Neutrino masses and mixing angles in a model with six Higgs triplets and A_4 symmetry

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We have considered a model [E. Ma and D. Wegman, Phys. Rev. Lett. **107**, 061803 (2011)], where masses and a mixing pattern for neutrinos are governed by six Higgs triplets and A_4 symmetry. In this model we have applied a certain diagonalization procedure through which we have shown that neutrino masses can have both normal or inverted hierarchy. We have also shown that current neutrino oscillation data can be explained in this model.

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I. INTRODUCTION

Neutrino masses and mixing angles play a vital role in our understanding about physics beyond the standard model (SM) [1]. For a review on neutrino masses and mixing angles, see Ref. [2]. One of the unknown facts about neutrino masses is that we do not know how these masses have been ordered. Data from experiments indicate that neutrino masses can be arranged in either normal or inverted hierarchy [2]. The problem related to neutrino mixing angles is explained below. From the fits to various neutrino oscillation data, three mixing angles and the CP-violating Dirac phase (δ_{CP}) in the neutrino sector have been found [3]. Out of the three mixing angles, the values of θ_{12} and θ_{23} are consistent with $\sin^2\theta_{12} = 1/3$ and $\sin^2\theta_{23} = 1/2$, respectively. The third mixing angle is small and it is found that $\sin^2\theta_{13} \sim 10^{-2}$ [3]. To a good approximation, the three neutrino mixing angles are close to the following pattern: $\sin^2\theta_{12} = 1/3$, $\sin^2\theta_{23} = 1/2$, $\sin^2\theta_{13} = 0$. This is known as tribimaximal (TBM) mixing [4]. From this we can infer that the mixing angles in the neutrino sector are not arbitrary but could emerge from a pattern. Based on this, one would like to know if there is any underlying physics that is responsible for the pattern among the neutrino mixing angles.

To address the above mentioned problem, several theoretical models based on discrete symmetries have been proposed. For a review on these models and related works, see Refs. [5,6]. Out of these, models based on A_4 symmetry [7,8] are elegant in explaining the mixing pattern in the neutrino sector. Among these various models of A_4 symmetry, here we particularly focus on one model [9], which is proposed by Ma and Wegman. In this model, six Higgs triplets are introduced along with the SM fields [9]. Neutrinos, in this model, acquire nonzero masses through the type II seesaw mechanism [10], where the neutral component of Higgs triplets get vacuum expectation values (VEVs). By choosing certain A_4 symmetric charges for SM fields and Higgs triplets, the mixing pattern among neutrinos has been explained in this model. Some details related to these are given in the next section.

The above mentioned model is versatile, and was proposed soon after the T2K Collaboration had found [11], for the first time, that the mixing angle θ_{13} is nonzero. This model has rich phenomenology, since it has six Higgs triplets. One can study correlation between neutrino oscillation observables and the phenomenology due to Higgs triplets in this model. We discuss phenomenological implications of this model in Sec. VI. But before we study on that phenomenology, we have found that there are few limitations about the results obtained in Ref. [9]. In the work of Ref. [9], results are obtained after assuming VEVs of some particular two Higgs triplets are equal and opposite. We elaborate on this assumption in the next section where we briefly describe their work. After making this assumption, one conclusion from the results of Ref. [9] is that the neutrino masses in this model can only be in normal hierarchy. In the present work, we have analyzed the same model as it is proposed in Ref. [9], but we make some assumptions about the VEVs of Higgs triplets which are different from those in Ref. [9]. Following from our assumptions, we have shown that not only is the normal hierarchy possible in this model, the inverted hierarchy for neutrino masses is also possible. Moreover, we have shown that this model is compatible with any currently acceptable values for neutrino mixing angles and δ_{CP}.

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In the model of Ref. [9], after the six Higgs triplets get VEVs, neutrinos acquire a mixing mass matrix in the flavor basis. This mass matrix should be diagonalized by a unitary matrix and from this we can find the neutrino mixing angles and $\delta_{\rm CP}$. In this work, in order to diagonalize this mass matrix, we develop an approximation scheme, after making some assumptions about the VEVs of the Higgs triplets. From our approximation scheme, we obtain the leading order expressions for the three neutrino mixing angles and $\delta_{\rm CP}$. The approximation scheme that is applied in this work can have similarities with that in other works of Ref. [12]. But a difference can be seen in the way the mixing angles and $\delta_{\rm CP}$ are computed in our work as compared to that in other works.

The paper is organized as follows. In the next section we describe the model of Ref. [9]. In Sec. III we explain the assumptions we make in our work and describe a procedure for diagonalizing the mixing mass matrix for the neutrinos. In Sec. IV we obtain leading order expressions for the neutrino mixing angles and $\delta_{\rm CP}$. In Sec. V we present numerical results of our work. In Sec. VI we describe the phenomenological implications of the model of Ref. [9]. We conclude in the last section.

