Measuring the Dynamics of the Achievement Gap Between Public and Private School Students During Early Life in India

October 14, 2020

Abstract

It is well documented that private school students outperform their public school counterparts in India. However, researchers have only focused on the achievement gap in levels without considering the underlying dynamics of how students move through the distribution of achievement over time. We bridge this gap here by exploring the dynamics of the public-private school achievement gap in India by applying nonparametric measures of distributional mobility to panel data on math and Peabody Picture Vocabulary test scores from the Indian state of Andhra Pradesh. We find that public school students are at least as mobile as private school students during early childhood. During preadolescence, however, public school students, relative to private school students, are significantly less upwardly mobile while at the same time more downwardly mobile through the distribution of test scores. These mobility patterns, taken together with the level gap in test scores, suggests one would expect to see very little convergence in achievement between private and public school students during the middle and high school years of schooling.

JEL: I21, I24, O12, O15
Keywords: Directional Rank Mobility, Private Schools, Public Schools, Staying Probability, Test Scores, Transition Probability

1 Introduction

The academic achievement gap between students attending public and private schools in India has been widely studied in the recent development literature (see for e.g. Kingdon, 1996; Muralidharan and Kremer, 2008; Desai et. al. 2009; French and Kingdon, 2010; Chudgar and Quin, 2012; Muralidharan and Sundararaman 2013; Singh, 2014; Singh 2015; Azam et al., 2016; ASER, 2018). Almost all studies so far have found evidence of private school students outperforming public school students on standardized tests, although there is considerable variation in the magnitude of the estimated gaps across the studies with some studies documenting large gaps in achievement while others finding evidence of only modest gaps. Summing up the findings of this literature, Singh (2014, p. 33) notes, "...private schools [in India] are associated with student achievement that are as high or higher even after accounting for all pre-existing differences in socioeconomic background." Similar conclusions are also reached by Kingdon (2017. p. 28): "literature indicates that children's learning levels in private schools are no worse than, and in many studies better than, those in government schools, after controlling rigorously for the differing home backgrounds of the children in these two types of school."

Given this backdrop, this study aims to explore the dynamics of the academic achievement gap between public and private school students during early childhood and preadolescence. In particular, and using panel data on test scores in math and Peabody Picture Vocabulary Test (PPVT) for a cohort of school-enrolled children collected by the Young Lives study (YLS) in the Indian state of Andhra Pradesh, we employ two types of nonparametric measures of distributional mobility, transitional probabilities and distributional rank mobilities (Buchinsky and Hunt, 1999; Bhattacharya and Mazumder, 2011; Mazumder, 2011), to estimate the likelihood that public and private school students will transition in a directional sense (upward, downward, etc.) through the distribution of academic achievement between 5 and 12 years of age. By exploring the underlying differences in distributional mobility between private and public school students, this paper contributes to the literature by providing a novel, yet complementary, measure to help better understand the progression of the public-private achievement gap during the formative years of schooling for Indian children.[1]

Understanding the dynamics of the achievement gap between public and private school students is fundamentally important for researchers and policymakers for at least two reasons. First, solely evaluating the achievement gap between public and private school students in levels and not taking into account student mobility patterns is unlikely to be meaningful and can result in both policymakers and researchers, alike, drawing incomplete conclusions regarding the seriousness of the public-private achievement gap. To see this, suppose a 'modest' achievement gap exists, yet mobilities are equally high between the students attending public and private schools. Under these conditions, it is possible that such mobility through the distribution will result in test scores being more uniformly distributed over time compared to the distribution of test scores measured at some singular point in time. In contrast, if the academic achievement gap is 'small' between public and private school students, but mobility is close to zero, then the achievement gap is permanent. What begins as an achievement gap in school has the potential to influence gaps in other important outcomes such as in skills, wages, health, and incarceration. In short, understanding the underlying dynamics of the achievement gap is needed to asses whether the 'public–private' achievement gap is transitory in nature or more of a persistent phenomena during childhood.[2]

Second, the documented public–private test score gap is unlikely to be robust to various scale transformations. In effect, the magnitude of the gap and how it evolves can vary solely on the basis of how underlying item responses, and thus test scores, are scaled (Bond and Lang, 2013; Jacob and Rothstein, 2016).[3] However, and as noted by McDonough (2015),

[1]We note upfront that our study is essentially descriptive in nature and may be considered as a "first pass" analysis of the dynamics of public-private achievement gap in India. Such "first pass" analysis is routinely carried out to examine the racial gaps in income mobility or gaps in academic achievement mobility in context of the US (see for e.g. Bhattacharya and Mazumder, 2011; Chetty et al. 2014; Mazumder, 2014; McDonough, 2014). While systematic investigation into the mechanisms underlying the evolution patterns documented here is important, it is outside the scope of current research, and perhaps is the next step in this research line.

[2]Similar arguments have been made by Kopczuk et al. (2010) and Glewwe (2012) in the literature on economic inequality to highlight the importance of studying income mobility.

[3]Bond and Lang (2013), in context of the black-white achievement gap in the US, rigorously show that by

distributional mobility measures are robust to monotonic scale transformations of underlying test scores so long as the rank order of students within the distribution of achievement is unchanged. Given the stable nature of distributional mobility when rank order is preserved, estimated gaps in mobility are also unchanged under such transformations. As such, scaling issues associated with achievement gaps in levels are not present when looking at gaps in distributional mobility.

Our results are compelling. We find that during early childhood, private school students are not significantly more (less) mobile than public school students in both the upwards (downwards) direction in math as well PPVT. During preadolescence, however, we find clear evidence of private school students being at an advantage relative to the public school students in terms of upward and downward mobility. Specifically, during preadolescence, we find that compared to private school students, public school students are significantly less upwardly mobile and more downwardly mobile in both subjects. Our results are robust to inclusion of various controls (e.g., parental perceptions about child ability, wealth, caste, etc.) that might be determinants of parental decisions regarding the type of school (public or private) a child should be enrolled in while at the same time being correlated with the child's achievement.

