

Muon $(g - 2)_\mu$ from the bulk neutrino field in a warped extra dimensional model

R. S. Hundi¹, Sourov Roy² and Soumitra SenGupta³

Department of Theoretical Physics,
Indian Association for the Cultivation of Science,
2A & 2B Raja S.C. Mullick Road,
Kolkata - 700 032, India.

Abstract

In the Randall-Sundrum model, a bulk neutrino field in the 5-dimensional space-time can give rise to tiny Dirac masses to neutrinos. In such a scenario, we have computed the contribution of the bulk neutrino field to the anomalous magnetic moment $(g - 2)_\mu$ of muon. We have computed this contribution in the 't Hooft-Feynman gauge and have found that the contribution has the right sign to fit the current discrepancy between the experiment and the standard model value of $(g - 2)_\mu$. We have also studied possible constraints on the model parameters by including contributions to $(g - 2)_\mu$ from other sources such as bulk gravitons.

¹E-mail address: tprsh@iacs.res.in

²E-mail address: tpsr@iacs.res.in

³E-mail address: tpssg@iacs.res.in

1 Introduction

The models in extra dimensions have been proposed for solving the hierarchy between the electroweak and Planck scales [1, 2]. Among these the Randall-Sundrum (RS) model assumes the existence of two 3-branes connected by one extra spatial dimension and the metric in this model is non-factorizable [2]. The extra spatial coordinate has been orbifolded by the symmetry S^1/Z_2 and one of the 3-branes can be identified as the visible brane and the other as the Planck brane. The warp factor in this model suppresses any Planck scale quantities into electroweak scale on the visible brane. In the RS model, all the standard model fields are assumed to be confined on the visible brane and only gravity propagates in the bulk of the five dimensions. Subsequently, the RS model was generalized to include other bulk fields in order to explain physical quantities such as neutrino masses and mixing pattern which normally cannot be explained within the standard model of elementary particles.

The deficit in the solar and atmospheric neutrino flux has given evidence for non-zero masses to neutrinos [3]. Fitting to the data of solar and atmospheric neutrino experiments, the following set of mass-square differences are obtained: $\Delta m_{\text{solar}}^2 = m_2^2 - m_1^2 \approx 7.6 \times 10^{-5} \text{ eV}^2$ and $\Delta m_{\text{atm}}^2 = |m_3^2 - m_{1,2}^2| \approx 2.5 \times 10^{-3} \text{ eV}^2$ [4]. Here, $m_{1,2,3}$ are the mass eigenvalues of the 3 active neutrinos. Apart from the mass-square differences an upper limit on the neutrino masses have been found through other experiments. Tritium β -decay puts an upper limit on the neutrino mass scale to be of the order of 2 eV [5]. Whereas from the cosmological observations, the sum of the three neutrino masses needs to be less than about 1 eV [6]. Since all the above experiments suggest tiny values for neutrino masses, perhaps a different mechanism should be operational for neutrino mass generation as compared to other fermion masses.

To explain the smallness of neutrino masses in the framework of RS model, an additional singlet neutrino field has been proposed, which like gravity field, propagates in the entire bulk of space-time [7]. The wave function of the bulk neutrino field is extended in the extra spatial dimension. The boundary conditions for this wave function can be chosen in such a way that it will have a very small overlap on our visible brane, resulting in tiny masses for neutrinos. The phenomenology of this model is determined through the Kaluza-Klein (KK) modes of the bulk neutrino field. Since the bulk neutrino field is singlet under the standard model gauge group, detection of KK modes of this field is challenging in the collider experiments. However, the loop effects due to these fields to any physically observable quantity can give us some hints about its existence. Here,

we study one of such observable quantities, namely the anomalous magnetic moment of muon.

At the tree level, the anomalous magnetic moment of muon, g -factor, has a value of 2 and radiative contributions give some corrections to it. Hence, it can be quantified as $a_\mu = (g - 2)_\mu/2$. For a review on $(g - 2)_\mu$, see [9, 10]. The world average value of $(g - 2)_\mu$ after the experiment E821 at the Brookhaven National Laboratory is as given below [8]

$$a_\mu^{\text{EXP}} = 11659208.0(6.3) \times 10^{-10}, \quad (1)$$

which is obtained with a precision of 0.54 parts per million. Various groups have computed the theoretical value for $(g - 2)_\mu$ in the standard model. Most of the groups have found a discrepancy between the experiment and the corresponding standard model value of $(g - 2)_\mu$ at about 3σ level [11]. Here, we take this difference as follows [10]

$$\Delta a_\mu = a_\mu^{\text{EXP}} - a_\mu^{\text{SM}} = (29 \pm 9) \times 10^{-10}. \quad (2)$$

The above difference would indicate existence of new physics. Moreover, since this difference is positive, the contribution due to new physics to $(g - 2)_\mu$ should yield a net positive value.

As explained before, in the model of Ref. [7], KK neutrinos can give some contribution to $(g - 2)_\mu$, which we have computed in this work. Previously, some work in this direction has been done in Ref. [12], where the authors have obtained a negative contribution to the $(g - 2)_\mu$ by adopting mass insertion approximation in the unitary gauge. In the present work, by carrying out an exact analysis in the mass eigenstate basis, we have computed the contribution from the bulk neutrino field to the $(g - 2)_\mu$ in the 't Hooft-Feynman gauge and scan the parameter space in the region consistent with experimental bounds. We have found that the bulk neutrino contribution to $(g - 2)_\mu$ has the right sign to fit the above mentioned discrepancy in $(g - 2)_\mu$. We have also incorporated contributions from other sources such as gravitons to the $(g - 2)_\mu$. Finally, we have studied the constraints that may arise from these various sources of $(g - 2)_\mu$ in the parameter space defined in [7]. Recently, in [13], constraints have been obtained from lepton flavor violation by considering a set of models in the RS frame work.

