revenue of SP 1 w.r.t. p_1 and equating the result to zero, we obtain

$$\begin{aligned} &\text{for} \quad p_1 < p_2, \quad p_2 < \frac{\alpha_l ab}{\beta_l x_0}, \\ &p_1^{\mathrm{BR}} = p_2 + \frac{-x_0\alpha_l abp_2 - \sqrt{\left(x_0\alpha_l abp_2/\beta_l\right)\left(x_0\beta_l p_2 - 2\alpha_l ab\right)^2}}{x_0\alpha_l ab - x_0^2\beta_l p_2} \\ &\text{for} \quad p_1 < p_2, \quad p_2 > \frac{\alpha_l ab}{\beta_l x_0}, \\ &p_1^{\mathrm{BR}} = p_2 + \frac{-x_0\alpha_l abp_2 + \sqrt{\left(x_0\alpha_l abp_2/\beta_l\right)\left(x_0\beta_l p_2 - 2\alpha_l ab\right)^2}}{x_0\alpha_l ab - x_0^2\beta_l p_2} \\ &\text{for} \quad p_1 > p_2, \quad p_2 < \frac{\alpha_l ab}{\beta_l x_0}, \\ &p_1^{\mathrm{BR}} = p_2 + \frac{x_0\alpha_l abp_2 + \sqrt{\left(x_0\alpha_l abp_2/\beta_l\right)\left(x_0\beta_l p_2 - 2\alpha_l ab\right)^2}}{x_0\alpha_l ab - x_0^2\beta_l p_2} \\ &\text{for} \quad p_1 > p_2, \quad p_2 > \frac{\alpha_l ab}{\beta_l x_0}, \\ &p_1^{\mathrm{BR}} = p_2 + \frac{x_0\alpha_l abp_2 - \sqrt{\left(x_0\alpha_l abp_2/\beta_l\right)\left(x_0\beta_l p_2 - 2\alpha_l ab\right)^2}}{x_0\alpha_l ab - x_0^2\beta_l p_2} \end{aligned} \tag{22}$$

In (22), the BR of AP 1, given the price of AP 2, has been obtained. The cases considered in (22) are $p_1 < p_2$ and $p_1 > p_2$. The intersection of the $p_1^{\mathrm{BR}}s$ for these two cases (when $p_1 = p_2$) results in an NE. The point of intersection for $p_2 > \alpha_l ab/\beta_l x_0$, obtained using (22), is given by $\sqrt{x_0\alpha_l abp_2/\beta_l\left(x_0\beta_l p_2 - 2\alpha_l ab\right)^2} = x_0\alpha_l abp_2$, which, after simplification, results in

$$\left(x_0\beta_l p_2 - 4\alpha_l ab\right)\left(x_0\beta_l p_2 - \alpha_l ab\right) = 0 \tag{23}$$

Considering the case when $p_2 > \alpha_l ab/x_0\beta_l$, $p_2 = 4\alpha_l ab/x_0\beta_l$ is the only possible solution of (23). Thus, by symmetry, an NE always exists at the price vector given by

$$\left(p_1, p_2\right) = \left(\frac{4\alpha_l ab}{x_0\beta_l}, \frac{4\alpha_l ab}{x_0\beta_l}\right) \tag{24}$$

5 Surplus analysis

From (3), (21) and (24), the mth SP's revenue at NE 'for duopoly' ($M_t = 2$) is given by

$$u_m(p_1, p_2) = \frac{4\alpha_l ab}{x_0\beta_l}\frac{x_0}{2} = \frac{2\alpha_l ab}{\beta_l} \tag{25}$$

which implies that for two competing SPs, the revenue at equilibrium is independent of the total number of connected users in the WLAN service area. Instead, the revenue depends on the maximum of α_n/β_n. Thus, a higher sensitivity to QoS and lower sensitivity to price among the users positively affect the SPs' revenues. The revenue can also be increased through better QoS by appropriately setting the values of parameter b. Since b depends on the minimal packet transmission error rate P_m^{t} given by (15), if

SPs compete over price and QoS, then they end up setting maximum b by minimising P_m^{t}. In the case when any of the SPs is technologically dominated by the other SP, it cannot meet the maximum value of b. This results in QoS dominance of the SP and loss in revenue. The WLAN infrastructure cost is small compared with the revenues. Hence, competing SPs will invest in upgrading to the latest technology. Thus, (25) reconfirms the assumption in (15) of equal QoS at the two SPs.