Overall, our results present a rather alarming picture of the disparity in academic performance between public and private school students. Coupled with the existence of a level gap in test scores, the divergent mobility patters observed in the data suggests that the documented gap in test scores between public and private school students will continue to persist as students make their way through middle and high school. Our results emphasize the need for policymakers to think about policies that could be implemented before the onset of preadolescence to promote higher upward mobility, while at the same time lower

selectively choosing the scale, the initial black-white gap for reading could range from one-ninth of a standard deviation all the way up to roughly half a standard deviation. Jacob and Rothstein (2016) illustrates this issue by using a simple example and notes that elaborate examples could even produce reversals of the sign of the gap depending on the transformation and assessment used. Further, Jacob and Rothstein (2016, p. 89) note, "this problem worsens if one considers changes over time."

downward mobility, for public school students.[4]

The rest of the paper unfolds as follows. In the next section we discuss the background and prior literature. Section 3 presents the mobility measures. The data is discussed in section 4. Section 5 discusses the results while the last section concludes.

2 Background and Literature Review

The education systems in the India generally follow the 8+2+2+3 system, i.e., eight years of elementary education (five years of primary school education and three years of middle school education), two years of secondary education, two years of higher secondary education, and three years of university education. Broadly, schools in India can be classified into four types: government schools, local body schools, private aided schools (private schools that receive aid from the government), and private unaided schools (private schools that do not receive aid from the government). Government schools are financed and managed completely by the public sector. As such, these schools are collectively referred to as 'public' or 'government' schools.

The proportion of private schools making up total enrollment has increased significantly over the previous two decades for both urban and rural areas in India. According to Kingdon (2017), between 2010-2011 and 2015-2016, student enrollment in private schools, across 20 Indian states, increased by 17.5 million, while enrollment in government schools fell by 13 million. Currently private schools account for about 35% of enrollment in these 20 states.

Over the last few years, a significant body of literature has emerged that examine the learning levels of private school students relative to public school students in India. One of the earliest studies on this topic examined differences in achievement between private

[4]It is worth emphasizing that although our findings are based on data from Andhra Pradesh, they are likely to have relevance beyond Andhra Pradesh and even beyond the Indian context. As noted by Singh (2015), the share of students enrolling in low-fee private schools has increased in several developing countries and in many of these countries (in Latin America, Asia, and Africa) these students at low-fee private schools outperform their government school counterparts. As such, findings presented here may also be of importance for these other developing countries.

and public school students using survey data collected in the Indian state of Uttar Pradesh (Kingdon 1996). After addressing endogenous selection into school types, the author finds that private schools provide more effective instruction in mathematics and marginally better instruction in teaching language.

Similar findings have also been reported in Tooley and Dixon (2005), Muralidharan and Kremer (2008), Desai et al. (2009), Goyal (2009) and French and Kingdon (2010). For example, Muralidharan and Kremer (2008), using survey data from the rural areas of 20 Indian states, find evidence of a "private school effect" of considerable magnitude. French and Kingdon (2010), on the other hand, using ASER individual level survey data for age group 6-14 and employing village and year fixed effects, find that the private school effect in basic test measures is about 0.17 standard deviations.

Among the more recent studies that examine the differential in private and public school achievement, Muralidharan and Sundararaman (2013) provide some experimental evidence of private school students achieving higher scores in Hindi and English relative to public school students. However, in Telugu (native language of Andhra Pradesh) and math there was no discernible difference in performance. When averaging the results across course subjects, the authors find that students attending private schools scored 0.23 standard deviations higher overall relative to public school students.

Using panel data from Young Lives Study in the Indian state of Andhra Pradesh, the same survey that we have drawn our data from, and employing value-added models of achievement production, Singh (2015) finds that among 8 and 10 year olds in rural Andhra Pradesh there is a positive relationship between being enrolled in a private school and English but no relationship between mathematics and private school enrollment. The author further finds that secondary school children (15 year olds) in private school in Andhra Pradesh outperform their government counterparts in mathematics by 0.2 standard deviations. However, Singh (2015) finds no impact in Telugu language.

Azam et al. (2016) provide additional evidence of private-public school achievement gaps from two Indian states: Orissa and Rajasthan. The authors use detailed secondary level achievement data from the two states in 2005 as part of a broader study conducted by the World Bank. Using the propensity score matching estimator, the authors find some evidence of a private school premium in Rajasthan. Specifically, the authors find that both rural and urban private school students perform higher compared to their public school peers. However, the authors find no discernible difference when looking at students in Orissa.

Chudgar and Quin (2012), using the IHDS data and corresponding achievement outcomes similar to Desai et al. (2012), find mixed results. Specifically, using regression techniques and controlling for various observables the authors find that both urban and rural private students outperform their urban and rural public counterparts. However, the private school premium becomes statistically insignificant after using propensity score matching to balance the data on observables between public and private students.

In sum, and at least for some subjects, the existing studies mostly find evidence of a ‘private school premium’ in academic achievement. None of these studies, however, compare the rates of mobility of public school students relative to private school students over the distribution of achievement during early life. By providing the first exploration of the dynamics of the differential in achievement from early childhood to preadolescence, our study complements the literature to provide a more complete understanding of the evolution of the achievement gap between public and private school students in India.

3 Measures of Academic Achievement Mobility

To explore the differences in achievement dynamics between public and private school students in India from early childhood to preadolescence, we borrow several metrics commonly used to measure income mobility. We discuss each of these metrics in detail.

3.1 Probability Transition Matrices

To begin with, we construct probability transition matrices capturing the entirety of student transition dynamics over time. Specifically, let y_i^t denote the test score for student i, $i = 1, ..., N$, in time t, $t = t_0, t_1$, $t_0 \neq t_1$, and let $F_{t_0}(\cdot)$ and $F_{t_1}(\cdot)$ denote the cumulative distribution function (CDF) of test scores for students in two distinct time periods t_0 and t_1. Further, let $F_{t_0,t_1}(y^{t_0}, y^{t_1})$ denote the bivariate joint CDF, where $y^t \equiv [y_1^t \ \cdots \ y_N^t]$.

To summarize and provide intuition to the movement through the distribution of test scores captured by $F_{t_0,t_1}(y^{t_0}, y^{t_1})$, we construct a $K \times K$ transition matrix, Π_{t_0,t_1}, given by

$$\Pi_{t_0,t_1} = \begin{bmatrix} \pi_{11} & \cdots & \cdots & \pi_{1K} \\ \vdots & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ \pi_{K1} & \cdots & \cdots & \pi_{KK} \end{bmatrix}. \tag{1}$$

where elements of (1) are represented by

$$\pi_{kl}^{t_0,t_1} = \frac{\Pr(y^{t_0} \in k, \ y^{t_1} \in l)}{\Pr(y^{t_0} \in k)} \qquad k, \ l = 1, ..., K, \tag{2}$$

where k and l denotes partitions in period t_0 and t_1 respectively. Thus, $\pi_{kl}^{t_0,t_1}$ gives the fraction of children in partition k in period t_0 who are in partition l in period t_1. Being completely immobile in a distributional sense implies $\pi_{kl}^{t_0,t_1}$ equals unity if $k = l$ and zero otherwise.