The paper is organized as follows. In the next section, we give a brief description of the warped extra dimensional model for neutrino masses [7]. In Sec. 3, we have computed the contribution of bulk neutrino field to $(g - 2)_\mu$ in the 't Hooft-Feynman gauge. In this section, we have also performed a detailed phenomenological study of our obtained expression for $(g - 2)_\mu$. In Sec. 4, we describe possible constraints on the model parameters

of [7] by including contributions to $(g - 2)_\mu$ from other sources such as gravitons. We conclude in Sec. 5.

2 Dirac neutrinos in the warped extra dimensional model

As described before, the model [7] is based on the RS model, where an additional singlet neutrino field is introduced in the bulk of the 5-dimensional space-time. The metric in this model is

$$ds^2 = e^{-2\sigma(\phi)} \eta_{\mu\nu} dx^\mu dx^\nu + r_c^2 d\phi^2, \quad (3)$$

where $\sigma(\phi) = kr_c|\phi|$, r_c is the compactification radius of the fifth dimension and k is an energy scale of the order of Planck scale, M_P . Here, ϕ is an angular coordinate representing the fifth dimension. Due to the S^1/Z_2 orbifold symmetry, ϕ varies from 0 to π and the 3-branes located at these points are called Planck and visible branes, respectively. In this model, the invariant action for a singlet bulk fermion Ψ is as given below.

$$S = \int d^4x \int d\phi \sqrt{G} \left\{ E_a^A \left[\frac{i}{2} (\bar{\Psi} \gamma^a \partial_A \Psi - \partial_A \bar{\Psi} \gamma^a \Psi) + \frac{\omega_{bcA}}{8} \bar{\Psi} \{ \gamma^a, \sigma^{bc} \} \Psi \right] - M_b \text{sgn}(\phi) \bar{\Psi} \Psi \right\}, \quad (4)$$

where G is the determinant of 5-dimensional metric G_{AB} , E_a^A is the inverse vierbein and ω_{bcA} is the spin connection. The small case letters, a, b, c , run over flat 5-dimensions and upper case Roman letters run over curved 5-dimensional space-time. Here, M_b is a bulk mass parameter which is $\mathcal{O}(M_P)$. The above action is invariant under $\phi \rightarrow -\phi$, which should follow due to the Z_2 orbifolding of the RS model. This symmetry is known as ϕ -parity which sets some boundary conditions on the wave functions of Ψ . The bulk field Ψ decomposes into KK modes in the 4-dimensional world, which can be written as

$$\Psi_{L,R}(x, \phi) = \sum_n \psi_n^{L,R}(x) \frac{e^{2\sigma}}{\sqrt{r_c}} f_n^{L,R}(\phi), \quad (5)$$

where $\Psi_{L,R} = \frac{1}{2}(1 \mp \gamma_5)\Psi$, $\psi_n^{L,R}(x)$ are the left- and right-handed KK modes in the 4-dimensions and $f_n^{L,R}(\phi)$ are its corresponding wave functions. In order to have the following canonically normalized action for the KK modes in 4-dimensions

$$S_{\text{eff}} = \sum_n \int d^4x \{ \bar{\psi}_n(x) i \gamma^\mu \partial_\mu \psi_n(x) - m_n \bar{\psi}_n(x) \psi_n(x) \}, \quad (6)$$

the wave functions $f_n^{L,R}$ should satisfy the following conditions

$$\int_\epsilon^1 dt f_m^{L*}(t) f_n^L(t) = \int_\epsilon^1 dt f_m^{R*}(t) f_n^R(t) = \delta_{mn}, \quad (\pm t \partial_t - \nu) f_n^{L,R}(t) = -x_n t f_n^{R,L}(t). \quad (7)$$

Here, m_n are the masses of KK modes. In the above equations, the following change of variables have been done : $t = \epsilon e^{\sigma(\phi)}$, $\epsilon = e^{-kr_c\pi}$ and $f_n^{L,R}(\phi) \rightarrow \sqrt{kr_c\epsilon} f_n^{L,R}(t)$. This change of variables imply that $t = 1$ corresponds to the visible brane. The unknown quantities in the above equation are :

$$\nu = \frac{M_b}{k}, \quad x_n = \frac{m_n}{k} e^{kr_c\pi}. \quad (8)$$

The ϕ -parity, which is described above, imposes the following boundary conditions on the wave functions : $f_m^{L*}(\epsilon) f_n^R(\epsilon) = f_m^{L*}(1) f_n^R(1) = 0$. The coupled differential equations of Eq. (7) can be exactly solved and for $n > 0$, the wave functions can be expressed in the form of Bessel functions. The wave function for the zeroth mode can be written as

$$f_0^{L,R}(t) = f_0^{L,R}(1) t^{\pm\nu}, \quad |f_0^{L,R}(1)|^2 = \frac{1 \pm 2\nu}{1 - \epsilon^{1 \pm 2\nu}}. \quad (9)$$

The zeroth mode wave function of the right-handed field is highly suppressed on the visible brane, for $\nu > \frac{1}{2}$. Because of this reason, by choosing the boundary condition as $f_n^L(\epsilon) = f_n^L(1) = 0$, we can achieve a very small overlap of the singlet bulk field on the visible brane, for $\nu > \frac{1}{2}$. Using this mechanism, in the next paragraph, we describe the neutrino masses in this model.

The invariant action for neutrino Yukawa interaction is

$$S_Y = - \int d^4x \sqrt{-g_{\text{vis}}} \{ \hat{Y}_5 \bar{L}_0(x) \tilde{H}_0(x) \Psi_R(x, \pi) + \text{h.c.} \}, \quad (10)$$

where L_0 is a left-handed lepton doublet and \tilde{H}_0 is the conjugate of the Higgs doublet. Here \hat{Y}_5 has mass dimensions of $\frac{1}{\sqrt{M_P}}$ and the metric on the visible brane is $(g_{\text{vis}})_{\mu\nu} = e^{-2\sigma(\pi)} \eta_{\mu\nu}$ and $g_{\text{vis}} = \det \left((g_{\text{vis}})_{\mu\nu} \right)$. Both the lepton and Higgs doublets need to have the following rescaling : $L_0 = e^{\frac{3}{2}\sigma(\pi)} L$, $H_0 = e^{\sigma(\pi)} H$, in order to have canonical kinetic terms for these fields. After substituting the KK mode expansion, eq. (5), the invariant action becomes

$$S_Y = - \int d^4x \sum y_n \bar{L} \tilde{H} \Psi_n^R(x) + \text{h.c.}, \quad y_n = \sqrt{k} \hat{Y}_5 f_n^R(1) = Y_5 f_n^R(1). \quad (11)$$