II. THE MODEL

The model we consider is an extension of the SM where the additional fields are two extra Higgs doublets and six Higgs triplets [9]. In this model, A_4 symmetry is imposed in addition to the SM gauge symmetry. The field content of this model in the neutrino sector and also their charge assignments under A_4 and electroweak symmetries are given in Table I. A_4 has the following four irreducible representations: $\underline{1}, \underline{1}', \underline{1}'', \underline{3}$. Under A_4, $SU(2)_L$ doublets and singlets of leptons are assigned as $L_i = (\nu_i, \ell_i) \sim \underline{3}$, $\ell_1^c \sim \underline{1}$, $\ell_2^c \sim \underline{1}'$, $\ell_3^c \sim \underline{1}''$. Here, $i = 1, 2, 3$. In the above mentioned model, altogether there are three Higgs doublets which we denote as Φ_i, $i = 1, 2, 3$. These doublets are assigned under $\underline{3}$ of A_4. With these charge assignments, the Yukawa couplings for charge leptons can be written as [7]

$$\mathcal{L} = h_{ijk}\overline{L_i}\ell_j^c\Phi_k + \text{H.c.} \tag{1}$$

Here, $i, j, k = 1, 2, 3$. h_{ijk} are Yukawa couplings, whose form is determined by A_4 symmetry, which can be seen in Ref. [7]. Assuming that the three Higgs doublets acquire the same VEV after the electroweak symmetry breaking, we get a mixing mass matrix for charged leptons. This mass matrix can be diagonalized with the following transformations on the charged lepton fields [7]:

$$\begin{aligned}
&\Psi_L \to U_L\Psi_L, \qquad \Psi_R \to U_R\Psi_R,\\
&\Psi_L = (\ell_1, \ell_2, \ell_3)^{\rm T}, \qquad \Psi_R = (\ell_1^c, \ell_2^c, \ell_3^c)^{\rm T},\\
&U_L = U_{CW} = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix},\\
&U_R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.
\end{aligned} \tag{2}$$

Here, $\omega = e^{2\pi i/3}$.

As stated before, neutrinos in this model acquire masses through the type II seesaw mechanism [10], when the six Higgs triplets get VEVs. Denoting these six Higgs triplets as $\xi_i, i = 1, \ldots, 6$, under A_4 their charges are assigned as follows: $\xi_1 \sim \underline{1}$, $\xi_2 \sim \underline{1}'$, $\xi_3 \sim \underline{1}''$, $\xi_j \sim \underline{3}$. Here, $j = 4, 5, 6$. After these Higgs triplets get VEVs, mass terms for neutrinos can be written as follows [9]:

$$\begin{aligned}
&\mathcal{L} = \bar{\Psi}^c{}_\nu\mathcal{M}_\nu\Psi_\nu + \text{H.c.}, \quad \Psi_\nu = (\nu_1, \nu_2, \nu_3)^{\rm T}, \quad \Psi_\nu^c = C\bar{\Psi}_\nu^{\rm T},\\
&\mathcal{M}_\nu = \begin{pmatrix} a+b+c & f & e \\ f & a+\omega b+\omega^2 c & d \\ e & d & a+\omega^2 b+\omega c \end{pmatrix}.
\end{aligned} \tag{3}$$

Here, C is the charge conjugation matrix. In the above equation, a, b, c, d, e, f come from $\langle\xi_1^0\rangle$, $\langle\xi_2^0\rangle$, $\langle\xi_3^0\rangle$, $\langle\xi_4^0\rangle$, $\langle\xi_5^0\rangle$, $\langle\xi_6^0\rangle$, respectively [9]. After applying the following transformation on Ψ_ν as

$$\begin{aligned}
&\Psi_\nu \to U_{CW}U_{\rm TBM}\Psi_\nu,\\
&U_{\rm TBM} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix},
\end{aligned} \tag{4}$$

the matrix $\mathcal{M}_\nu$ of Eq. (3) would transform to

$$\begin{aligned}
&\mathcal{M}_\nu'\\
&= \begin{pmatrix} a-(b+c)/2+d & (f+e)/\sqrt{2} & (b-c)\sqrt{3}/2 \\ (f+e)/\sqrt{2} & a+b+c & i(e-f)/\sqrt{2} \\ (b-c)\sqrt{3}/2 & i(e-f)/\sqrt{2} & -a+(b+c)/2+d \end{pmatrix}.
\end{aligned} \tag{5}$$

TABLE I. Relevant fields in the neutrino sector in the model of Ref. [9]. Charge assignments of these fields under A_4 and electroweak symmetries are also given. Here, $i = 1, 2, 3$ and $j = 4, 5, 6$.

Field	L_i	ℓ_1^c	ℓ_2^c	ℓ_3^c	Φ_i	ξ_1	ξ_2	ξ_3	ξ_j
A_4	$\underline{3}$	$\underline{1}$	$\underline{1}'$	$\underline{1}''$	$\underline{3}$	$\underline{1}$	$\underline{1}'$	$\underline{1}''$	$\underline{3}$
$SU(2)_L$	2	1	1	1	2	3	3	3	3
$U(1)_Y$	$-\frac{1}{2}$	-1	-1	-1	$\frac{1}{2}$	1	1	1	1

The above matrix would be in diagonal form if $e = f = 0$ and $b = c$ and in this case, from the transformations of charged leptons and neutrinos, we notice that $U_{\rm TBM}$ is the unitary matrix which diagonalizes the neutrino mass matrix in a basis where charged lepton masses are already diagonalized. Hence $U_{\rm TBM}$ can be identified as the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix. We can parametrize the PMNS matrix ($U_{\rm PMNS}$) in terms of neutrino mixing angles, which we have given in Sec. IV. After equating $U_{\rm TBM}$ with $U_{\rm PMNS}$ we can find that the neutrino mixing angles fit the TBM pattern, which we have described in the previous section. But in the above mentioned case, where $e = f = 0$ and $b = c$, the angle θ_{13} would become zero and this possibility is ruled out by the oscillation data. Hence, in order to get $\theta_{13} \neq 0$, at least some of e, f and $b - c$ should have nonzero values.