The aggregate consumer surplus at NE for duopoly is expressed using (2), (18), (21) and (24) as

$$cs = \sum_{m\in M}\sum_{n\in N}\left(\alpha_n b\left(1+\frac{2a}{x_0}\right) - \beta_n\frac{4\alpha_l ab}{\beta_l x_0}\right)x_{mn} \qquad (26)$$

Since $\alpha_l/\beta_l = \max_{n\in N}\{\alpha_n/\beta_n\}$, the cs in (26) is maximised when $\alpha_n/\beta_n = \alpha_l/\beta_l \forall n \in N$. This represents the case of homogeneous user demand, with the corresponding cs given by

$$cs = \left(\alpha_l b\left(1+\frac{2a}{x_0}\right) - \frac{4\alpha_l ab}{x_0}\right)x_0 = \left(\alpha_l b x_0 - 2\alpha_l ab\right) \qquad (27)$$

which implies that the maximum cs depends on QoS sensitivity α_l, the actual QoS b and the total number of connected users in the system x_0, but not on the price sensitivity of the users β_l. Since price competition results in the NE price being inversely proportional to β_l, it effectively cancels out the impact of price sensitivity on the cs.

The 'social welfare' (SW), denoted as w, is defined by Mascolell *et al.* [15]

$$w \triangleq \sum_{m\in M}\sum_{n\in N}\alpha_n\left(1-\mathrm{PLR}(x_m)\right)x_{mn} \qquad (28)$$

From (15) and (28), the SW for an arbitrary user demand distribution at M_t APs is

$$\begin{aligned} w &= \sum_{m\in M}\sum_{n\in N}\left(\alpha_n b\left(1+\frac{a}{\sum_{n\in N} x_{mn}}\right)x_{mn}\right) \\ &= \alpha_l \sum_{m\in M}\left(b\left(1+\frac{a}{\sum_{n\in N} x_{mn}}\right)\sum_{n\in N}\left(\frac{\alpha_n}{\alpha_l}\right)x_{mn}\right) \end{aligned} \qquad (29)$$

Since, α_n/α_l is ≤ 1, substituting $\Sigma_{n\in N}(\alpha_n x_{mn})/\alpha_l \leq \Sigma_{n\in N} x_{mn}$ in (29), and using the fact that $\Sigma_{m\in M}\Sigma_{n\in N} x_{mn} = x_0 \leq x_t$, we obtain an upper bound on the maximum possible SW as

$$w^{\max} = \alpha_l\left(bx_t + abM_t\right) \qquad (30)$$

where M_t is the number of APs. Note that (29) offers some interesting insights as follows. For a constant user demand and fixed number of Aps, the maximum SW is directly related to the maximum possible QoS sensitivity. The bound in (29) is attained only in the case of homogeneous users, that is, $\alpha_n = \alpha^{\max} \forall n \in N$. In the case of heterogeneous demand, some loss in SW is expected. $w^{\max}$ increases as more and more users get connected. This implies maximisation of coverage area, that is, enabling network access to more and more users, and has a positive impact on the SW. The increase in number of APs M_t in a particular area improves the available opportunity for users to connect thereby de-congesting networks and increasing SW.

We next consider the case when $\alpha_n = \alpha_l \forall n \in N$, that is, homogeneous user demand. From (12), we conclude that, at NE, $(x_1, x_2) = (x_0/2, x_0/2)$. Thus, the SW at NE from (29) is

$$w^{\mathrm{equ}} = \left(\alpha_l b\left(1+\frac{2a}{x_0}\right)\right)x_0 = \alpha_l\left(x_0 b + 2ab\right) \qquad (31)$$

From (30) and (31), for homogeneous user demand, the PoA at duopoly, (as in [9]), is obtained as

$$\mathrm{PoA}^{\mathrm{duopoly}} = \left(\frac{x_0 b + 2ab}{x_t b + 2ab}\right) \qquad (32)$$

Clearly competing SPs like to maximise individual revenue, resulting in x_0 close to x_t. Hence, a PoA very close to unity is obtained for WLAN SPs duopoly in presence of homogeneous user demand. This is consistent with the results obtained in [4]. Even for heterogeneous user demand with reasonable variance in $\alpha_n \forall n \in N$, a PoA close to unity is obtained as shown in [9].