Here, Y_5 is an order one parameter. In the basis, $\Psi_L^\nu = (\nu_L, \Psi_1^L, \dots, \Psi_n^L)$ and $\Psi_R^\nu = (\Psi_0^R, \Psi_1^R, \dots, \Psi_n^R)$, we have

$$S_Y = - \int d^4x \bar{\Psi}_L^\nu M \Psi_R^\nu + \text{h.c.}, \quad M = \begin{pmatrix} y_0 & y_1 & \cdots & y_n \\ 0 & m_1 & \cdots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & m_n \end{pmatrix}. \quad (12)$$

Here, the vacuum expectation value of the Higgs field is $v = 174$ GeV. Now, define

$$\Psi_L^\nu = U^L \nu_L^{\text{kk}}, \quad \Psi_R^\nu = V^R \nu_R^{\text{kk}}, \quad (13)$$

then the physical masses are given by

$$(U^L)^\dagger M V^R = M_{\text{diag}}. \quad (14)$$

While diagonalizing the matrix M , the masses m_n of KK modes are determined by the equation $J_{\nu-\frac{1}{2}}(x_n) = 0$. However, the masses m_n are not the physical masses of KK modes, since they get correction due to mixing with the left-handed lepton doublet after the electroweak symmetry breaking. After diagonalizing the matrix M , the lowest mass eigenvalue can be shown to be proportional to y_0 which yields a very small value due to small overlap of the wave function $f_0^R(1)$. The mass of light neutrino can be shown to be [7]

$$m_\nu = \sqrt{2\nu - 1} Y_5 \epsilon^{(\nu-\frac{1}{2})} v. \quad (15)$$

So far we have assumed the presence of only one bulk neutrino field and it generates one light Dirac neutrino mass. As explained in the previous section, to fit the neutrino oscillation data, we need at least two non-zero neutrino mass eigenstates. Hence, in the model of Ref. [7] we have to propose more than one bulk neutrino field. However, it has been argued in [7] that to cancel the anomalies related to ϕ -parity, only even number of singlet bulk fields can be introduced into the model. So, by introducing two bulk neutrino fields with slightly different bulk mass parameters, M_b , we can fit both the solar and atmospheric neutrino mass scales. In this picture the third neutrino has exactly zero mass. However, by introducing 4 bulk neutrino fields in an analogous way, we can generate all the three light neutrino masses.

3 Contribution of KK neutrinos to $(g - 2)_\mu$

We have shown in the previous section that to generate neutrino masses, the KK modes of bulk neutrino field have some mixing with the left-handed lepton doublet. This mixing would lead to gauge as well as Yukawa type interactions of muon with the physical KK modes of neutrinos. As a result of this, the KK modes of neutrinos give some contribution to $(g - 2)_\mu$, which we will describe shortly. It is to be noticed here that we have computed the contribution of bulk neutrino to $(g - 2)_\mu$ in the 't Hooft-Feynman gauge, hence the interactions of muon with Nambu-Goldstone states are necessary. For the sake

of completeness, below we describe interaction terms involving the physical KK modes of bulk neutrino field. The interaction terms of other fields are unchanged from that of the standard model.

The coupling of $\mu W^+ \nu_i^{\text{kk}}$:

In this model, the interaction term of W -boson with charged lepton and neutrino would come from the 5-dimensional invariant action of the kinetic energy of the lepton doublet. For our particular case of muon field, these interactions are as given below.

$$S_K \ni \int d^4x \frac{g}{\sqrt{2}} [\bar{\nu}_{\mu L} \gamma^\mu \mu_L W_\mu^+ + \bar{\mu}_L \gamma^\mu \nu_{\mu L} W_\mu^-]. \quad (16)$$

After using Eq. (13) we get

$$S_K \ni \int d^4x \frac{g}{\sqrt{2}} \sum_i \left[(U_{1i}^L)^* \bar{\nu}_i^{\text{kk}} \gamma^\mu \frac{1 - \gamma_5}{2} \mu W_\mu^+ + U_{1i}^L \bar{\mu} \gamma^\mu \frac{1 - \gamma_5}{2} \nu_i^{\text{kk}} W_\mu^- \right]. \quad (17)$$

The coupling of $\mu G^+ \nu_j^{\text{kk}}$:

Without loss of generality, we can go to a basis where charged lepton Yukawa couplings are diagonalized. In this basis, the necessary Lagrangian for our purpose is

$$\mathcal{L} = -Y_\mu \bar{L}_\mu H \mu_R - \sum_j y_j \bar{L}_\mu \tilde{H} \Psi_j^R + \text{h.c.} \quad (18)$$

Here L_μ is the left-handed lepton doublet containing muon. The second term in the above equation is from Eq. (11). We can use $Y_\mu = m_\mu/v$ where m_μ is the mass of muon. After using Eq. (13), we can show that

$$\begin{aligned} \mathcal{L} \ni & -\frac{m_\mu}{v} \sum_j \bar{\nu}_j^{\text{kk}} G^+ (U_{1j}^L)^* \frac{1 + \gamma_5}{2} \mu + \sum_{i,j} \bar{\nu}_j^{\text{kk}} G^+ y_i^* (V_{i,j}^R)^* \frac{1 - \gamma_5}{2} \mu \\ & -\frac{m_\mu}{v} \sum_j \bar{\mu} G^- U_{1j}^L \frac{1 - \gamma_5}{2} \nu_j^{\text{kk}} + \sum_{i,j} \bar{\mu} G^- y_i V_{ij}^R \frac{1 + \gamma_5}{2} \nu_j^{\text{kk}}. \end{aligned} \quad (19)$$