Based on the observations made in the previous paragraph, in Ref. [9], θ_{13} has been shown to be nonzero by assuming $e = -f \neq 0$ and $b - c \neq 0$. But by considering this possibility it has been concluded that the neutrinos can only have normal mass hierarchy. Although we should assume e and f to be nonzero, in general there need not be any constraint between them. In this work we consider nonzero values for e, f, and $b - c$, but otherwise do not assume any relation between e and f.

III. DIAGONALIZATION PROCEDURE AND NEUTRINO MASSES

In this section we explain our methodology of diagonalizing the matrix $\mathcal{M}_\nu$ of Eq. (3). As explained in the previous section that after applying the transformation of Eq. (4) on $\mathcal{M}_\nu$ of Eq. (3), we have got the mixing mass matrix among neutrinos which is given by $\mathcal{M}'_\nu$. We notice that $\mathcal{M}'_\nu$ is nearly diagonal if we assume e, f, and $b - c$ are small values. After assuming that these are small, we can expect that $\mathcal{M}'_\nu$ can be diagonalized by a unitary matrix which is nearly equal to the unit matrix. This unitary matrix can be parametrized, up to first order, as [12]

$$U_\epsilon = \begin{pmatrix} 1 & \epsilon_{12} & \epsilon_{13} \\ -\epsilon_{12}^* & 1 & \epsilon_{23} \\ -\epsilon_{13}^* & -\epsilon_{23}^* & 1 \end{pmatrix}. \tag{6}$$

In the above equation, ϵ_{12}, ϵ_{13}, ϵ_{23} are small and complex.

In the above described methodology, in order to diagonalize the matrix $\mathcal{M}_\nu$ of Eq. (3), we are applying the following transformation on the neutrino fields

$$\Psi_\nu \to U_{CW} U_{\rm TBM} U_\epsilon \Psi_\nu. \tag{7}$$

Now, from the transformations of charged leptons and neutrinos, we notice that the PMNS matrix in this model would be

$$U_{\rm PMNS} = U_{\rm TBM} U_\epsilon. \tag{8}$$

As we explained before, $U_{\rm PMNS}$ can be parametrized in terms of neutrino mixing angles. Hence, from the above relation we may hope to get θ_{13} to be nonzero for some particular values of ϵ parameters. As mentioned before, these ϵ parameters need to be small, since in our diagonalization procedure we have assumed that e, f, and $b - c$ of $\mathcal{M}'_\nu$ should be small. Here we quantify how small these parameters need to be. As mentioned previously, the neutrino oscillation data predicts that $\sin^2\theta_{13} \approx 2 \times 10^{-2}$ which is very small in comparison to unity. So we can take $\sin\theta_{13} \approx 0.15$ to be a small value. Based on this observation, we assume that the real and imaginary parts of ϵ parameters are at most the order of $\sin\theta_{13}$. By making this assumption, we show later that we get consistent results in our work.

As explained previously, we are applying the transformation of Eq. (7) on $\mathcal{M}_\nu$ of Eq. (3). As a result of this, we notice that, effectively the matrix $\mathcal{M}'_\nu$ is diagonalized by U_ϵ. The relation for the diagonalization of $\mathcal{M}'_\nu$ can be expressed as

$$\mathcal{M}'_\nu = U_\epsilon^* \cdot {\rm diag}(m_1, m_2, m_3) \cdot U_\epsilon^\dagger. \tag{9}$$

Here, m_1, m_2, m_3 are masses of neutrinos. Neutrino masses can be estimated from the global fits to the neutrino oscillation data [3]. From these global fits we know that there are two mass-square differences among the neutrino masses, which are given below [3].

$$m_{\rm sol}^2 = m_2^2 - m_1^2 = 7.39 \times 10^{-5}\,{\rm ev}^2,$$
$$m_{\rm atm}^2 = \begin{cases} m_3^2 - m_1^2 = +2.525 \times 10^{-3}\,{\rm eV}^2 & \text{(normal hierarchy)} \\ m_3^2 - m_2^2 = -2.512 \times 10^{-3}\,{\rm eV}^2 & \text{(inverted hierarchy)} \end{cases}. \tag{10}$$

In the above we have given the best fit values. Here $m_{\rm sol}$ and $m_{\rm atm}$ represent solar and atmospheric mass scales, respectively. To fit the above mass-square differences we can take neutrino masses as

$$m_1 \lesssim m_{\rm sol}, \quad m_2 = \sqrt{m_1^2 + m_{\rm sol}^2}, \quad m_3 = \sqrt{m_1^2 + m_{\rm atm}^2} \ \text{(NH)}$$
$$m_3 \lesssim m_{\rm sol}, \quad m_2 = \sqrt{m_3^2 - m_{\rm atm}^2}, \quad m_1 = \sqrt{m_2^2 - m_{\rm sol}^2} \ \text{(IH)}. \tag{11}$$

Here, NH (IH) indicate normal (inverted) hierarchy. In the case of IH, by taking $m_3 = m_{\rm sol}$ we would get $\sum m_\nu = m_1 + m_2 + m_3 = 0.11$ eV. This value is just below the upper bound on the sum of neutrino masses obtained by Planck, which is 0.12 eV [13]. On the other hand, in the case of NH, even if we take $m_1 = m_{\rm sol}$ we would get $\sum m_\nu = 0.07$ eV, which is reasonably below the above mentioned upper bound.