To derive public-private differences in these transition probabilities, we refine (2) by conditioning $X = x$, where X denotes covariates of interest. By doing such (2) simply becomes

$$\pi_{kl}^{t_0,t_1} = \int \pi_{kl}^{t_0,t_1}(x) dF(x \mid y^{t_0} \in k), \ k, \ l = 1, ..., K, \tag{3}$$

where the covariate of interest is type of school (i.e., whether public or private) that a student attends.

3.1.1 Staying Probabilities, Upward Transition Probabilities, and Downward Transition Probabilities

The elements of a typical transition matrix as given by (1) can be categorized into staying probabilities (SP), upward transition probabilities (UTP) and downward transition probabilities (DTP). For a particular k, the SP is given by $\pi_{kl}^{t_0,t_1}$, $k = l$, the set of UTP is given by $\{\pi_{kl}^{t_0,t_1}\}_{l>k}$ and the set of DTP is given by $\{\pi_{kl}^{t_0,t_1}\}_{l<k}$. For example, for $k = 3$, the SP is given by $\pi_{33}^{t_0,t_1}$, the set of UTP is given by $\{\pi_{34}^{t_0,t_1}, \pi_{35}^{t_0,t_1}, ..., \pi_{3K}^{t_0,t_1}\}$ and the set of DTP is given by $\{\pi_{31}^{t_0,t_1}, \pi_{32}^{t_0,t_1}\}$.

3.2 Directional Rank Mobilities

Useful and informative as they are, transition probabilities neither provide any information on movements *within* partitions of the discretized CDF. Accordingly, we include additional measures of mobility that track student movements, if any, *within* partitions. Bhattacharya and Mazumder (2011) and Mazumder (2011) introduced these measures in the context of intergenerational income mobility between fathers and sons and labeled them *upward* rank mobility (URM) and *downward* rank mobility (DRM), respectively. These directional rank mobility measures capture the likelihood that a student's rank in the overall test score distribution when his/her age is t_1 surpasses, or falls below, his/her rank in the test score distribution at age t_0 by a constant amount.

Again let Y^{t_0} and Y^{t_1} denote a student's test score with given CDF's $F_{t_0}(\cdot)$ and $F_{t_1}(\cdot)$. URM over time, then, can be expressed by

$$\theta_{kl,\delta}^{t_0,t_1} = \frac{\Pr(y^{t_0} \in k,\ F_{t_1}(y^{t_1}) - F_{t_0}(y^{t_0}) > \delta)}{\Pr(y^{t_0} \in k)} \tag{4}$$

where δ is some chosen constant representing the threshold defining upward mobility. In words, $\theta_{kl}^{t_0,t_1}(\delta)$ captures the probability of a student in the terminal period exceeding their initial percentile rank by at least δ, conditional on residing in the k^{th} percentile range of the test score distribution in the initial period.

The same intuition holds for constructing DRM measures by reversing the logic in (4) yielding

$$\psi_{kl,\delta}^{t_0,t_1} = \frac{\Pr(y^{t_0} \in k,\ F_{t_0}(y^{t_0}) - F_{t_1}(y^{t_1}) > \delta)}{\Pr(y^{t_0} \in k)}. \tag{5}$$

Similar to (2), we can condition (4) and (5) on school-type and obtain the conditional URM and DRM. To obtain the mobility measures in practice, we first construct the empirical CDF at each time period by pooling all students (who are roughly the same age) from public and private schools who have valid test scores. We then find each child's ranking within the overall CDF and track how they move through the test score distribution from one time period to the next. For this paper we focus on the dynamics of public and private school students. For inference, we bootstrap the standard errors.

3.3 Directional Rank Mobilities versus Transitional Probabilities

The difference between directional rank mobilities and directional transition probabilities is directional mobilities capture movements through the distribution that are possibly missed when looking at staying/transition probabilities. The reason for this is that in order to determine mobility using standard transition probabilities one must impose arbitrary lower and upper bounds, call them a and b, that are the same for all students. Thus, in order to be counted as having 'stayed' in a particular partition, a student must remain in the portion of the distribution bounded by the non-unique values of a and b irrespective of their initial starting point between those values. Relative to students starting out in the distribution close to these bounding values, a higher proportion of students amassed at the midpoint of that interval between a and b will naturally have a higher likelihood of lacking mobility since it is those students who have to make larger absolute movements, in either direction, to be considered mobile. This becomes an especially salient point when focusing on the extremes of the test score distribution, which are of considerable interest.

Given the nature of the public-private achievement gap, it is reasonable to think that within any range of the distribution, public school students will be closer to a and private school students will be closer to b, where $a < b$ (i.e., public school students tend to have lower test scores than private school students). Such ordering defined by arbitrary thresholds has the potential to distort actual student mobility patterns since some students are prone to either not progressing or regressing due solely to their initial position in the achievement distribution. Directional rank mobilities attempt to circumvent this problem by requiring a student to surpass, or fall behind, their initial rank in the test score distribution by a fixed amount δ. While the chosen value of δ can be viewed as equally arbitrary, one can iterate through values of δ to get a truer sense of mobility. For example, $\delta = 0$ considers *any* movement through the distribution as being mobile. As δ becomes larger a student will need to increase or decrease their rank in the overall distribution more, in absolute value, to be considered mobile. We iterate through values of $\delta = \{0.00, 0.05, 0.10\}$.

