Optimal Number of Cognitive Users in K-out-of-M Rule

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Abstract—In this letter, we obtain a generalized expression for the optimal number of cognitive users (CUs) for the K-**out-of-**M **rule** that minimizes the Bayes risk at the fusion center (FC) over noisy control channels. We show many existing and new are special cases of the proposed solution. Numerical results are presented using energy detector. However, the expressions for optimal M obtained in this letter are applicable to any detector used in cooperative spectrum sensing.

Index Terms—Cognitive radio, Bayes risk function, noisy control channel, number of cognitive users.

I. INTRODUCTION

Cognitive radio (CR) has been proposed to overcome the spectrum shortage problem and the spectrum underutilization of current radio spectrum by allowing the cognitive users (CUs) to access spectrum of the licensed or primary user (PU) under sufficient protection to the PU [1], [2]. To do so, the CUs must continuously sense to identify the free spectrum and must be able to detect the presence of the PU signal [3]. Well-known detectors such as conventional energy detector (ED) [4], improved energy detector (IED) [5], etc., have been studied to determine the presence of the PU.

However, spectrum sensing using one CU may results in poor detection performance due to multipath and shadowing and may results in interference to the PU. In order to improve the reliability in detecting the PU signal, *cooperative spectrum sensing* (CSS) [6], [7] can be employed. The idea of CSS is to use multiple CUs and combine their observations at the fusion center (FC) using K-out-of-M rule [8]. The K-out-of-M rule decides the presence of PU if at least K out of M CUs must detect the PU signal. However, in practice, the control channels between the CUs and the FC are noisy [9], [10]. This will deteriorate the reliability of decisions transmitted from the CUs to the FC. A detailed survey on spectrum sensing and CSS which also highlights the research challenges and unsolved problems are presented in [11], [12].

Most of the recent work focuses on optimizing K of the K-out-of-M rule aiming for different objectives such as minimizing the Bayes risk function [8, p. 94], minimizing the *total error rate* [13], [14], maximizing the energy efficiency [15]. In [16], optimal K is found to maximize the CUs network throughput while satisfying protection constraint to the PU. In [17], an algorithm for the optimal K is presented to maximize the global detection probability subject to a constraint on global false alarm probability. In [18], the optimal K and the

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optimal detection threshold of the multi-hop CR network are derived. Some studies on optimizing the M are as follows; the optimum value of M that minimizes the total error rate for (i) OR rule is obtained in [19], [20] (ii) for AND and (iii) MAJORITY rule in [21]. In [15], the optimal M to maximize the energy efficiency is obtained through an exhaustive search algorithm. In this letter, we formulate the general optimization problem (GOP) for finding the optimal M that minimizes the Bayes risk function and then we show that most existing works [19]–[22] are special cases of GOP. Finally, we present the solutions for GOP and its special cases.

The outline of this paper is as follows. In Section II, we describe the system model for the CSS. In Section III, we formulate the GOP and its spacial cases for finding the optimal M. In Section IV, we present the solutions for the formulated problems. Section-V presents the numerical results using energy detector followed by conclusions in Section VI.

II. SYSTEM MODEL

We consider a centralized CSS model [14] composed of M CUs, a PU and a FC. Each CU conducts the spectrum sensing over the *sensing channel* and makes a binary decision regarding the presence of PU. Let $\mathcal{H}_0$ and $\mathcal{H}_1$ denote the hypotheses for the absence and presence of the PU, respectively. The local false alarm and missed detection probabilities of the kth CU are given, respectively, by

$$P_f^k = \Pr\{d_k = 1|\mathcal{H}_0\}, \; P_m^k = \Pr\{d_k = 0|\mathcal{H}_1\},$$

where $d_k \in \{0,1\}$ is the binary decision of the kth CU indicating the hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$, respectively. The local decisions from M CUs are transmitted to the FC over noisy control channels. The FC makes the final decision on the status of the PU by adopting the K-out-of-M rule [8]. Following [13], we assume that the CR network is homogenous, this implies $P_f^k = P_f, P_m^k = P_m, \forall k$. Let P_e^k denote the error probability of a control channel between the kth CU and the FC. We assume that all control channels are identical, which implies $P_e^k = P_e$. Under these assumptions, the global false alarm and missed detection probabilities for the K-out-of-M rule are given, respectively, by [10]

$$P_F(K,M) = \Pr(D = 1|\mathcal{H}_0) = \sum_{i=K}^{M} \binom{M}{i} P_{fe}^i (1 - P_{fe})^{M-i}, \tag{1}$$

$$
\begin{aligned}
P_M(K,M) &= \Pr(D = 0|\mathcal{H}_1) \\
&= 1 - \sum_{i=K}^{M} \binom{M}{i} (1 - P_{me})^i P_{me}^{M-i}, \tag{2}
\end{aligned}
$$

where $D \in \{0, 1\}$ is the final decision drawn by the FC that the PU is absent and present, respectively and

$$P_{fe} = P_f (1 - P_e) + (1 - P_f) P_e, \qquad (3)$$
$$P_{me} = P_m (1 - P_e) + (1 - P_m) P_e. \qquad (4)$$