In the diagonalization procedure that we have described above, to find the neutrino masses we need to solve the relations in Eq. (9). We notice here that the matrix $\mathcal{M}'_\nu$ contains all the model parameters related to neutrino masses. From Eq. (9) it is clear that these model parameters are related to mass eigenvalues of neutrinos and ϵ parameters. In the next section we will show that these ϵ parameters can be determined from the neutrino mixing angles and $\delta_{\rm CP}$, whose values are found the oscillation data [3]. As for the mass eigenvalues of neutrinos, we have described above that they are chosen from mass-square differences which are also found from the oscillation data. Now, after using Eq. (9) we can proceed to find the model parameters of $\mathcal{M}'_\nu$ in terms of observables from oscillation data. Before doing that let us mention that the oscillation data predict that there is a hierarchy between the two neutrino mass-square differences. In fact, from the global fits to oscillation data, we notice that $\frac{m_{\rm sol}^2}{m_{\rm atm}^2} \sim \sin^2\theta_{13} \sim 10^{-2}$ [3]. As mentioned previously, quantities which are of the order of $\frac{m_{\rm sol}^2}{m_{\rm atm}^2}$ or $\sin^2\theta_{13}$ are very small in comparison to unity and so we neglect them in our analysis. As a result of this, we compute terms which are up to first order in $\sin\theta_{13} \sim \frac{m_{\rm sol}}{m_{\rm atm}}$, in the right-hand side of Eq. (9). We do this computation in both the cases of NH and IH. In either of these cases, the mass eigenvalues of neutrinos in terms of model parameters are found to be same and are given below.

$$m_1 = a + d - \frac{b+c}{2}, \qquad m_2 = a + b + c,$$
$$m_3 = -a + d + \frac{b+c}{2}. \tag{12}$$

However, relations for other model parameters are found to be dependent on neutrino mass hierarchy. These relations are given below.

$$\text{NH}:\ e + f = 0, \quad \frac{\sqrt{3}}{2}(b - c) = m_3\epsilon_{13}^*,$$
$$\frac{i}{\sqrt{2}}(e - f) = m_3\epsilon_{23}^*.$$
$$\text{IH}:\ \frac{e+f}{\sqrt{2}} = -m_1\epsilon_{12} + m_2\epsilon_{12}^*,$$
$$\frac{\sqrt{3}}{2}(b - c) = -m_1\epsilon_{13}, \quad \frac{i}{\sqrt{2}}(e - f) = -m_2\epsilon_{23}. \tag{13}$$

Using the above relations, we can see that the diagonal elements of the matrix $\mathcal{M}'_\nu$, up to first order approximation, would be the same as the mass eigenvalues of neutrinos, whereas the off-diagonal elements in $\mathcal{M}'_\nu$ are related to neutrino masses and ϵ parameters. Previously we have assumed that the real and imaginary parts of ϵ parameters are around $\sin\theta_{13}$. As a result of this, the relations in Eq. (13) suggest that the off-diagonal elements of the matrix $\mathcal{M}'_\nu$ are suppressed by $\mathcal{O}(\sin\theta_{13})$ as compared to the neutrino mass eigenvalues. This result is consistent with the assumption we made before that e, f, and $b - c$ should be small values.

Using the relations of Eqs. (12) and (13), depending on the case of NH or IH, we can determine all the model parameters in terms of neutrino mass eigenvalues and the ϵ parameters. As stated previously, these ϵ parameters can be found from the neutrino mixing angles and $\delta_{\rm CP}$, which is the subject of the next section. So we can state that by appropriately choosing the model parameters we can explain either the normal or inverted hierarchy mass spectrum for neutrinos in this model. Here it is worth mentioning that in the case of NH, we have $e = -f$. This is exactly what it is assumed in Ref. [9] and as a result of this it has been concluded that neutrinos can only have normal mass hierarchy. So our results are agreeing with that of Ref. [9] in the case of NH. But in addition to this, we have shown that the inverted mass hierarchy for neutrinos can also be possible in this model.

IV. NEUTRINO MIXING ANGLES

In the previous section we have explained that in order to get θ_{13} to be nonzero, we have chosen to follow a certain diagonalization procedure through which we have shown that the PMNS matrix in our model could be given by Eq. (8). The PMNS matrix can be parametrized in terms of neutrino mixing angles and a Dirac CP-violating phase, $\delta_{\rm CP}$. After using this parametrization in Eq. (8) we can get relations among neutrino mixing angles, $\delta_{\rm CP}$ and ϵ parameters. In this section, we will solve these relations and show that all the three neutrino mixing angles get deviations away from the TBM pattern and hence $\theta_{13} \neq 0$.