4 Data

4.1 The Young Lives Survey

The data used here comes from the Young Lives Survey (YLS). The YLS is a comprehensive survey aimed at understanding drivers of childhood poverty in Ethiopia, India, Peru, and Vietnam. Specifically in this paper, we focus on school-aged children from 2002 to 2014 in Andhra Pradesh.[5] Andhra Pradesh is one of the largest states in India and as of 2011 had a population of over 84 million inhabitants. The state is divided into the three regions of Coastal Andhra, Rayalaseema, and Telangana. Each state is then further divided into districts and sub-districts (mandals) with these sub-districts serving as the primary sampling

[5]For more details see Galab et al. (2003) and/or visit www.younglives.org.uk.

units for data collection.[6] As well, the children of Andhra Pradesh are divided into cohorts of younger and older children. For our study here, we focus on the children who belong to the younger cohort of Young Lives.[7]

The data was collected over five waves with the first round of data collection taking place in 2002, the second in late 2006/early 2007, the third in late 2009/early 2010, the fourth in late 2013/early 2014, and the final round in late 2016/early 2017.[8] As noted above, the YLS is comprehensive in nature and collects extensive information on 2,011 children who were aged 6 to 21 months (the Younger Cohort born between January 2001 and June 2002) and 1,008 children aged 7.5 to 8.5 years (the Older Cohort born between January 1994 and June 1995) for the first survey round in 2002. Information for some of the children could not be obtained after the first round due to various reasons such as death of the child, unwillingness regarding the participation in the survey, etc. In the fourth round, 1915 out of 2011 children in younger cohort and 952 out of 1008 children in older cohort were interviewed.[9]

4.1.1 Test Score Data

Data on children's test scores were collected in each round of the survey. As noted by Singh (2015), each administered test varied in an attempt to capture various dimensions of learning and to mimic the formal school curriculum in Andhra Pradesh. Age appropriateness of the tests were considered in each round to ensure that the material being tested was appropriate given the children's age and stage of development. For children belonging to the younger

[6] In June 2014, Andhra Pradesh was bifurcated into two states named as Andhra Pradesh and Telangana. Since then the YLS continued in both the states.

[7] For more details on survey methodology see `http://doc.ukdataservice.ac.uk/doc/5307/mrdoc/pdf/5307sampling_india.pdf`; last accessed 21 August 2019.

[8] In addition to this, school surveys for a randomly selected sub-sample taken from the younger cohort were conducted in 2010 and late 2016/early 2017. Note, data for the latest round (2016-17) was not available in the public domain when we carried out this study.

[9] Notable attrition occurred between the first and the second round. See Outes-Leon and Dercon (2008) for more on attrition in the YLS.

cohort, YLS conducted math[10] and PPVT[11] in all rounds of the survey except the first round, since the children were too young (6 to 18 months old) to respond to any academic test. Additionally, a writing (English) test was introduced in the third round whereas a language test was conducted in the fourth round.

Note, the tests used to capture learning by Young Lives are noted as being more comprehensive along the dimension of learning relative to historical surveys (see ASER data and India Human Development Survey 2005 data). Specifically, and as noted by Singh (2015), these prior measures only capture very basic skills along narrow domain of childhood learning and thus are not well-suited to broadly measure student achievement.

4.2 Analytic Sample

Using the YLS test data, we estimate transition probabilities and directional rank mobilities for public and private school student for the two periods from 2006/07 to 2009/10 (when the children were between 5 and 8 years of age) and from 2009/10 to 2013/14 (when the children were between 8 and 11 years of age). In other words, we examine the evolution of student achievement from early childhood to the end of preadolescence. We consider test scores of children in math and PPVT since these are available for all the three rounds. For both math and PPVT, we convert the raw test scores to item response theory (IRT) test scores.[12]

4.2.1 Sample Construction

In 2006/07, 1950 children were surveyed. Of those, 874 report having attended school with 789 of those children having information on school type. Out of these 789 children, 595 were in public school and 194 were in private school. In 2009/10, 1930 children were surveyed. Of

[10] In the second round, Cognitive Development Assessment (CDA) is conducted where quantity based questions are asked from the children. A sample question of CDA is "Look at the plates of cupcakes. Point to the plate that has a few cupcakes… Point to the plate that has a few cupcakes". We consider this test also as a form of Mathematics test as knowledge of numbers is required to answer to the questions in CDA.

[11] The PPVT was initiated in 1959 to analyze the verbal intelligence of a child. It also helps in evaluating the scholastic aptitude for the children in school going age.

[12] Note, survey weights are unavailable in the public-use YLS data.

these 1930 students, 1051 were in public school and 863 were in private school, 8 children were not going to school regularly or not going to school at all, while the school type information is missing for the remaining 8 children.[13] In 2013/14, 1915 children were surveyed. Out of 1915 children, information on the type of school they attend is available for 1858 children. Out of these 1858 children, 1102 were in public school and 756 were in private school.[14] Table 1 provides summary statistics of the variables used in the analysis for 745 children in 2006/07, 1767 children in 2009/10, and 1751 children in 2013/14 with valid information on the relevant variables. Kernel density distributions of test scores in math and PPVT for the three different rounds of survey are presented in appendix Figure A1.

For our analysis, we focus only on the sample of students who did not drop out from school or who did not switch from a public to a private school or from a private to a public school between two successive rounds of the survey.[15] Out of the 595 school students who were in public schools in 2006/07, 124 students moved to private schools and 4 students dropped out between 2006/07 and 2009/10. Thus, the number of public school students who remained in the public school between 2006/07 and 2009/10 is 467. Of the 194 students who were in private schools in 2006/07, 25 students moved to public schools and 3 students

[13] There are 1132 students who were not in school in second round but were in school by the next round. Out of these 1132 students, 573 were in private and 559 in public school in third round.

[14] Note, in our analysis, while public schools refer to pure public schools, private school students refer to children going to pure private schools, community schools (run by NGOs, charitable organizations, religious organizations, etc.) and public aided private schools. However, in all the rounds, among the students classified as private school students, the majority are indeed students of pure private schools. Specifically, in the 2006/07 wave, 87% of children classified as private school students, attend pure private schools, in 2009/10 wave, 98% of students classified as private school students attend pure private schools, and in 2013/14 wave, 82% of students classified as private school students attend pure private schools.

[15] If we were to simply include all students, including those who switched from public to private schools, and vice versa, then our mobility estimates may be in part due to measurement error. Having said this we realize that if switching is based on test performance, then we may additionally have a selection problem. To get some sense of whether this is a problem, we compare test scores between switchers and non-switchers in previous rounds of the data. In doing so we do not find statistically significant differences in test scores (2006/07) between switchers (children who switched their school from 2006/07 to 2009/10) and non-switchers (children who stayed in the same type of school from 2006/07 to 2009/10). Similarly, we do not find statistically significant difference in test scores (2009/10) between switchers (children who switched their school from 2009/10 to 2013/14) and non-switchers (children who stayed in the same type of school from 2009/10 to 2013/14). Additionally, the population distributions, not just the averages, are remarkably similar between these switcher and non-switcher groups. While selection into switching on the basis of performance is understandably a concern, we do not think that it is a major issue here.

dropped out between 2006/07 and 2009/10. Thus, the number of private school students who remained in the private school between 2006/07 and 2009/10 is 166. Thus, our analytic sample for the analysis of the mobility dynamics during early childhood includes 467 public school students and 166 private school student.