The Bayes risk function for the K-out-of-M rule is given by [8, eq. 2.2.1]

$$\mathcal{R}(K, M) = \sum_{m=0}^{1} \sum_{n=0}^{1} \beta_{mn} P_n \Pr(D = m | \mathcal{H}_n)$$
$$= \beta_F P_F(K, M) + \beta_M P_M(K, M) + \beta_C, (5)$$

where $\beta_F = P_0 (\beta_{10} - \beta_{00})$, $\beta_M = P_1 (\beta_{01} - \beta_{11})$ and $\beta_C = \beta_{00} P_0 + \beta_{11} P_1$. $\beta_{mn}, \forall m, n \in \{0, 1\}$, is the cost incurred by declaring the final decision $D = m$ by the FC when the true hypothesis about the PU is $\mathcal{H}_n$, and where P_0 and P_1 denote the *a priori* probabilities of the PU being absent and present, respectively.

III. PROBLEM FORMULATION

The general optimization problem (GOP) can now be formulated as

$$\begin{aligned}
&\underset{M}{\text{minimize}} && \mathcal{R}(K, M) \\
&\text{subject to} && \mathbb{C}_1 : P_{fe} = P_{fe}^0, \\
& && \mathbb{C}_2 : P_{me} = P_{me}^0, \\
& && \mathbb{C}_3 : K = K_0,
\end{aligned} \qquad (6)$$

where $\mathcal{R}(K, M)$ is given in (5). P_{fe}^0, P_{me}^0 and K_0 are the equality constraints.

A. Special Cases of GOP

Now we present special cases of the GOP.
- GOP-I: Substituting $\beta_{10} = \beta_{01} = 1$ and $\beta_{00} = \beta_{11} = 0$ in (6), we get the optimization problem for finding the optimal M that minimizes the *average probability of error* of the K-out-of-M rule.
- GOP-II: Substituting $\beta_{10} = \beta_{01} = 2$, $\beta_{00} = \beta_{11} = 0$ and $P_0 = P_1 = 0.5$ in (6), we get the optimization problem for finding the optimal M that minimizes the *total error rate* of the K-out-of-M rule [22].
- GOP-III: Substituting $\beta_{10} = \beta_{01} = 2$, $\beta_{00} = \beta_{11} = 0$, $P_0 = P_1 = 0.5$ and $K = 1$ in (6), we get the optimization problem for finding the optimal M that minimizes the *total error rate* for the OR rule [19], [20].
- GOP-IV: Substituting $\beta_{10} = \beta_{01} = 2$, $\beta_{00} = \beta_{11} = 0$, $P_0 = P_1 = 0.5$ and $K = M$ in (6), we get the optimization problem for finding the optimal M that minimizes the *total error rate* for the AND rule [21].
- GOP-V: Substituting $\beta_{10} = \beta_{01} = 2$, $\beta_{00} = \beta_{11} = 0$, $P_0 = P_1 = 0.5$ and $K = \lceil M/2 \rceil$ in (6), we get the optimization problem for finding the optimal M that minimizes the *total error rate* of the MAJORITY rule [21].

Note that GOP-II to GOP-V have been addressed in literature while GOP-I is a new optimization problem. Also note that the minimization of *total error rate* and *average probability of error* result in different optimal solutions.

IV. SOLUTION OF THE GOP

Theorem 1. *The solution for GOP in (6), denoted as $M_{\mathcal{R}}^*$, is given by*

$$M_{\mathcal{R}}^* = \left\lceil K\beta + K - 1 + \frac{\ln \frac{P_1(\beta_{01} - \beta_{11})}{P_0(\beta_{10} - \beta_{00})}}{\ln \frac{1 - P_{fe}}{P_{me}}} \right\rceil, \quad \beta = \frac{\ln \frac{1 - P_{me}}{P_{fe}}}{\ln \frac{1 - P_{fe}}{P_{me}}}, \qquad (7)$$

where $\lceil . \rceil$ represents the ceiling function and P_{fe}, P_{me} are given by (3), (4) respectively.