To express the PMNS matrix in terms of neutrino mixing angles and $\delta_{\rm CP}$, we follow the PDG convention, which we have given below [14].

$$U_{\rm PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\rm CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\rm CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{\rm CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{\rm CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{\rm CP}} & c_{23}c_{13} \end{pmatrix}. \tag{14}$$

Here, $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$. We use the above form of $U_{\rm PMNS}$ in Eq. (8) and determine the neutrino mixing angles and $\delta_{\rm CP}$ in terms of ϵ parameters. Since these ϵ parameters are complex we can write them as

$$\epsilon_{ij} = {\rm Re}(\epsilon_{ij}) + i{\rm Im}(\epsilon_{ij}), \qquad i,j = 1,2,3. \tag{15}$$

Here, ${\rm Re}(\epsilon_{ij})$ and ${\rm Im}(\epsilon_{ij})$ are real and imaginary parts of ϵ_{ij}.

As explained above, we use the form for $U_{\rm PMNS}$ of Eq. (14) in Eq. (8). After equating the 13 elements in the matrix relation of Eq. (8), we get the following relation for $\sin\theta_{13}$:

$$s_{13} = \left(\sqrt{\frac{2}{3}}\epsilon_{13} + \frac{1}{\sqrt{3}}\epsilon_{23}\right)e^{i\delta_{\rm CP}}. \tag{16}$$

Since the sine of an angle is real, we need to demand that the imaginary part of the right-hand side of the above relation should be zero. After doing this we get

$$\begin{aligned} s_{13} = &\left(\sqrt{\frac{2}{3}}{\rm Re}(\epsilon_{13}) + \frac{1}{\sqrt{3}}{\rm Re}(\epsilon_{23})\right)\cos\delta_{\rm CP} \\ &- \left(\sqrt{\frac{2}{3}}{\rm Im}(\epsilon_{13}) + \frac{1}{\sqrt{3}}{\rm Im}(\epsilon_{23})\right)\sin\delta_{\rm CP}. \end{aligned} \tag{17}$$

$$\begin{aligned} &\left(\sqrt{\frac{2}{3}}{\rm Re}(\epsilon_{13}) + \frac{1}{\sqrt{3}}{\rm Re}(\epsilon_{23})\right)\sin\delta_{\rm CP} \\ &+ \left(\sqrt{\frac{2}{3}}{\rm Im}(\epsilon_{13}) + \frac{1}{\sqrt{3}}{\rm Im}(\epsilon_{23})\right)\cos\delta_{\rm CP} = 0. \end{aligned} \tag{18}$$

From the above two equations we can see that both $\sin\theta_{13}$ and $\delta_{\rm CP}$ can be determined in terms of ϵ_{13} and ϵ_{23} parameters. Hence, by choosing some particular values for these ϵ parameters we may hope to get consistent values for $\sin\theta_{13}$ and $\delta_{\rm CP}$. We present these numerical results on ϵ parameters in the next section. But before doing that we will apply the above described method to obtain expressions for other sine of the angles, which is explained below.

As stated before, we are neglecting terms of the order of s_{13}^2 in comparison to unity; hence, we have $c_{13} = \sqrt{1-s_{13}^2} = 1 + \mathcal{O}(s_{13}^2) \approx 1$. Now that we have known c_{13}, by equating 12 and 23 elements of the matrix relation of Eq. (8), we determined s_{12} and s_{23} in terms of ϵ parameters. Here again we need to demand that the sine of an angle should be real. After doing this we get the following relations:

$$\begin{aligned} s_{12} &= \frac{1}{\sqrt{3}} + \sqrt{\frac{2}{3}}{\rm Re}(\epsilon_{12}), \\ s_{23} &= -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{6}}{\rm Re}(\epsilon_{13}) + \frac{1}{\sqrt{3}}{\rm Im}(\epsilon_{23}), \end{aligned} \tag{19}$$

$${\rm Im}(\epsilon_{12}) = 0, \qquad {\rm Im}(\epsilon_{13}) = \sqrt{2}{\rm Im}(\epsilon_{23}). \tag{20}$$

In the above we have shown that the sine of the three neutrino mixing angles and $\delta_{\rm CP}$ can be obtained in terms of ϵ parameters after equating the 12, 13, and 23 elements of the matrix relation of Eq. (8). In our analysis we have three complex ϵ parameters whose real and imaginary parts will give us six independent parameters. But from Eq. (20) we can see that ${\rm Im}(\epsilon_{13})$ and ${\rm Im}(\epsilon_{23})$ are not independent parameters and ${\rm Im}(\epsilon_{12}) = 0$. As a result of this the following four parameters can be used to determine the three neutrino mixing angles and $\delta_{\rm CP}$: ${\rm Re}(\epsilon_{12})$, ${\rm Re}(\epsilon_{13})$, ${\rm Re}(\epsilon_{23})$, and ${\rm Im}(\epsilon_{13})$.