Out of the 1051 children who were in public schools in 2009/10, 91 students moved to private schools and 54 students dropped out of school between 2009/10 and 2013/14. Thus, the number of public school students who remained in public schools between 2009/10 and 2013/14 is 906. Of the 863 students who were in private schools in 2009/10, 186 students moved to public schools and 14 students dropped out between 2009/10 and 2013/14. Thus, the number of private school students who remained in private schools between 2006/07 and 2009/10 is 663. Our analytic sample for the analysis of mobility dynamics during children's preadolescence consists of 906 public school students and 663 private school students. Lastly, approximately 85% of students in the early childhood sample are also in the preadolescent sample.

5 Results

We now turn to the analysis of the achievement gap between public and private school students using various mobility metrics introduced in Section 3. We begin with the analysis based on the estimated transition matrices. We then discuss the results obtained using directional rank mobility measures.

5.1 Transition Matrices

5.1.1 Staying Probabilities

The estimates of SP for public and private school students in math and PPVT are plotted in Figure 1 with our primary interest being how students transition conditional on initially residing in the extremes of the test score distributions. In both the figures, panel A shows the staying probabilities estimated using the 2006/07 and 2009/10 waves of the data, whereas panel B shows the transition probabilities estimated using the 2009/10 and 2013/14 waves of the data. Thus, panel A depicts staying probabilities during early childhood, while panel B shows the estimates of staying probabilities during preadolescence. The X-axis varies the sample used based on the quartile range of IRT test scores, while the Y-axis shows the staying probability. The green lines show the estimates for the public school students, while the maroon lines show the estimates for private school students. The black line plots the difference in the probabilities by school type, along with confidence bands.

We begin by discussing the results for math. Compared to private school students, public school students have a 14 percentage point higher chance of remaining in the lowest quartile range of test scores during early childhood and a 12 percentage point higher probability of staying in the lowest quartile range of the test score distribution during preadolescence. However, the difference is statistically significant only during preadolescence. As we move to the right and gradually increase the quartile range of test scores, the difference in staying probabilities between public and private school students falls and becomes negative in the top two quartile ranges. At the highest quartile range of the test score distribution, we find that, compared to public school students, private school students are 3 percentage points more likely to remain in this quartile range during early childhood and 34 percentage points more likely to remain during preadolescence. Again, the difference is statistically significant only during preadolescence.

For PPVT, we observe a similar pattern. Compared to private school students, public school students have an 8 percentage point higher likelihood of staying in the lowest quartile range of the test score distribution during early childhood and a 25 percentage point higher probability of remaining in the lowest quartile range of the test score distribution during preadolescence. The difference is statistically significant during preadolescence but not during early childhood. As we move to the right, the difference in staying probabilities between public and private school students falls and becomes negative in the third and fourth quartile ranges. We find that private school students, compared to the public school students, are 27 percentage points more likely to remain in the top of the test score distribution during early childhood and 18 percentage points more likely to remain in the top of test score distribution during preadolescence. However, unlike in math, these estimated differences are highly statistically significant.

5.1.2 Upward Transition Probabilities

Figure 2 plots estimates of upward transition probabilities for public and private school students in math and PPVT . Similar to staying probabilities, we are primarily interested how students transition with respect to initially residing in the extremes of the test score distributions. As such, we plot the estimates of the upward transition probabilities (and the differences) for those public and private school students whose test scores were at or below the first quartile in the initial period. Estimates and a complete set of figures of the upward transition dynamics for students whose test scores initially fall within the second or third quartile ranges can be found in Tables A1 and A2 and in Figures A2 and A3 in the Appendix.

In math, we observe that UTP for public school students is much smaller than the UTP for private school students across all the quartile ranges during early childhood. For example, the likelihood of public school students moving upwards from the first quartile range to the third quartile range is 15 percentage points; the corresponding figure for private school students is 19 percentage points. However, none of these differences in UTP are statistically significant. During preadolescence, two out of the three differences in UTP are statistically insignificant. The only statistically significant difference in UTP between public and private school students is observed when moving from the first to the second quartile range with the UTP of private school students being higher than that of public school students by 9 percentage points.

Turning attention to the comparison of UTP between public and private school students in PPVT during early childhood, we find no statistical difference in the rates of upward mobility between public and private school students initially residing in the bottom quartile range of the test score distribution when tracking movements to the second or third quartiles over time. However, we do observe statistically significant differences in UTP when considering movement from the lowest to the highest quartile range with private school students having a 22 percentage point higher UTP compared to public school students. During preadolescence, we find that public school students have significantly lower UTP relative to private school students when considering movement between first and third or fourth quartile ranges. However, when considering movement from the first to the second quartile range, the public school students seem to have a slight advantage over private school students, although these difference in UTP are not statistically significant.

5.1.3 Downward Transition Probabilities

Figure 3 plots estimates of downward transition probabilities for public and private school students. We plot the estimates of the downward transition probabilities (and the differences) for only those public and private school students whose test scores resided in the fourth quartile range of the test score distribution in the initial period. As earlier with UTPs, a complete set of DTP estimates and figures having conditioned on initial test scores residing in the second or third quartile ranges of the test score distributions can be found in Tables A1 and A2 and Figures A4 and A5 in the Appendix.

Looking at the DTP estimates for math, we notice that for both public and private school students, the DTP estimates decline as we move towards the right during early childhood with the DTP for public school students always being at least as high as the DTP for private school students. However, the difference in the DTPs between public and private school students are never statistically significant. During preadolescence, for both public and private school students, the DTPs declines as we move to the right during early childhood and the DTP for public school students are always higher than the DTP for private school students. However, these difference in the DTPs between public and private school students are highly statistically significant. For example, public school students, compared to private school students, are 14, 15, and 5 percentage points more likely to transition out of the top quartile range to the third, second, and first quartile ranges, respectively.