Proof: The optimal M that satisfies (6) can be obtained by setting the difference of $\mathcal{R}$ with respect to M and equating to zero. i.e,

$$\mathcal{R}(K, M+1) - \mathcal{R}(K, M) = 0.$$
$$\Rightarrow \beta_F (P_F(K, M+1) - P_F(K, M)) +$$
$$\beta_M (P_M(K, M+1) - P_M(K, M)) = 0. \qquad (8)$$

To further simplify (8), we consider a function $\mathcal{F}$ in terms of $P(K, M) = \sum_{i=K}^{M} \binom{M}{i} \alpha^i (1 - \alpha)^{M-i}$ which is given by

$$\mathcal{F}(M) = P(K, M+1) - P(K, M)$$
$$= \sum_{i=K}^{M+1} \binom{M+1}{i} \alpha^i (1-\alpha)^{M-i+1} - \sum_{i=K}^{M} \binom{M}{i} \alpha^i (1-\alpha)^{M-i}$$
$$= \alpha^{M+1} +$$
$$\sum_{i=K}^{M} \left[\binom{M+1}{i} \alpha^i (1-\alpha)^{M-i+1} - \binom{M}{i} \alpha^i (1-\alpha)^{M-i} \right]. \qquad (9)$$

Substituting $\binom{M+1}{i} = \binom{M}{i} + \binom{M}{i-1}$ into (9) and rearranging, we have

$$\mathcal{F}(M) = \alpha^{M+1} + \sum_{i=K}^{M} \binom{M}{i-1} \alpha^i (1-\alpha)^{M-i+1} +$$
$$\sum_{i=K}^{M} \binom{M}{i} \alpha^i \left[(1-\alpha)^{M+1-i} - (1-\alpha)^{M-i} \right]$$

$$= \alpha^{M+1} + \sum_{i=K}^{M} \binom{M}{i-1} \alpha^i (1-\alpha)^{M-i+1}$$
$$- \sum_{i=K}^{M} \binom{M}{i} \alpha^{i+1} (1-\alpha)^{M-i}.$$

By expanding the summation terms in the above equation, we have

$$\mathcal{F}(M) = \alpha^{M+1} + \binom{M}{K-1} \alpha^K (1-\alpha)^{M-K+1}$$
$$+ \binom{M}{K} \alpha^{K+1} (1-\alpha)^{M-K} + \dots + \binom{M}{M-1} \alpha^M (1-\alpha)$$
$$- \binom{M}{K} \alpha^{K+1} (1-\alpha)^{M-K} - \dots - \binom{M}{M-1} \alpha^M (1-\alpha)$$
$$- \binom{M}{M} \alpha^{M+1}$$

$$= \binom{M}{K-1} \alpha^K (1-\alpha)^{M-K+1}. \qquad (10)$$

By substituting (10) into (8), we have

$$\beta_F P_{fe}^K (1 - P_{fe})^{M-K+1} - \beta_M (1 - P_{me})^K P_{me}^{M-K+1} = 0.$$

Substituting $\beta_F = P_0 (\beta_{10} - \beta_{00})$ and $\beta_M = P_1 (\beta_{01} - \beta_{11})$ in the above equation and rearranging, we get

$$M = K\beta + K - 1 + \frac{\ln \frac{P_1(\beta_{01}-\beta_{11})}{P_0(\beta_{10}-\beta_{00})}}{\ln \frac{1-P_{fe}}{P_{me}}}, \ \beta = \frac{\ln \frac{1-P_{me}}{P_{fe}}}{\ln \frac{1-P_{fe}}{P_{me}}}. \quad (11)$$

Since M is an integer value, therefore we take *ceiling function* for the M in (11) to obtain (7). ∎

Note that (7) is a function of P_f, P_m and P_e. Therefore, (7) is applicable to any detector used in the CSS. Now, we present the solution of optimal M for the special cases of GOP as follows.