The interaction terms, as described above, generate some contribution to $(g-2)_\mu$ at one loop level, which are shown in Fig. 1. The amplitude of any of the Fig. 1 is of the form $i\mathcal{M} = -ie\bar{u}(p')\Gamma^\mu u(p)\epsilon_\mu$, where ϵ_μ is the polarization of photon and e is the positron charge. After using the Gordon identity, $\bar{u}(p')\gamma^\mu u(p) = \bar{u}(p') \left[\frac{(p+p')^\mu}{2m_\mu} + \frac{i\sigma^{\mu\nu}q_\nu}{2m_\mu} \right] u(p)$, where $q = p' - p$, we can put the interesting part of the amplitude as

$$\Gamma^\mu = \gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m_\mu} F_2(q^2) + \dots \quad (20)$$

The contribution to $(g-2)_\mu$ is $\Delta a_\mu = \frac{(g-2)_\mu}{2} = F_2(0)$.

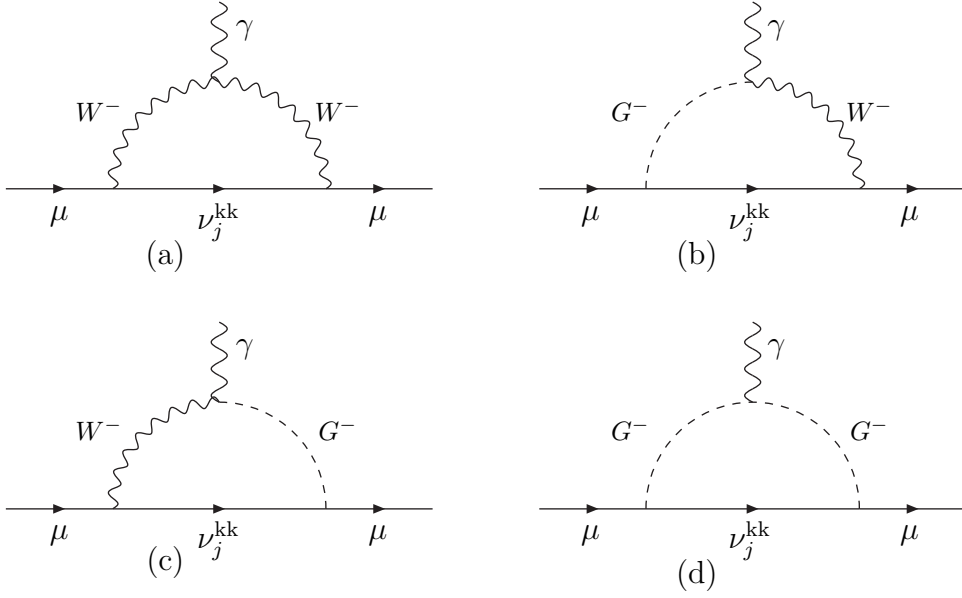


Figure 1: The contribution of KK neutrinos to $(g - 2)_\mu$. The momentum convention is that the outgoing muon has p' , the incoming muon has p and the momentum on the photon line is $q = p' - p$, which is incoming.

The contribution from Fig. 1(a) to $(g - 2)_\mu$ is

$$\begin{aligned} \Delta a_\mu^{(1)} &= \frac{m_\mu^2}{32\pi^2} \sum_{j=2} g^2 U_{1j}^L (U_{1j}^L)^* \frac{1}{(m_j^{\text{kk}})^2} F_a(x_j), \quad x_j = \frac{m_W^2}{(m_j^{\text{kk}})^2}, \\ F_a(x) &= \frac{1}{(1-x)^4} \left[-\frac{31}{6} + \frac{19}{2}x - \frac{11}{2}x^2 + \frac{7}{6}x^3 - (3-x)\ln(x) \right]. \end{aligned} \quad (21)$$

The sum of the contributions from Figs. 1(b) and 1(c) to $(g - 2)_\mu$ is as follows

$$\begin{aligned} \Delta a_\mu^{(2)} &= \frac{m_\mu^2}{32\pi^2} \sum_{j=2} g^2 U_{1j}^L (U_{1j}^L)^* \frac{1}{(m_j^{\text{kk}})^2} F_b(x_j), \quad x_j = \frac{m_W^2}{(m_j^{\text{kk}})^2}, \\ F_b(x) &= \frac{1}{(1-x)^3} \left[-\frac{3}{2} + 2x - \frac{1}{2}x^2 - \ln(x) \right]. \end{aligned} \quad (22)$$

The contribution to $(g-2)_\mu$ from Fig. 1(d) is

$$\begin{aligned}
\Delta a_\mu^{(3)} &= \frac{m_\mu^2}{16\pi^2} \sum_{i=2, j=2, k=2} \left\{ - \left[\frac{m_\mu^2}{v^2} U_{1j}^L (U_{1j}^L)^* + y_i y_k^* V_{ij}^R (V_{kj}^R)^* \right] \frac{1}{(m_j^{\text{kk}})^2} F_c(x_j) \right. \\
&\quad \left. + \frac{1}{vm_j^{\text{kk}}} [U_{1j}^L y_k^* (V_{kj}^R)^* + (U_{1j}^L)^* y_k V_{kj}^R] F_d(x_j) \right\}, \quad x_j = \frac{m_W^2}{(m_j^{\text{kk}})^2}, \\
F_c(x) &= \frac{1}{(1-x)^4} \left[\frac{1}{3} + \frac{1}{2}x - x^2 + \frac{1}{6}x^3 + x \ln(x) \right], \\
F_d(x) &= \frac{1}{(1-x)^3} \left[\frac{1}{2} - \frac{1}{2}x^2 + x \ln(x) \right]. \tag{23}
\end{aligned}$$

It is to be noticed that in the contribution to $(g-2)_\mu$ from all the plots of Fig. 1, the summation in indices over i, j, k is from 2, since we have to subtract the light neutrino contribution which exists in the standard model.