Thus, with PoA close to unity, for both monopoly and duopoly, the primary issue with WLAN SPs price competition is not the users SW, instead it is the surplus transfer occurring because of competition. A monopoly is very desirable from SPs' perspective as it results in high revenues, whereas a duopoly/oligopoly is beneficial for the WLAN users as it ensures high cs. However, monopoly is undesirable from users' perspective as it ensures negligible cs, whereas SPs competition drives down their revenues to negligible levels. Thus, we propose a regulation of the price charged by the SPs to be in the range of $\left[p_{\mathrm{reg}}^{\min}, p_{\mathrm{reg}}^{\max}\right]$. This will force the monopolist to a maximum price of $p_{\mathrm{reg}}^{\max}$ and also ensure that SPs price competition will not drive down the prices below $p_{\mathrm{reg}}^{\min}$. The values of $p_{\mathrm{reg}}^{\max}$ and $p_{\mathrm{reg}}^{\min}$ can be varied by a regulator based on the user demand patterns, number of SPs in the market and the requisite surplus distribution between the SPs and the users. The values of $p_{\mathrm{reg}}^{\max}$ and $p_{\mathrm{reg}}^{\min}$ are numerically calculated in the numerical result section to ensure that consumer surplus and SPs' revenues are upper and lower bounds by the monopoly and duopoly scenarios.

6 Numerical results

Given a price vector $\boldsymbol{p} = (p_1, \ldots, p_{M_t})$, the corresponding user demand is obtained through numerical search in MATLAB such that it satisfies (5) and (6). As in [13], the corresponding NE price vector is obtained by finding the intersection of the BR functions, for all possible price vectors. Using (2), the corresponding aggregate consumer surplus at NE, cs^{equ}, is obtained. The SPs revenue at NE is obtained using (3). The computations have been performed for 1 (monopoly), 2 (duopoly) and 3 (oligopoly) APs. The minimal packet transmission error rate P_m^{t} is taken as 0.01 for all the APs. As in [11], K is set to 9.334. The simulations are performed for the total number of users in the system x_0 varying from 4 to 72. The QoS sensitivity α_n for homogeneous users is varied from 0.5 to 4.5, while the price sensitivity β_n is varied from 0.25 to 2.5. For heterogeneous user demand, four types of users are considered in the simulation. type 1 users have $\alpha_1 = 0.75$

IET Commun., 2013, Vol. 7, Iss. 18, pp. 2113–2121
doi: 10.1049/iet-com.2013.0296

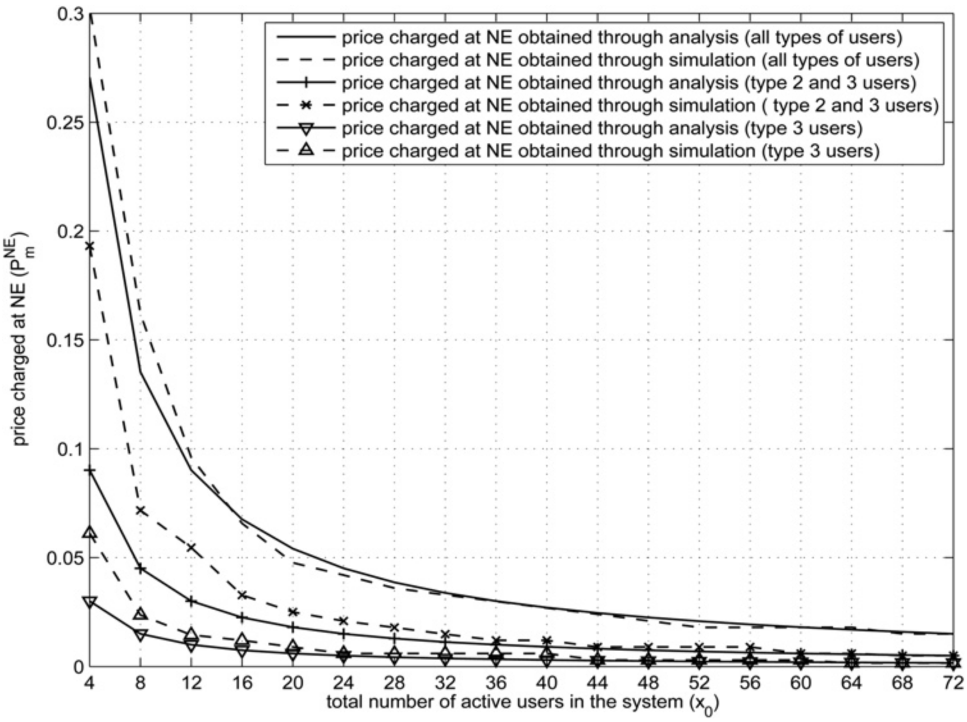

Fig. 3 *Variation of the price charged at NE p_m^{NE} with total number of connected users x_0 for various user demand distributions*

and $\beta_1 = 0.25$, type 2 users have $\alpha_2 = 0.75$ and $\beta_2 = 0.75$, type 3 users have $\alpha_3 = 0.25$ and $\beta_3 = 0.75$ and type 4 users have $\alpha_4 = 0.25$ and $\beta_4 = 0.25$.