A. Solutions for the Special Cases of GOP

The solution for GOP-I, denoted as $M^*_{P_E}$ and can be obtained by direct substitution of $\beta_{10} = \beta_{01} = 1$, $\beta_{00} = \beta_{11} = 0$ in (11), we get $M^*_{P_E} = \left\lceil K\beta + K - 1 + \frac{\ln(P_1/P_0)}{\ln \frac{1-P_{fe}}{P_{me}}} \right\rceil$. The solution for GOP-II, denoted as $M^*_{P_T}$ and can be obtained by direct substitution of $\beta_{10} = \beta_{01} = 2$, $\beta_{00} = \beta_{11} = 0$ and $P_0 = P_1 = 0.5$ in (11), we get [22] $M^*_{P_T} = \lceil K\beta + K - 1 \rceil$. The solution for GOP-III, denoted as $M^{OR}_{P_T}$ and can be obtained by direct substitution of $\beta_{10} = \beta_{01} = 2$, $\beta_{00} = \beta_{11} = 0$, $P_0 = P_1 = 0.5$ and $K = 1$ in (11), we get [19], [20] $M^{OR}_{P_T} = \lceil \beta \rceil$. The solution for GOP-IV, denoted as $M^{AND}_{P_T}$ and can be obtained by substituting $\beta_{10} = \beta_{01} = 2$, $\beta_{00} = \beta_{11} = 0$, $P_0 = P_1 = 0.5$ and $K = M$ in (11) and rearranging, we get [21] $M^{AND}_{P_T} = \lceil 1/\beta \rceil$.

The solution for GOP-V is a bit involved and can be obtained by substituting $\beta_{10} = \beta_{01} = 2$, $\beta_{00} = \beta_{11} = 0$, $P_0 = P_1 = 0.5$ and $K = \lceil M/2 \rceil$ in (11), we have

$$M = \lceil M/2 \rceil \beta + \lceil M/2 \rceil - 1$$

$$\Rightarrow \lceil M/2 \rceil = \frac{M+1}{\beta+1}.$$

Using mathematical definition of *ceiling function*, above equation can be written as,

$$\frac{M+1}{\beta+1} - 1 < \frac{M}{2} \le \frac{M+1}{\beta+1}. \qquad (12)$$

Considering left hand side inequality of (12), we have

$$\frac{M+1}{\beta+1} - 1 < \frac{M}{2}, \ \Rightarrow M < \frac{2\beta}{1-\beta} \qquad (13)$$

Note that M is non-negative integer value, therefore (13) is valid when $\beta < 1$ which implies $P_{me} < P_{fe}$. Considering right hand side inequality of (12), we have

$$\frac{M}{2} \le \frac{M+1}{\beta+1}, \Rightarrow M \le 2/(\beta - 1). \qquad (14)$$

Combining (13) and (14) we have [21]

$$M^{MAJ}_{P_T} = \begin{cases} \lceil 2\beta/(1-\beta) \rceil, & P_{me} < P_{fe}, \\ \lceil 2/(\beta-1) \rceil, & P_{me} \ge P_{fe}. \end{cases} \qquad (15)$$

V. NUMERICAL RESULTS USING ENERGY DETECTOR

We present the numerical results using energy detector (ED) as an example for analyzing our results obtained in this paper. The P_f and P_m of a CU using ED over additive white Gaussian noise (AWGN) channel are given, respectively, by [4]

$$P_f = \frac{\Gamma\left(\mu, \frac{\lambda}{2}\right)}{\Gamma(\mu)}, \ P_m = 1 - Q_\mu\left(\sqrt{2\gamma}, \sqrt{\lambda}\right), \qquad (16)$$

where the standard notations $\Gamma(.,.)$, $\Gamma(.)$ and $Q_\mu(.,.)$ denotes the upper incomplete gamma function, gamma function and generalized Marcum Q-function of order $\mu - 1$, respectively and μ denote the time-bandwidth product, λ and γ represents the sensing threshold and received *signal-to-noise ratio* (SNR) of a CU, respectively. Note that, in (16) P_f is a decreasing function and P_m is a increasing function with λ, respectively.
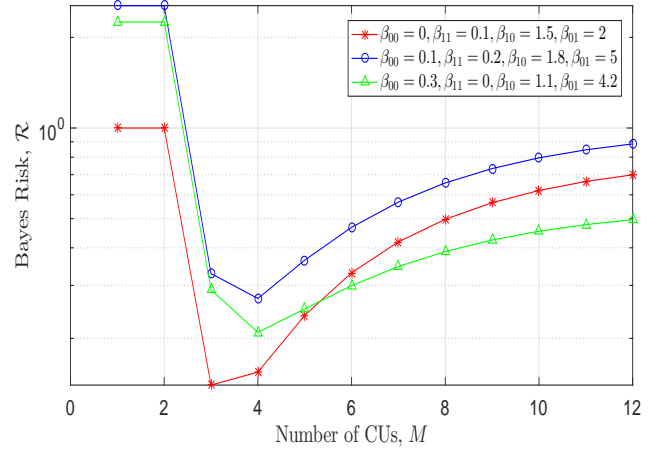