Here we comment on our results on the contribution of bulk neutrino to $(g-2)_\mu$. As already explained before that we have computed the contribution from Fig. 1 in the 't Hooft-Feynman gauge. The first three diagrams of Fig. 1 have given positive contributions, while the Fig. 1(d) has given both positive as well as negative contributions. In our numerical analysis, which we present below, we have found that in most of the parameter space the contribution from Fig. 1(d) is dominant and it gives a net positive contribution. Specifically, we have seen that the negative and positive contributions are comparable to each other but the magnitude of the former one is at least an $\mathcal{O}(1)$ less than the later one in $\Delta a_\mu^{(3)}$. Another comment is that, since there are two insertions of Yukawa couplings in Fig. 1(d) we would expect the contribution to $(g-2)_\mu$ to increase with Y_5 . In our numerical analysis we have found that this is true but there are some exceptions to this, which we will explain in our numerical results.

The total contribution of one bulk neutrino field to the $(g-2)_\mu$ is

$$\Delta a_\mu^N = \Delta a_\mu^{(1)} + \Delta a_\mu^{(2)} + \Delta a_\mu^{(3)} \tag{24}$$

The above contribution is mainly dependent on ν , kr_c and the 5-dimensional bulk mass parameter M_b , which altogether determine the KK masses of the bulk neutrino field. Apart from this, Δa_μ^N also depends on the dimensionless parameter Y_5 which determine the elements of the unitary matrices U^L and V^R . On the other hand, the light neutrino mass eigenvalue due to one bulk neutrino field, depends on ν , kr_c and Y_5 . As described in Sec. 1, from the neutrino oscillation data we have some idea on the magnitude of neutrino masses. For instance, in the hierarchical pattern of neutrinos at least two mass

eigenvalues should be : $m_{\text{solar}} = \sqrt{\Delta m_{\text{solar}}^2} \approx 0.01$ eV and $m_{\text{atm}} = \sqrt{\Delta m_{\text{atm}}^2} \approx 0.05$ eV. In this case, the third neutrino can have either zero mass or its mass should be less than of the order of m_{solar} . In the case of degenerate neutrinos, all the three neutrinos should have a nearly equal mass and the common mass eigenvalue should be sufficiently larger than m_{atm} . Since an upper bound from the cosmological observations indicate that the sum of the three neutrino masses should be less than about 1 eV [6], we take the common mass to be as $m_{\text{deg}} \approx 0.3$ eV. By fixing the mass eigenvalues of light neutrinos, we may eliminate ν as independent variable. The parameter Y_5 should be $\mathcal{O}(1)$ and here we take its value in the range 0.1 to 2.5. Here the lower limit of 0.1 is due to the naturalness argument and the upper limit is due to perturbativity constraints. In order not to hit the Landau pole, the Yukawa couplings y_n should be less than $\sqrt{4\pi}$. For $n > 0$, $|f_n^R(1)| = \sqrt{2}$ and hence Y_5 should be less than $\sqrt{2\pi} \approx 2.5$. As for the bulk mass parameter M_b , we take its value close to the Planck scale. We take the values of kr_c to be around 12, since for this set of values we can produce TeV scale masses on the visible brane from the exponential warping. Finally, in our numerical analysis, we have allowed a 50×50 mixing mass matrix of Eq. (12). After diagonalizing this matrix we get 49 KK neutrino mass modes which we have summed in the Δa_μ^N . We have checked that the above mentioned number of KK modes in Δa_μ^N is sufficient and its value vary insignificantly by further increasing the number of KK modes.

In Fig. 2 we have plotted the contribution due to one bulk neutrino field to $(g-2)_\mu$. In Fig. 2(a) we have fixed $kr_c = 12.5$. In this plot, the upper three curves are due to a bulk mass of $M_b = 5 \times 10^{18}$ GeV and the remaining three lower curves are for $M_b = 10^{19}$ GeV. The three curves are arising depending on whether the light neutrino mass eigenvalue fits the solar, or atmospheric, or degenerate mass eigenvalue. The value of ν is not shown in these plots, since as explained before it is not an independent parameter. The meaning of curves in Fig. 2(b) are same as that of Fig. 2(a), except that kr_c has taken a value 12.0 in Fig. 2(b). For one particular value of Y_5 , the contribution due to a bulk neutrino field which fits the solar, or atmospheric, or degenerate neutrino is nearly the same. This is evident in both the Figs. 2(a) and 2(b), where the three curves due to different neutrino mass eigenvalues are close to each other. We can understand the reason for this as follows. For fixed values of Y_5 , kr_c and M_b , the change in the value of Δa_μ^N from one curve to the other curve can happen only due to the change in the neutrino mass eigenvalue. The ratio of neutrino mass eigenvalues between degenerate and solar cases could be at most by a factor of $\mathcal{O}(10)$. From Eq. (15), it can be seen that the neutrino mass eigenvalue is related to ν dominantly through exponential factor, so we need only a change of ~ 0.1

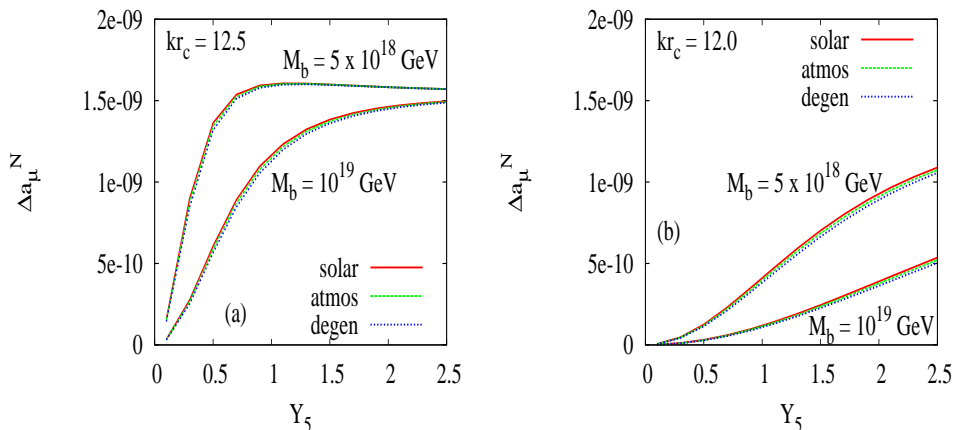


Figure 2: Δa_μ^N has been plotted as a variable of Y_5 . In these plots the three curves which are closely stacked together represents whether the light neutrino mass fits the solar, or atmospheric, or degenerate mass. The values of kr_c and M_b have been varied in both these plots.