For PPVT, our results are similar to the results for math during early childhood. Specifically, we do not find any evidence of a statistically significant difference in DTP between public and private school students who reside in the top quartile range of the test score distribution. For preadolescence, however, our results are different from what was obtained for math. In PPVT, public school students do not seem to always have a greater chance of sliding down the test score distribution once at the top. For example, compared to private school students, public school students have a 9 percentage point greater probability of moving from the fourth to second quartile range while at the same time having roughly an equal probability of transitioning down to the very bottom quartile range of the test score distribution.

5.2 Directional Rank Mobilities

5.2.1 Upward Rank Mobilities

Figures 4 plots the URM estimates when $\delta = 0$ and Figure 5 plots the URM estimates when $\delta = 0.10$. These estimates, along with that of DRM, are also available in a tabular form in the Appendix (Tables A3 and A4). We begin by discussing the URM estimates for math. We find that during early childhood, when $\delta = 0$, the URM estimates of public school students and private school students are very similar with none of the differences exhibiting statistical significance. When we set $\delta = 0.10$, the URM estimates of both public and private school students fall with the estimated differences between the two groups being even smaller relative to the case when $\delta = 0$ (and still statistically insignificant as before). During preadolescence, however, our results are dramatically different. When $\delta = 0$, we observe that the URM estimates for private school students always exceed the URM estimates for public school students. The differences are large, ranging between 10 to 29 percentage points, as well as being statistically significant. When $\delta = 0.10$, the URM estimates for both public and private school students, as well as the differences between them, fall slightly. However, three out of the four differences continue to remain statistically significant.

We next turn our attention to the estimates of URM for PPVT. The results are very similar to those for math. During early childhood, as in case of math, we do not find evidence of any differences in the URM estimates between private and public school students in any of the quartile ranges of the test score distributions. This holds true for both $\delta = 0$ and $\delta = 0.10$. However, during preadolescence, we find clear evidence of private school students being significantly more upwardly mobile than public school students across the entire test score distribution when $\delta = 0$ and in three out of four quartile ranges when $\delta = 0.10$.

5.2.2 Downward Rank Mobilities

Next we turn to the estimates of DRM. Figure 6 plots the DRM estimates when $\delta = 0$ and Figure 7 plots the DRM estimates when $\delta = 0.10$. For math, and during early childhood, the DRM estimates of public school students and private school students are not significantly differently in any of the quartile ranges. When $\delta = 0.10$, the DRM estimates for public school students, as well as private school students, falls relative to the case when $\delta = 0$, and the differences between the two groups is attenuated across all quartile ranges. However, during preadolescence when $\delta = 0$, the DRM estimates for public school students exceed the DRM estimates for private school students. The differences are remarkably large as well as statistically significant. When we set $\delta = 0.10$, the DRM estimates for both the public and private school students falls slightly; the differences in DRM estimates between public and private school students continue to remain large and statistically significant across all quartile ranges.

For PPVT, as with math test scores, we do not find evidence of any difference in the DRM estimates between private and public school students in any of the quartile ranges of the test score distributions during early childhood. This holds true for $\delta = 0$ as well as $\delta = 0.10$. During preadolescence, public school students, compared to private school students, seem to have significantly higher chances of moving downwards across all the quartiles of the test score distribution when $\delta = 0$. When $\delta = 0.10$, the magnitude of the differences in the DRM estimates remain similar compared to the case when $\delta = 0$. The differences at the very top and very bottom of the test score distribution, however, fail to exhibit statistical significance.

5.3 Implications of the Public-Private School Achievement Gap

As discussed above, the mobility patterns of public and private school students appear to be similar during early childhood but divergent in nature during preadolescence. Indeed, during preadolescence, we observe a pattern of public school students descending down through the distribution of achievement while at the same time struggling to climb up through the distribution compared to private school students. This naturally raises the question, what are the implications of such mobility patterns?

To examine this question, and using the complete matrices characterizing the transition dynamics for both public and private school students, we derive the Markov chain steady-state distributions given these underlying mobility dynamics. We do this separately for the early childhood phase and preadolescence phase. This allows us to examine the public-private gap in academic achievement assuming that the estimated transition probabilities are a permanent feature of the educational system in Andhra Pradesh. The Markov-chain steady-state distributions, by subject and school type, are displayed in Figure 8.

The results show that in steady state, in math, if the estimated transition probabilities for early childhood persisted over the long run, there would be some, but not extreme, differences in the proportion of public and private school students settling across the test score distribution. For example, 27% of public school students would settle in the lowest quartile range of the test score distribution while approximately 20% the private school students would settle in the same quartile range over the long run. As well, roughly 23% of public school students would settle in the highest quartile range of the test score distribution compared to 28% of the private school students. The results, however, are dramatically different when assuming the estimated transition probabilities for preadolescence persist over the long run. In this case, we find that approximately 35% of public school students settle at the bottom of the test score distribution compared to only 11% of private school students. On the other hand, 47% of the private school students settle in the very top of the test score distribution compared to only 13% of public school students.

For PPVT, the difference in the proportion of public and private school students settling across the different quartiles of the test score distribution are relatively more uniform during early childhood as compared to preadolescence. For example, during early childhood, approximately 28% of public school students and 18% of private school students settle in the lowest quartile range of the distribution. In the highest quartile range, the corresponding estimates are 21% and 38% for public and private school students, respectively. During preadolescence, around 32% of the public school students settle in the bottom compared to 16% of private school students. In the top quartile range, the proportion of public and private school students settling in this region are 16% and 38%, respectively.

These results suggest that the mobility patterns prevailing during preadolescence are likely to lead to large and significant gaps in achievement between public and private school students over the long run. The mobility patterns prevailing during early childhood, on the other hand, might not give rise to significant gaps in academic achievement between public and private school students.

5.4 Conditional Mobility Gap

Our results suggest that private (public) school students are more upwardly (downwardly) mobile than public (private) school students with this being particularly true during preadolescence. However, this might not be a fair comparison because of the self-selection associated with children who attend private schools. As noted by Wadhwa (2018, p. 18), “it is well known that children who go to private schools come from relatively affluent backgrounds and tend to have more educated parents. This affords them certain advantages that aid learning. These advantages are not available to children who are from less advantaged families and are more likely to attend government schools." In the preceding analysis, we have not controlled for any factors that could potentially determine children's selection into schools. If we control for these factors that affect learning, the mobility gap in math or PPVT levels during preadolescence between children attending different types of schools may disappear. In order assess whether this is the case (in other words, whether selection on observables into schools are driving our results), we implement a linear probability model (LPM) and estimate the mobility gap between public and private school students conditional on various covariates that could potentially influence parental decisions to enroll children in private or public schools. Note, conditioning mobility measures on multiple covariates is difficult to do nonparametrically given the small sample sizes that arise as more and more right-hand side variables are introduced.