Plots of the price charged at NE $p_m^{\rm NE}$ against the total number of connected users in the system x_0, obtained through (24) and simulations, are shown in Fig. 3. The scenarios considered are, equal number of users of all types, users of only 2 types (types 2 and 3) and homogeneous users (type 3 users only). The expression in (24) is based on an approximated PLR in (14). Hence, Fig. 3 clearly indicates that, with an increase in total number of connected users, as the error of approximation in (14) reduces, the $p_m^{\rm NE}$ obtained in (24) converges towards the value obtained via simulation. Note that in Fig. 3 the NE price is inversely proportional to the number of connected users; the same is obtained in (24). Fig. 3 illustrates the result that $\max_{n \in N} \alpha_n / \beta_n$ is the equilibrium characterising user type; in the presence of all four user types, type 1 users are the equilibrium characterising ones; in the presence of only types 2 and 3, type 2 users dominate over type 3 users.

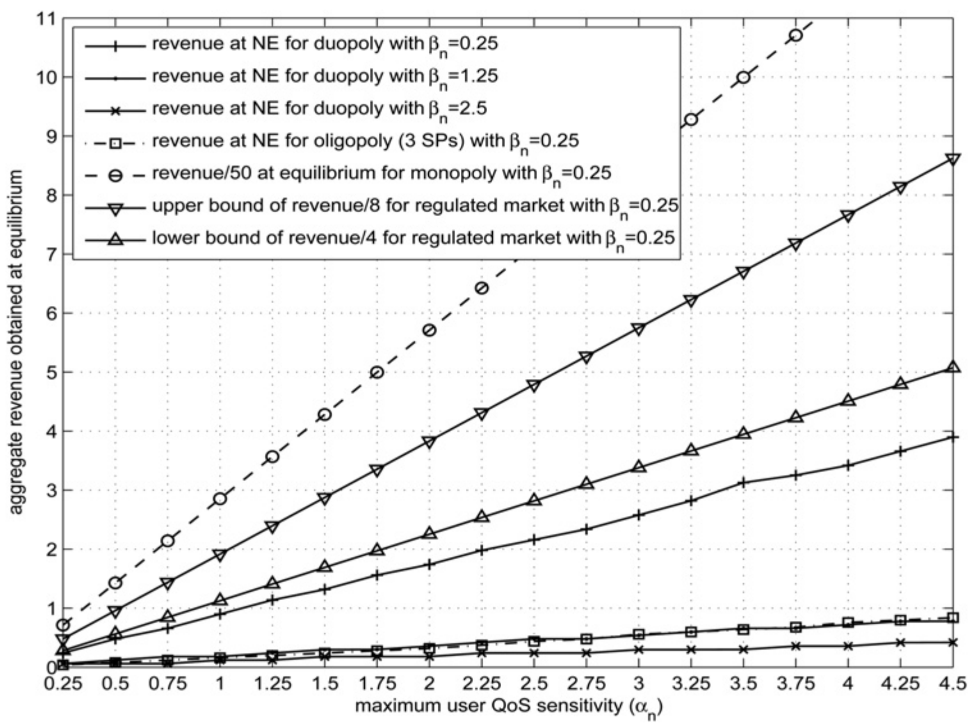

Fig. 4 *Variation of aggregate revenue of SPs with user QoS sensitivity α_n for varying user price sensitivity β_n*