in ν to produce a change of factor $\mathcal{O}(10)$ in the mass eigenvalue. These exact numerical values in the change of ν due to change in the neutrino mass eigenvalue can be seen in Tab. 1, where ranges of ν are given for extreme values of Y_5 . Such a small change in ν produces a slight change in the KK masses of neutrino fields, and hence a slight change to the Δa_μ^N . However, a closer examination would reveal that the amount of Δa_μ^N where solar mass is fitted is slightly more than that for atmospheric mass which is even slightly more than that for degenerate mass. The change in these three cases could be at most in the first decimal place of Δa_μ^N value. The values of ν in these three cases would be different even though Y_5 could be same. We have presented the ranges of ν for the cases of $kr_c = 12.5$ and 12.0 in Tab. 1. The ranges of ν in Tab. 1 are for extreme values of Y_5

	$kr_c = 12.5$	$kr_c = 12.0$
m_{sol}	$\nu \sim (1.22 - 1.31)$	$\nu \sim (1.25 - 1.34)$
m_{atm}	$\nu \sim (1.18 - 1.26)$	$\nu \sim (1.21 - 1.30)$
m_{deg}	$\nu \sim (1.13 - 1.22)$	$\nu \sim (1.16 - 1.25)$

Table 1: Ranges of ν for different values of kr_c and for different masses of light neutrino mass eigenvalues. In a particular range of ν , the left- and right-end corresponds to $Y_5 = 0.1$ and $Y_5 = 2.5$, respectively.

of 0.1 and 2.5. Since the mass eigenvalue of neutrino, eq. (15), does not depend on the bulk mass parameter M_b , the range of ν is also independent of this parameter. From the plots of Fig. 2, we have realized that there is some sensitivity between the light neutrino mass eigenvalue and the corresponding Δa_μ^N value. However, for the same value of Y_5 , this sensitivity is so small that we may not determine the neutrino mass eigenvalue based on the Δa_μ^N value.

Next, we present results on how the Δa_μ varies by changing the kr_c as well as the bulk mass parameter M_b , which is given in Fig. 3. In the plots of Fig. 3, we have fixed the

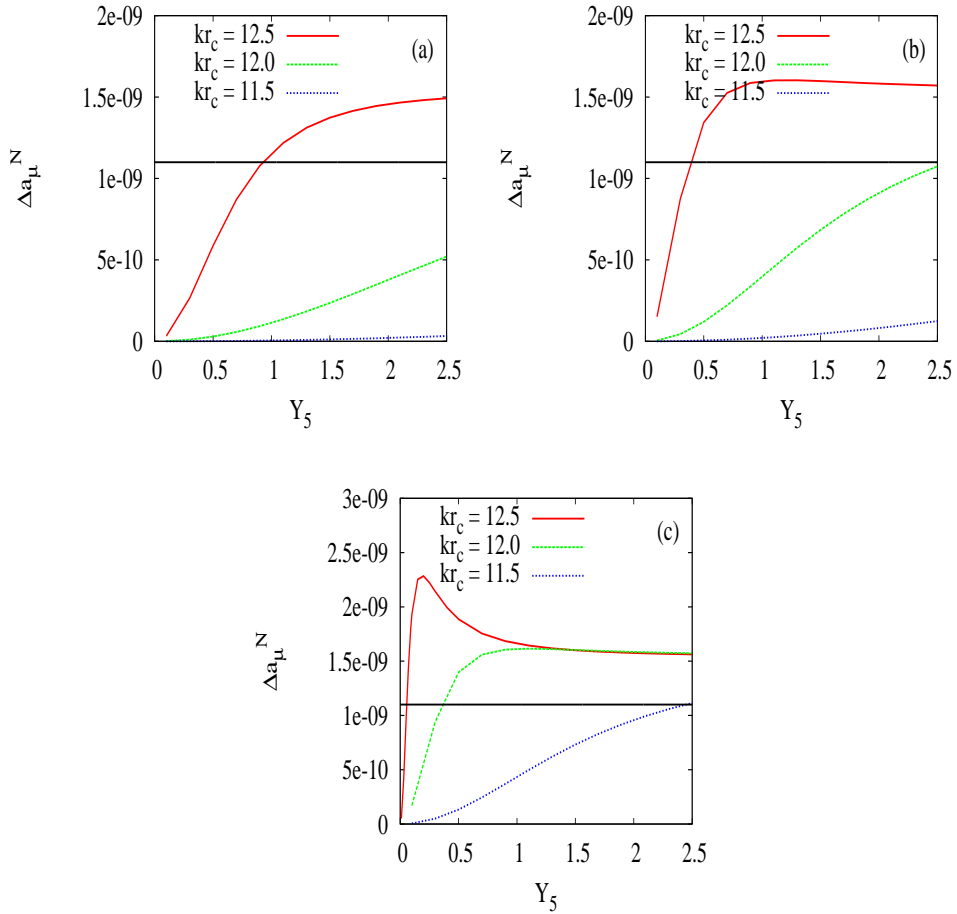


Figure 3: Δa_μ^N has been plotted against Y_5 for different values of kr_c . In the top-left and top-right plots, the value of M_b has been taken as 10^{19} GeV and 5×10^{18} GeV, respectively. In the lower-middle plot, $M_b = 10^{18}$ GeV. The horizontal line in all these plots indicates the 2σ lower limit of Δa_μ , see the text for details.