Specifically, to examine the conditional URM, we estimate

$$y_{it} = \alpha + \gamma S_i + \mathbf{x}_{it}\beta + \varepsilon_{it} \tag{6}$$

where y_{it} is equal to 1 if $F_t(Y_t) - F_{t-1}(Y_{t-1}) > \delta$, 0 otherwise; S_i is a school dummy equal to 1 if school type is public, 0 otherwise; $\mathbf{x}_{it}$ is a vector of covariates, ε_{it} is an i.i.d. error term and $t = 1, 2$. As such, the estimate for γ is equal to the public-private gap in URM controlling for $\mathbf{x}_{it}$. To assess the gap in the DRM conditional on various covariates, we simply construct y_{it} equal to 1 if $F_{t-1}(Y_{t-1}) - F_t(Y_t) > \delta$ and 0 otherwise. We estimate (6) separately for both math and PPVT.

The specific covariates that are used in this exercise include the child's gender, family size, caste affiliation, household wealth index, a dummy indicating whether the child lives in a urban or rural area, and a variable capturing parental perceptions about a child's ability (specifically, the parents or caregivers are asked the number of years of schooling they think that their child would complete). All these covariates could potentially be affecting a child's achievement as well as be determining his/her parents' decision to enroll him/her in a public or a private school (e.g., a parent who has high perceptions about his/her child's ability, might enroll the child in a private school instead of public school; the parent might also be providing various resources to the child which might be positively affecting the child's achievement). Given our ability to control for a host of such covariates (thanks to the rich nature of YLS data), the threat of omitted variable bias or selection driving our results is significantly reduced. However, the results presented below should still be interpreted with caution since the underlying associations are not necessarily causal in nature. All estimates for the public-private gap in directional mobility measures, as well as the point estimates for the various observables, can be found in Tables 2 and 3.[16]

We find that during preadolescence, conditional on the covariates, public school students are significantly less mobile in the upward direction and significantly more mobile in the downward direction relative to private school students in both math and PPVT. Specifically, when $\delta = 0$ public school students have approximately a 17 percentage point less likelihood of moving upwards and 17 percentage point less likelihood of moving downwards in the distribution of math test scores. When $\delta = 0.10$, these gaps reduce slightly: public school students now are approximately 15 percentage points less likely to move upwards and 12 percentage points more likely to move downwards through the distribution of test scores compared to private school students. The results are similar for PPVT. When $\delta = 0$, public school students, compared to private school students, have approximately a 7 percentage point less chance of moving upwards and a 7 percentage point more likelihood of moving downwards through the test score distribution. When $\delta = 0.10$, and again compared to private school students, public school students are roughly 9 percentage point less likelihood of moving upwards through the test score distribution and approximately 2 percentage points

[16]We also include the coefficient estimates on the public school indicator without any controls to provide a sense of how these estimates change as additional covariates are added to the model.

more likely to move downward. With the exception of moving downward through the distribution of PPVT test scores, these results for preadolescence are statistically significant at conventional levels.

For early childhood, our findings indicate no benefit of private school students over public school students in terms upward or downward mobility. In fact, for both math and PPVT, private school students, compared to public school students, appear to be less upwardly mobile and more downwardly mobile. For example, when $\delta = 0$, in math, private school students are approximately 20 percentage points less likely to move in the upwards direction and 22 percentage points more likely to move in the downwards direction compared to public school students. In PPVT, private school students, compared to public school students, are approximately 11 percentage points less likely to move in the upwards direction and 11 percentage points more likely to move in the downwards direction. The results are similar when $\delta = 0.10$ (although now public school students and private school students do not differ in terms of downward mobility in PPVT). These results are in sharp contrast to the results obtained for the preadolescence phase. As well, but with the exception of downward mobility estimates for PPVT when $\delta = 0.10$, the results are statistically significant at conventional levels.

Coupled with the previous findings, these results suggest that during preadolescence private school students, relative to public school students, seem to be significantly more upwardly mobile while at the same time significantly less downwardly mobile. During early childhood, however, we do not find any evidence of private school students having higher likelihoods of moving upwards and/or lower chances of moving downwards compared to the public school students. In fact, our analysis of the conditional mobility gaps indicate that during early childhood, if anything, the public school students are more upwardly mobile and at the same time downwardly mobile through the distribution of test scores compared to private school students.

6 Conclusion

The achievement gap in academic performance between public and private school students in India has been widely studied. Almost all existing studies find evidence of private school students outperforming public school students on standardized tests. However, researchers have only focused on the achievement gap in levels without considering the underlying dynamics of how students move through the distribution of achievement over time. However, ignoring how students evolve through the test score distribution over time may skew the perceived severity, or lack thereof, of the existing gap in performance between these two groups of students.

In this study we explore the dynamics of the public-private school achievement gap in India by applying nonparametric measures of distributional mobility to panel data on math and Peabody Picture Vocabulary test scores from the Indian state of Andhra Pradesh. We find that during early childhood, public school students are at least as mobile as private school students in both the upward and downward direction. However, during preadolescence, relative to private school students, public school students are both significantly less upwardly mobile and more downwardly mobile through the distribution of test scores. Given the steady-state distributions associated with derived transition matrices during preadolescence, we show that if the observed differences in mobility were allowed to persist indefinitely, then the gap in academic achievement between public and private school students is likely to remain a permanent problem in the educational system in India.

Note, our findings are not meant suggest that convergence in academic achievement between public and private school students is impossible. Instead, our results emphasize the need for policymakers to think creatively about interventions that promote higher upward mobility, while at the same time lower downward mobility, for public school students, while preserving the quality of education for all students. Finally, it is worth emphasizing that focusing solely on the gap in distributional mobility without considering the test score gap in levels is equally misleading. Rather, taking into account both the level differences in performance between public and private school students, as well as differences in mobility measures, better equips policymakers with the insights needed to design effective education-based interventions relative to any single measure by itself.

7 Compliance with Ethical Standards

The authors have no potential conflicts of interest to report. However, one of the authors acknowledges funding under a seed grant from his/her institution. Further, the current research does not involve human participants and/or animals and thus informed consent does not apply.