In the matrix relation of Eq. (8) we have equated 12, 13, and 23 elements and found relations for the three neutrino mixing angles and $\delta_{\rm CP}$ in terms of ϵ parameters. By now we have used all the available ϵ parameters in determining the neutrino mixing angles and $\delta_{\rm CP}$. These relations for neutrino mixing angles and $\delta_{\rm CP}$ can be used in other elements of the matrix relation of Eq. (8) and then we may expect to get some constraints among the ϵ parameters. Below we will demonstrate that no constraints among these ϵ parameters will happen. Let us equate the 11 elements of the matrix relation of Eq. (8) and this would lead to

$$c_{12}c_{13} = \sqrt{\frac{2}{3}} - \frac{1}{\sqrt{3}}\epsilon_{12}^*. \tag{21}$$

We can check that the above relation is satisfied self consistently up to first order in ϵ parameters, after using Eqs. (17), (19), and (20). Similarly, we have checked that the relations we would get by equating other elements of the matrix relation of Eq. (8) are satisfied self-consistently up to first order in ϵ parameters after using Eqs. (17)–(20). As a result of this, we do not get any additional constraints on the ϵ parameters.

V. RESULTS

In the previous section we have explained that the three neutrino mixing angles and $\delta_{\rm CP}$ can be determined by ${\rm Re}(\epsilon_{12})$, ${\rm Re}(\epsilon_{13})$, ${\rm Re}(\epsilon_{23})$, and ${\rm Im}(\epsilon_{13})$. In this section we will show that for some particular values of these ϵ parameters, the three neutrino mixing angles and $\delta_{\rm CP}$ can be fitted to the observed values as found from the oscillation data [3]. For this purpose in Table II we mention

TABLE II. 3σ ranges in the cases of both NH and IH for the square of the sine of the three neutrino mixing angles and the *CP*-violating Dirac phase [3].

	NH	IH
$\sin^2\theta_{12}$	$0.275 \to 0.350$	$0.275 \to 0.350$
$\sin^2\theta_{23}$	$0.418 \to 0.627$	$0.423 \to 0.629$
$\sin^2\theta_{13}$	$0.02045 \to 0.02439$	$0.02068 \to 0.02463$
$\delta_{\rm CP}/^o$	$125 \to 392$	$196 \to 360$