light neutrino mass eigenvalue to be the atmospheric scale m_{atm} . The choice of neutrino

mass scale do not make much difference in numerical values which we have argued around Fig. 2. From the plots of Fig. 3 we can understand that by increasing the value of kr_c the exponential warping would decrease the KK masses of neutrinos and hence the contribution to Δa_μ^N would increase. It can also be noticed from Fig. 3 that the lower the bulk mass parameter the larger the contribution is to the Δa_μ^N , which is evident since the masses of KK neutrinos would become lower. For low values of Y_5 , the contribution to Δa_μ^N is low and it is increasing and in some cases it may be saturated for large enough Y_5 . We have noticed that all the curves would be saturated for some large enough Y_5 . For example, the curves for $kr_c = 12.0$ in Figs. 3(a) and 3(b) would saturate for Y_5 around 50 and 20, respectively. The dependence of Δa_μ^N on Y_5 is somewhat complicated as can be noticed from the theoretical expression given above. However, we can notice that Fig. 1(d) has two insertions of Yukawa couplings and hence we can predict that $\Delta a_\mu^{(3)}$ should increase with Y_5 . On the other hand, Y_5 also affect the contributions of the first three plots of Fig. 1 by determining the elements of U^L and V^R and also the physical KK masses of neutrinos. Numerically we have seen that for large value of Y_5 , the contribution from Fig. 1(d) is always 2 orders greater than that due to other plots of Fig. 1. The contribution from Fig. 1(d) is given in the form of $\Delta a_\mu^{(3)}$, from which we can see that there is a partial cancellation due to positive and negative contributions of $\Delta a_\mu^{(3)}$. We have numerically seen that $\Delta a_\mu^{(3)}$ goes to a saturation value for large enough Y_5 . Since in the plots of Fig. 3, we have fixed the neutrino mass eigenvalue to atmospheric scale, and the mass eigenvalue has an exponential dependence on ν , we have found that after large enough Y_5 the change in ν would be far less compared to the change in Y_5 . Hence the values of m_n which determine the KK masses of neutrinos would almost be saturated. The elements of U^L and V^R would change with Y_5 , however, numerically we have seen that the net sum of the various KK modes is saturated after large enough value of Y_5 . In the plot of Fig. 3(c), for the case of $kr_c = 12.5$ and $M_b = 10^{18}$ GeV the amount of Δa_μ^N would peak at around $Y_5 \sim 0.1$. In this particular case the contribution from $\Delta a_\mu^{(1)}$ is significantly dominant at around $Y_5 \sim 0.1$. However, in this case the lowest KK mass of neutrino is around 30 GeV. In other cases where the lowest KK mass is at least few 100 GeV, $\Delta a_\mu^{(3)}$ would give the dominant contribution. We have noticed these facts purely from numerical values.

The results described above show that the contribution from a single bulk neutrino field in five dimensions can easily fit, depending on the values of parameters, the 2σ

deviation of Δa_μ . From Eq. (2) we take the 2σ deviation as

$$\Delta a_\mu = (1.1 - 4.7) \times 10^{-9}. \quad (25)$$

However, to be specific, the results of Fig. 3 indicate that the contribution to Δa_μ is most likely towards the lower end of the above 2σ deviation or even less than this depending on the values of Y_5 . In the case of $kr_c = 11.5$, the contribution to Δa_μ is much below the 2σ limit for $M_b = 10^{19}$ GeV or 5×10^{18} GeV. But keeping $M_b = 10^{18}$ GeV and for $Y_5 \sim 2$, we get a value of $\Delta a_\mu \sim 10^{-9}$. Since this is only from one bulk neutrino field and in a realistic scenario we need at least two bulk neutrino fields, so we can fit the above 2σ deviation by adjusting the M_b and Y_5 , even for a low value of $kr_c = 11.5$.

In the above analysis we have presented our results due to the existence of one bulk neutrino field. By introducing a second bulk neutrino field, the additional main parameters that the second field would carry are its 5-dimensional bulk mass parameter, M_b , and its Yukawa coupling Y_5 to the muon. We can convince ourselves that the expression for Δa_μ due to this second field would be same as that of the first field, but replace the above said parameters accordingly. Now, consider a realistic scheme where there are two bulk neutrino fields which fits both the solar and atmospheric neutrino mass scales. In order to fit these neutrino mass scales, we may choose their Yukawa couplings to be nearly same but the values of ν would be slightly different for these fields, which can be understood from Tab. 1. This would result in slight difference in the respective values of their bulk parameters M_b . Hence the total contribution to $(g - 2)_\mu$ due to these two bulk fields would almost be the twice of the contribution from a single bulk field.

4 Constraints on the model parameters by including graviton contribution

In the previous section, we have shown that the contribution due to bulk neutrino field can fit the 2σ deviation of Δa_μ . The fits in the previous section would in fact set limits on the parameter space of the model in [7], if this is the only source for $(g - 2)_\mu$. However, the contribution due to the bulk neutrino may get further constraints due to the presence of other sources in the 5-dimensional warped model, such as from graviton or radion fields. Since gravity exists in the whole space-time, we cannot ignore its contribution to the Δa_μ [14]. Similarly, the length of the fifth dimension should be dynamically generated and this leads to the presence of radion field. We have found that the contribution from gravitons

to Δa_μ is significantly larger than that of the radion contribution [15]. Below, we show how the contribution from the gravitons to Δa_μ would lead to some constraints on the model parameters of [7].