Fig. 1. Bayes risk versus Number CUs, M for $P_0 = P_1 = 0.5$, $\lambda = 11$, $K = 3$, $\mu = 5$, $P_e = 0.01$ and SNR = 10 dB, using ED.
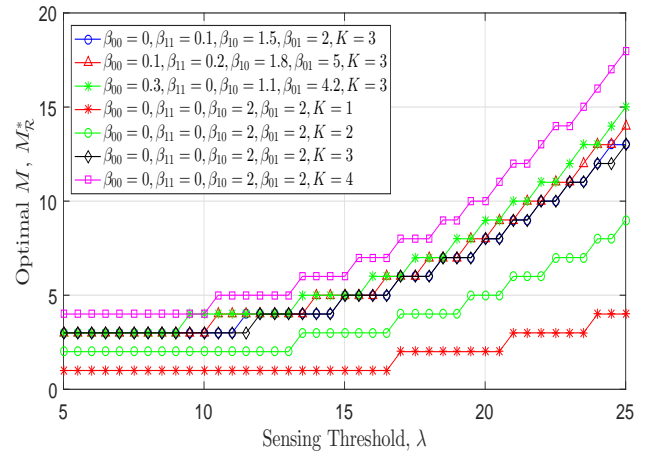

Fig. 2. Optimal M versus sensing threshold, λ for $P_0 = P_1 = 0.5$, $\mu = 5$, $P_e = 0.01$ and SNR = 10 dB, using ED.

Fig. 1 plots the Bayes against number of CUs for various combination of cost values using ED. It can be noticed that, for

each combination of cost values, the Bayes risk first decreases and then increases as M increases which suggests an optimal value of M. Fig. 2 plots the solution of optimal M versus sensing threshold. From Fig. 2, note that at $\lambda = 16.8$, we get $P_{fe} = P_{me} = 0.08$ for $P_e = 0.01$. Therefore, when $\lambda \leq 16.8$ implies $P_{me} \leq P_{fe}$ and when $\lambda > 16.8$ implies $P_{me} > P_{fe}$. From Fig. 2, now we make the following observations for the case when $\beta_{00} = 0, \beta_{11} = 0, \beta_{01} = 2, \beta_{10} = 2$.

- *OR Rule*: Cooperation using OR rule is beneficial and there exists an optimal M for $K = 1$. This can can be achieved when $\lambda > 16.8$, which implies $P_{me} > P_{fe}$.
- *AND Rule*: Cooperation using AND rule is beneficial and there exists an optimal M when $K = M_{\mathcal{R}}^*$. This can be achieved when $\lambda < 16.8$, which implies $P_{me} < P_{fe}$.
- *MAJORITY Rule*: Cooperation using MAJORITY rule is beneficial and there exists an optimal M when $K = \lceil M_{\mathcal{R}}^*/2 \rceil$. This can be achieved when $\lambda \approx 16.8$, which implies $P_{me} \approx P_{fe}$ (where the relation between P_{fe} and P_{me} can be $P_{me} \geq P_{fe}$ or $P_{me} > P_{fe}$).

Fig. 3 plots the optimal M versus SNR for three values of P_e. It can be noticed that, the optimal M decreases with SNR. For a fixed SNR the optimal M decreases with P_e, because the error in the control channel reduces the reliability of decisions received at the FC.
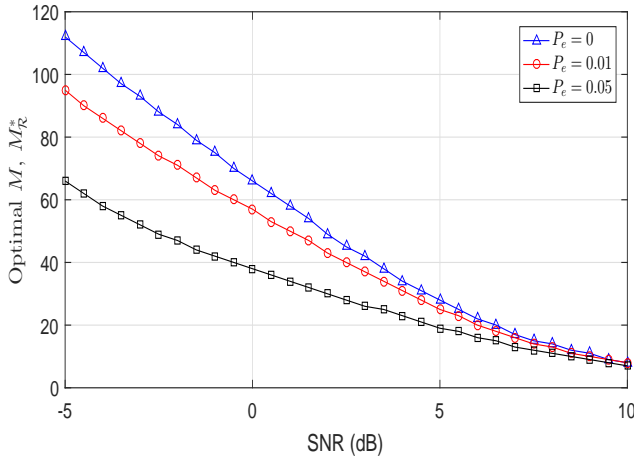

Fig. 3. Optimal M versus SNR for $\beta_{00} = 0.1, \beta_{11} = 0.2, \beta_{10} = 2, \beta_{01} = 3, P_0 = P_1 = 0.5, \mu = 5$ and $K = 3$, using ED.

VI. CONCLUSIONS

In this letter, we obtained a generalized expression for optimal number of CUs in the presence of control channel errors that minimizes the Bayes risk. We show that many existing results are special cases of the generalized expression. Also the results are valid for any detector.

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