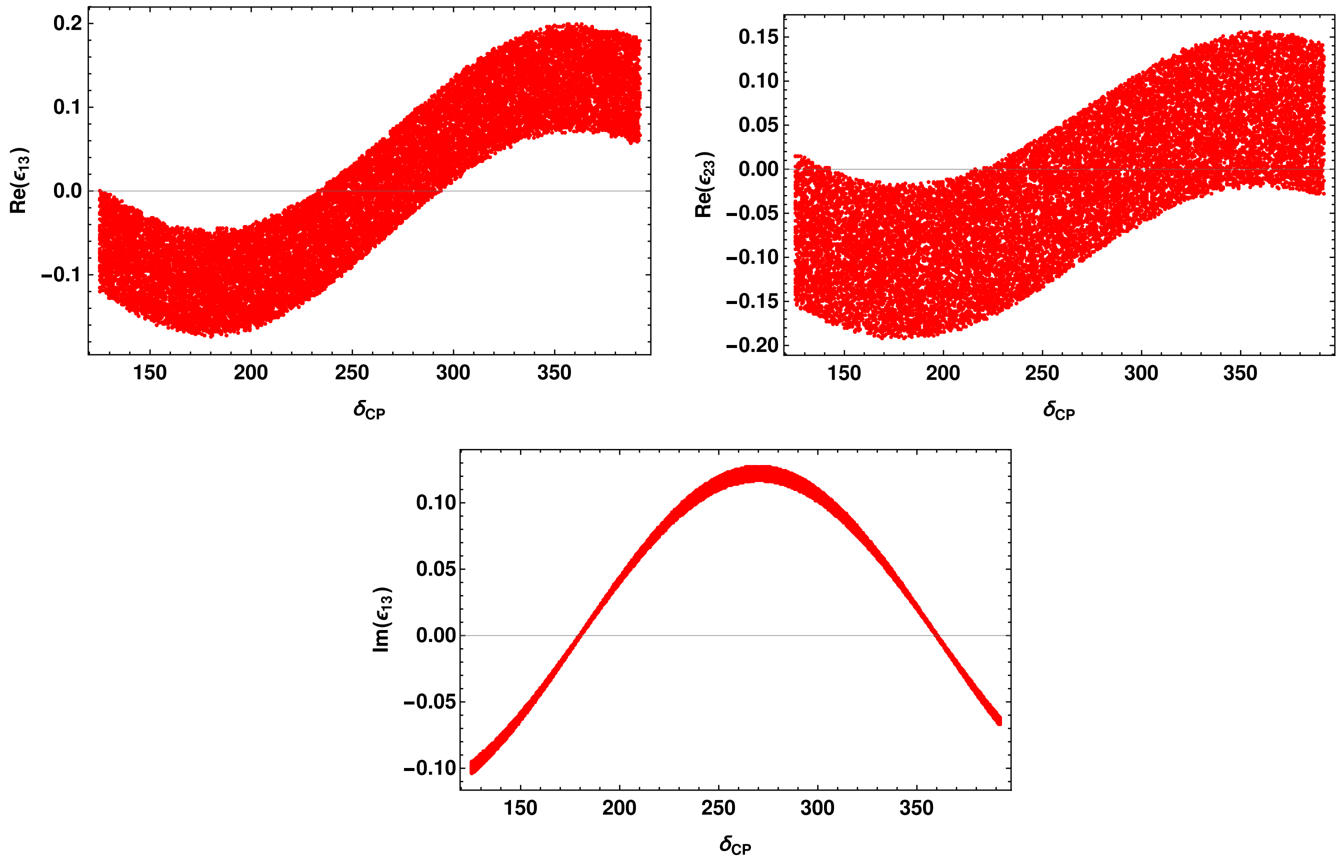

FIG. 1. Allowed regions for $\mathrm{Re}(\epsilon_{13})$, $\mathrm{Re}(\epsilon_{23})$, and $\mathrm{Im}(\epsilon_{13})$ are shown in the case of NH. δ_{CP} is expressed in degrees. In all the above plots, 3σ ranges for $\sin^2\theta_{13}$ and $\sin^2\theta_{23}$ have been used.

the 3σ ranges obtained in the cases of NH and IH for the neutrino mixing angles and δ_{CP}.

From the relations of Eqs. (17)–(20), we can obtain all ϵ parameters in terms of neutrino mixing angles and δ_{CP}. Using the 3σ range for $\sin^2\theta_{12}$, we found the allowed range for $\mathrm{Re}(\epsilon_{12})$ to be -6.19×10^{-2} to 1.77×10^{-2}. We can see that the magnitude of these allowed values are below $s_{13}\approx0.15$. From the 3σ ranges of $\sin^2\theta_{13}$, $\sin^2\theta_{23}$, and δ_{CP} we can get allowed regions for $\mathrm{Re}(\epsilon_{13})$, $\mathrm{Re}(\epsilon_{23})$, and $\mathrm{Im}(\epsilon_{13})$. These allowed regions are plotted in Fig. 1 in the case of NH. From this figure we can see that the values for $|\mathrm{Re}(\epsilon_{13})|$ and $|\mathrm{Re}(\epsilon_{23})|$ can be at most 0.2, which is just at the order of $s_{13}\approx0.15$. In fact, $|\mathrm{Re}(\epsilon_{13})|$ and $|\mathrm{Re}(\epsilon_{23})|$ get maximum values when δ_{CP} is around 180° or 360°. Otherwise, these parameters can take values even less than 0.2. As for the $|\mathrm{Im}(\epsilon_{13})|$, we notice from Fig. 1 that this parameter can take a maximum of 0.13 when δ_{CP} is around 270°.

We notice from Table II that the 3σ ranges for the neutrino mixing angles do not change much between NH and IH cases. The only significant difference is that δ_{CP} has a narrow allowed region in the case of IH as compared that of NH. Because of this, we can expect that the numerical limits quoted for $\mathrm{Re}(\epsilon_{12})$, $\mathrm{Re}(\epsilon_{13})$, $\mathrm{Re}(\epsilon_{23})$, and $\mathrm{Im}(\epsilon_{13})$ in the case of NH would almost be the same even in the case of IH. This we have seen after computing the above mentioned parameters in the case of IH. In fact, we have found that the allowed regions shown in Fig. 1 do not change significantly in the case of IH, except for the fact that in IH the axis of δ_{CP} varies from 196° to 360°.

From the numerical results described above we can see that all the ϵ parameters, in the case of NH and IH, are less than or of the order of s_{13}. This justifies the assumption we have made for diagonalizing the neutrino mass matrix in Sec. III. This justification also vindicates one of our results that both NH and IH cases are possible in the model of Ref. [9]. Here we comment on the fact that the calculations done in this work are up to first order in s_{13}. By including second order terms we expect the relations mentioned in Eqs. (12)–(13) and (17)–(20) get corrections with terms which are of $\mathcal{O}(s_{13}^2)$. Since these second order terms contribute very small values in the numerical analysis, we do not expect any changes in the qualitative conclusions made in this work.

VI. PHENOMENOLOGICAL IMPLICATIONS OF THE MODEL

As stated in Sec. I, neutrinos in the model of Ref. [9] acquire masses through the type II seesaw mechanism.

Hence, in this model, the lepton number is violated by two units and the neutrinos are Majorana particles. As a result of this, one implication of this model is the existence of neutrinoless double-beta decay. The rate of this decay is related to effective Majorana mass, which is given below

$$m_{ee} = \left| \sum_{i=1}^{3} m_i U_{ei}^2 \right|. \tag{22}$$

Here, U_{ei} are elements in the first row of the PMNS matrix, which is given in Sec. IV. So far the above mentioned decay has never been observed in experiments and as a result of that the following upper bound on m_{ee} has been set: 61–165 meV [15]. In the expression for m_{ee}, m_i indicate the three mass eigenvalues of neutrinos. In our analysis, these mass eigenvalues are related to model parameters through Eq. (12). The elements U_{ei} depend on neutrino mixing angles and $\delta_{\rm CP}$. Using our results obtained in Sec. IV we can express U_{ei} in terms of ϵ parameters, which are related to model parameters via Eq. (13). Hence, in our work, the quantity m_{ee} can be expressed in terms of model parameters. Using the above mentioned fact that m_{ee} has an upper bound from experiments, we can get constraints on model parameters in both NH and IH cases. We study these constraints in our future work.