Regarding the graviton contribution to $(g - 2)_\mu$, it has been computed in the case of RS model in [16, 14]. However, the expression given in [16] has been estimated by assuming all the standard model fields in the bulk of five dimensions. Since in the model of [7], only the singlet bulk neutrino and gravity are allowed in the bulk of space-time, we confine to the results given in [14], where it is claimed to be done for the case of the original RS model. In [14], an expression for the contribution of KK gravitons to the $(g - 2)_\mu$ at one loop level is given, which is as follows

$$\Delta a_\mu^g = \frac{5}{16\pi^2} \left(\frac{m_\mu}{\Lambda_\pi} \right)^2 n_c, \quad (26)$$

where n_c is the number of KK gravitons and $\Lambda_\pi = e^{-kr_c\pi} M_P$. It has also been argued in [14] that the number of KK gravitons are bounded by some unitarity constraints due to $\gamma\gamma$ elastic scattering, where it has been shown to be $n_c \leq 10 - 100$. By taking the Planck scale as $M_P = 10^{19}$ GeV, the contribution due to a single KK graviton are as follows : $(kr_c, \Delta a_\mu^g) = (12.2, 6.8 \times 10^{-9}), (12.1, 3.64 \times 10^{-9}), (12.0, 1.94 \times 10^{-9}), (11.7, 2.95 \times 10^{-10}), (11.3, 2.39 \times 10^{-11})$. Comparing these values with the 2σ range of Eq. (25), anything above the $kr_c = 12.1$ can be ruled out purely from the KK graviton contribution to $(g-2)_\mu$. For $kr_c = 12.0$, only two KK gravitons can exist. Whereas, for $kr_c = 11.7$ and 11.3 , of the order of 10 and 100 number of KK gravitons can exist, respectively. Although it may be inappropriate to assume the existence of only one or two KK gravitons in the universe, nevertheless, by the unitarity bounds of [14] it seems to be consistent. In these cases we show how this assumption would put bounds on the bulk neutrino parameters of [7].

For $kr_c = 12.1$, there can exist only one KK graviton to fit the 2σ deviation of Δa_μ . In this case, an amount of $\Delta a_\mu^{\max} - \Delta a_\mu^g = (4.7 - 3.64) \times 10^{-9} = 1.06 \times 10^{-9}$ can be shared by contributions due to other sources. It has been shown in [15] that for radion mass greater than about 200 GeV, its contribution to Δa_μ would be less than 10^{-9} . For simplicity, we ignore this contribution by assuming radion mass to be greater than 200 GeV. Then in the degenerate masses of light neutrinos, since at least 4 bulk neutrinos should exist, the contribution from any single bulk neutrino should be $\Delta a_\mu^N \leq 2.65 \times 10^{-10}$. On the other hand, if the neutrino mass pattern is hierarchical then at least two bulk neutrinos should exist. By assuming that each bulk neutrino contributes by the same amount to $(g - 2)_\mu$, we can then put a bound $\Delta a_\mu^N \leq 5.3 \times 10^{-10}$. An upper bound on Δa_μ^N would

M_b	Degenerate neutrinos		Hierarchical neutrinos			
	Y_5	ν	Y_5^a	ν^a	Y_5^s	ν^s
10^{19} GeV	1.2	1.22	1.9	1.28	1.8	1.32
5×10^{18} GeV	0.6	1.2	0.9	1.26	0.9	1.31
10^{18} GeV	0.1	1.15	0.14	1.21	0.14	1.25

Table 2: Upper limits on Y_5 and ν for different values of M_b and in different patterns of neutrino masses. In the case of hierarchical neutrinos, the suffix a and s represent for atmospheric and solar neutrinos, respectively.

imply an upper limit on Y_5 and thereby an upper bound on the corresponding ν of the bulk neutrino. In Tab. 2 we show conservative bounds on these parameters for various values of bulk parameters M_b . In the case of $kr_c = 12.0$, two KK gravitons can exist to fit the $(g - 2)_\mu$. The above analysis done for $kr_c = 12.1$ can be repeated for $kr_c = 12.0$ and we can get bounds on the parameters Y_5 and ν accordingly. However, if we include the contribution of the order of 10 KK gravitons then kr_c should be close to 11.7. In this case, by comparing the curves for $kr_c = 11.5$ of Fig. 3, for M_b close to 10^{18} GeV, a single bulk neutrino can contribute an order of 10^{-9} to $(g - 2)_\mu$. So for this choice of M_b we can get upper limits on Y_5 and ν . However, if the number of KK gravitons is of the order of 100 then kr_c should be around 11.3. For this low value of kr_c , a single bulk neutrino contribution would be at most 6×10^{-10} even for $M_b = 10^{18}$ GeV. Hence the constraints on the bulk neutrino parameters would be less stringent in this case. Finally, to comment on the radion contribution to $(g - 2)_\mu$, the constraints on the bulk neutrinos which are described above may become even severe if the radion mass is less than about 200 GeV.

In the previous paragraph we have described constraints on the bulk neutrino parameters which are arising purely from the 2σ deviation of Δa_μ . However, these constraints can be even stringent by including other observable quantities such as $BR(\mu \rightarrow e\gamma)$. The current experiments have not found the decay $\mu \rightarrow e\gamma$ and put an upper limit on its branching ratio as $BR(\mu \rightarrow e\gamma) < 2.4 \times 10^{-12}$ at 90% CL [17]. In the context of the warped model [7], this decay channel has been studied in [18]. To satisfy the upper bound on $BR(\mu \rightarrow e\gamma)$ and to get an appreciable contribution from Δa_μ^N , we may have to fine-tune the Yukawa couplings. It is interesting to study these effects but this is out of the scope of the current work. Finally, we comment that in this work we have studied correlation between neutrino mass eigenstate and $(g - 2)_\mu$ for a single generation. We hope to extend this to include all the generations in a future work. Some related work in

the context of different models has been studied in [19].

5 Conclusions

The RS model, which is based on the warp geometry, is elegant to explain the hierarchy between the Planck and electroweak scales [2]. Subsequently it was shown that such model can offer possible explanation of the origin of small neutrino masses when the right handed component of the neutrino is allowed to propagate in the bulk [7]. The signals of such a model lies in the detection of KK modes of the bulk neutrino field. In this work, we have studied an indirect signal of these KK modes, *i.e.* the virtual effects of these KK modes to the anomalous magnetic moment of muon, $(g - 2)_\mu$. We have computed the contribution of bulk neutrino to $(g - 2)_\mu$ at one loop level in the 't Hooft-Feynman gauge. Comparing our result with the experimental limits, we have found that this contribution comes with the right sign so as to fit with experimental data. Moreover, in some of the parameter space of the model in [7], the 2σ discrepancy of $(g - 2)_\mu$ can be accommodated. We have also studied how the bulk neutrino contribution would be restricted by including the graviton contribution to $(g - 2)_\mu$.

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