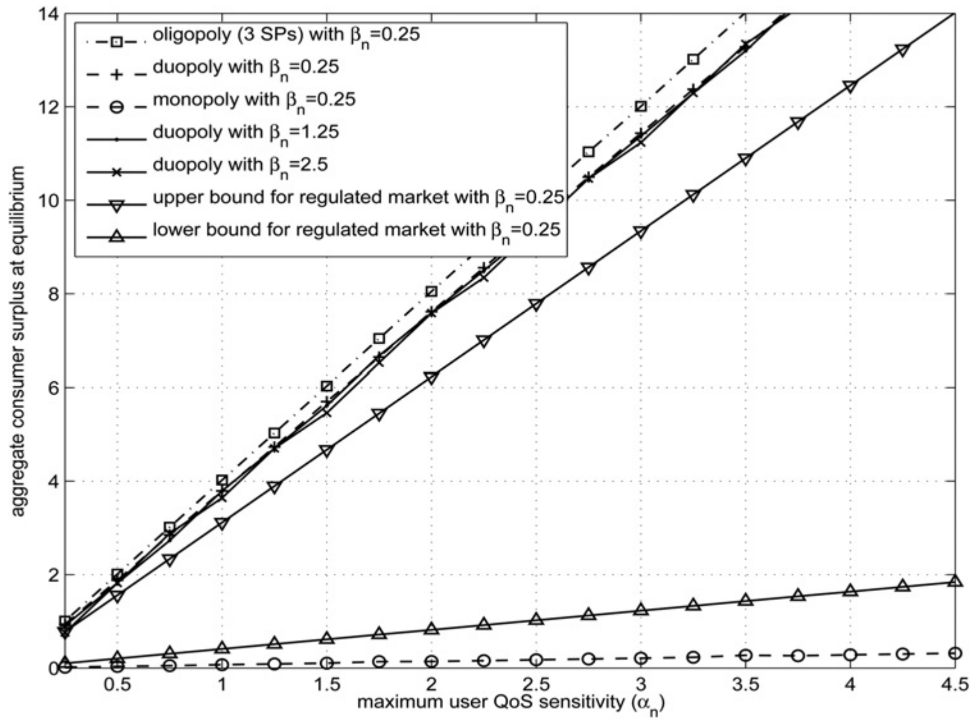

Fig. 5 *Variation of aggregate consumer surplus cs with user QoS sensitivity α_n for varying user price sensitivity β_n*

Thus, the p_m^{NE} curves from simulations are consistent with the result obtained in (24).

Fig. 4 contains plots of the aggregate revenue of the SPs for monopoly, duopoly and oligopoly for various values of maximum user QoS sensitivity. For Fig. 4, homogeneous user demand is considered. Price sensitivities of 0.25 (low price sensitivity), 1.25 (moderate price sensitivity) and 2.5 (high price sensitivity) are considered. The total number of connected users is set to 40. It is found that increase in competition from duopoly to oligopoly further reduces the revenue. Furthermore, an increase in user QoS sensitivity α_l results in almost linear increase in revenue. A decrease in revenue because of increasing user price sensitivity β_l is consistent with the results obtained. Note that for monopoly the revenue is orders of magnitude higher than the revenue in duopoly scenario. This is a very desirable outcome for the SPs; although, as shown in the next figure, the users suffer a lot as they obtain almost zero aggregate consumer surplus.

The variations of aggregate consumer surplus with price and QoS sensitivity are shown in Fig. 5. For Fig. 5, we consider 40 homogeneous users. The *cs* is clearly independent of the price sensitivity of the users. Fig. 5 indicates that an increase in SPs' competition from monopoly to duopoly and oligopoly results in a significant transfer of surplus from SPs to the users. In contrast with the monopoly, this is a very desirable outcome from the perspective of users. However, the minuscule revenue generated at equilibrium in duopoly and oligopoly may force the WLAN SPs to shut operations.

It is observed from Figs. 4 and 5 that there is a huge gap between the revenue and *cs* for monopoly and duopoly/oligopoly scenarios. Thus, as stated in the previous section, some sort of regulation is imperative for moderating the WLAN market. Through numerical search, we consider the values of $p_{\mathrm{reg}}^{\min} = 6.25ab\alpha_l/\beta_l$ and $p_{\mathrm{reg}}^{\max} = 42.5ab\alpha_l/\beta_l$. The revenue and *cs* for the same are shown in Figs. 4, 5 respectively. Note that the suggested price regulation results in optimal operation of the WLAN market with a balanced amount of surplus shared among the WLAN SPs and users. This shows the importance of price regulation in a WLAN market.

7 Conclusion

For an oligopoly price competition of SPs, we have shown the existence of WE in user demand distribution and the existence of equilibrium characterising user type. We have proved the existence of NE in the presence of heterogeneous user demand for duopoly of WLAN SPs. We have shown that, at NE in a duopoly/oligopoly scenario, a significant transfer of surplus occurs from SPs to users. In the case of a monopoly, the surplus predominantly remains with the SP. Hence, through the price regulation presented in this paper, policy makers/network regulators can ensure a just distribution of surplus among the WLAN users and SPs.

8 Acknowledgment

This work was supported in part by the IDRC Research Grant RP02253.

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