In a type II seesaw mechanism [10], charge lepton flavor violating decays such as $\mu \to 3e$ and $\mu \to e\gamma$ are driven by charged components of the scalar triplet Higgs [16]. These decays happen due to Yukawa couplings of triplet Higgs with lepton doublets. Since in the model of Ref. [9], the type II seesaw mechanism is responsible for neutrino mass generation, one can expect the above mentioned flavor violating decays to happen in this model as well. We have seen that the structure with six triplet Higgses of this model can explain the consistent neutrino mixing pattern. Now, these triplet Higgses can also drive the above mentioned flavour violating decays. Hence, in this model there can exist a correlation between neutrino mixing angles and the flavor violating decays. These flavor violating decays are not observed in experiments and hence the branching ratios of these decays are bounded from above [14]. Using these experimental constraints, one can study the bounds on the masses of triplet Higgses. We can expect that these bounds may depend on the neutrino mixing angles, since there is a correlation between neutrino oscillation observables and the decay rates of these flavor violating processes. This is an interesting phenomenology that one can study in this model.

It is described in Sec. II that in the model of Ref. [9], three doublet and six triplet Higgses are proposed. The general form of the scalar potential in this model can be written as

$$V = V_1(\Phi_i) + V_2(\xi_k) + V_3(\Phi_i, \xi_k). \tag{23}$$

Here, $i = 1, 2, 3$ and $k = 1, \ldots, 6$. The full terms in $V_1(\Phi_i)$, which depend only on the three doublet Higgses, are given in Ref. [7]. Terms in the scalar potentials of $V_2(\xi_k)$ and $V_3(\Phi_i, \xi_k)$ can be found in the following way. The general form of invariant scalar potential under electroweak symmetry, containing one doublet and triplet Higgses, is given in Ref. [17]. Now, this potential needs to be generalized with three doublet and six triplet Higgses, along with the imposition of the additional symmetry A_4. The resultant form of that potential give full terms in $V_2(\xi_k) + V_3(\Phi_i, \xi_k)$. We notice here that in the full scalar potential of Eq. (23), there can exist many terms as compared to that in a model with one doublet and triplet Higsses. Hence, we can expect lot more parameters to be there in the scalar potential of Ref. [9]. After minimizing the potential of Eq. (23), Φ_i and ξ_k get VEVs, which need to satisfy certain relations in order to get a consistent neutrino mixing pattern in the model of Ref. [9]. The minimization conditions for the part of $V_1(\Phi_i)$ are studied in Ref. [7]. Now, the minimization conditions for the scalar potential of Eq. (23) can be studied, and we believe, due to large number of parameters in V, these conditions can be satisfied. One needs to know if this minima corresponds to local or global minimum. We study these detailed topics in our future work.

In Eq. (23), from the scalar potential part of $V_3(\Phi_i, \xi_k)$, we can see that there are interaction terms between doublet and triplet Higgses. This part of the potential can give mixing masses between these two kinds of Higgses, after Φ_i and ξ_k acquire VEVs. The VEVs of these fields spontaneously break the electroweak and A_4 symmetries of the model. After this breaking, the following fields remain in the theory: six doubly charged, eight singly charged, and 17 neutral scalars. Out of these 17, nine will be scalars and the rest are pseudoscalars. One among the nine neutral scalars can be identified as the Higgs boson of SM. The masses of non-SM scalars can be chosen to be around 1 TeV by appropriately choosing the parameters of the scalar potential of Eq. (23). Collider signals of these scalars are briefly discussed below. But before that, from the interaction terms in the scalar potential of Eq. (23), we notice that there can be trilinear couplings involving one neutral and two charged scalars. These couplings may give an additional contribution to the Higgs diphoton decay rate in the model of Ref. [9]. Since the measured value related to this decay rate in the LHC experiment [18] is consistent with the SM prediction, we may get some constraints on the above mentioned couplings of this model.

The doublet and triplet Higgses of this model have gauge interactions. Moreover, they have Yukawa interactions with lepton fields. Through these interaction terms, all the non-SM scalars of this model can be produced at the LHC experiment through vector boson fusion and subsequently they decay to SM fields. One can see that the doubly charged scalars of this model can decay into $\ell^\pm\ell^\pm$ and $W^\pm W^\pm$. Singly charged scalars of this model can decay into $\ell^\pm\nu$, $W^\pm Z$ and $W^\pm\gamma$. Neutral scalars of this model can decay into $\ell^+\ell^-$, $\nu\nu$, W^+W^- and ZZ. If kinematically allowed, through the interaction terms of Eq. (23), a doubly charged scalar can decay into a pair of singly charged scalars. We notice here that an analysis of the collider signals of this model is really interesting and worth doing. For related works on the phenomenology of A_4 based neutrino mass models, see Ref. [19].

VII. CONCLUSIONS

In this work we have analyzed a model which is proposed in Ref. [9]. In this model, neutrinos acquire masses and mixing pattern mainly due to the presence of six Higgs triplets and A_4 symmetry. In order to explain the mixing pattern among neutrinos, we have followed a certain approximation procedure for diagonalizing the neutrino mass matrix of this model. We then have shown that both NH and IH cases are possible for neutrino masses in this model. Following our approximation procedure, we have computed leading order expressions for neutrino mixing angles and $\delta_{\rm CP}$. Using these expressions we have shown that the current oscillation data can be explained in this model.

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