References

[1] Azam, M., Kingdon, G., and Wu, K. B. (2016). “Impact of private secondary schooling on cognitive skills: evidence from India,” *Education Economics*, 24, 465-480.

[2] Bhattacharya, D., and Mazumder, B. (2011). “A nonparametric analysis of black-white differences in intergenerational income mobility in the United States,” *Quantitative Economics*, 2, 335–379.

[3] Bond,T.N., and Lang, K.(2013). “The evolution of the black-white test score gap in grades K-3: The fragility of results,” *Review of Economics and Statistics*, 95, 1468–1479.

[4] Buchinsky, M., and Hunt, J. (1999). “Wage mobility in the United States,” *Review of Economics and Statistics*, 81, 351–368.

[5] Chetty, R., Hendren, N., Kline, P., Saez, E. and Turner, N. (2014). “Where is the land of Opportunity? The Geography of Intergenerational Mobility in the United States,” *Quarterly Journal of Economics*, 129, 1553–1623.

[6] Chudgar, A., and Quin, E. (2012). “Relationship Between Private Schooling and Achievement: Results from Rural and Urban India.” *Economics of Education Review*, 31, 376-390.

[7] Desai, S., Dubey, A., Vanneman, R., and Banerji, R. (2009). “Private schooling in india: A new educational landscape,” In *India Policy Forum,* 5, 1-58.

[8] French, R. and Kingdon, G. (2010). “The relative effectiveness of private and government schools in Rural India: Evidence from ASER data,” Department of Quantitative Social Science Working paper, 10-03.

[9] Galab, S., Reddy, M. G., Antony, P., McCoy, A., Ravi, C., Raju, D. S., Mayuri, K., & Reddy, P. P. (2003). "Young lives preliminary country report: Andhra Pradesh, India," Young Lives Project, London, September.

[10] Glewwe, P. (2012). "How Much of Observed Economic Mobility is Measurement Error? IV Methods to Reduce Measurement Error Bias, with an Application to Vietnam," *World Bank Economic Review*, 26, 236-264.

[11] Goyal, S. (2009). "Inside the house of learning: the relative performance of public and private schools in Orissa," *Education Economics*, 17(3), 315-327.

[12] Jacob, B., and Rothstein, J. (2016). "The Measurement of Student Ability in Modern Assessment Systems," *Journal of Economic Perspectives*, 30, 85–108.

[13] Kingdon, G. G. (1996). "The Quality and Efficiency of Public and Private education: A case study of Urban India," *Oxford Bulletin of Economics and Statistics*, 58 (1), 57–82.

[14] Kingdon, G.G. (2017). "The Private Schooling Phenomenon in India: A Review," IZA DP No. 10612.

[15] Kopczuk, W., Saez, E., and Song, J. (2010), "Earnings Inequality and Mobility in the United States: Evidence from Social Security Data Since 1937," *Quarterly Journal of Economics*, 125, 91-128.

[16] Mazumder, B. (2011). "Black-white differences in inter generational economic mobility in the U.S.," FRB of Chicago Working Paper, No.2011-10.

[17] McDonough, I.K. (2015). "Dynamics of the black–white gap in academic achievement," *Economics of Education Review*, 47, 17-33.

[18] Muralidharan, K., and Kremer, M. (2008). "Public and Private Schools in Rural India." in Chakrabarti, R. and Petersen, P. (eds.), *School choice international: Exploring public–private partnerships*, Boston, MA: MIT Press.

[19] Muralidharan, K. and Sundararaman, V. (2013). "The Aggregate Effect of School Choice: Evidence from a Two-Stage Experiment in India." *Quarterly Journal of Economics*, 130, 1011-66.

[20] Outes-Leon, I., and Dercon, S. (2008). "Survey attrition and attrition bias in Young Lives," Young Lives Technical Note 5. Young Lives, University of Oxford.

[21] Pratham (2005). *Annual Status of Education Report*, New Delhi: Pratham Documentation Center.

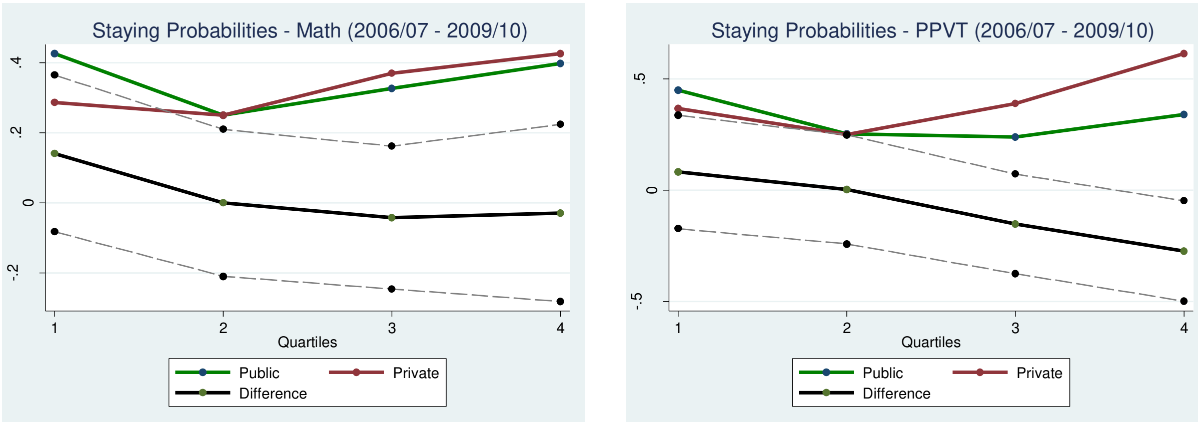
[22] Singh, A. (2014). "Test score gaps between private and government sector students at school entry age in India," *Oxford Review of Education*, 40, 30-49.

[23] Singh, A. (2015). "Private school effects in urban and rural India: Panel estimates at primary and secondary school ages", *Journal of Development Economics*, 113 C, 16-32.

[24] Tooley, J., and Dixon, P. (2005). "Private School is Good for the Poor: A Study of Private Schools Serving the Poor in Low-Income Countries." CATO Institute, Washington, DC.

[25] Wadhwa, W. (2018). "Equity in learning?," Annual Status of Education Report (Rural) 2018, 17–19, Pratham, New Delhi, India.

Panel A: 2006/07 - 2009/10



Panel B: 2009/10 - 2013/14

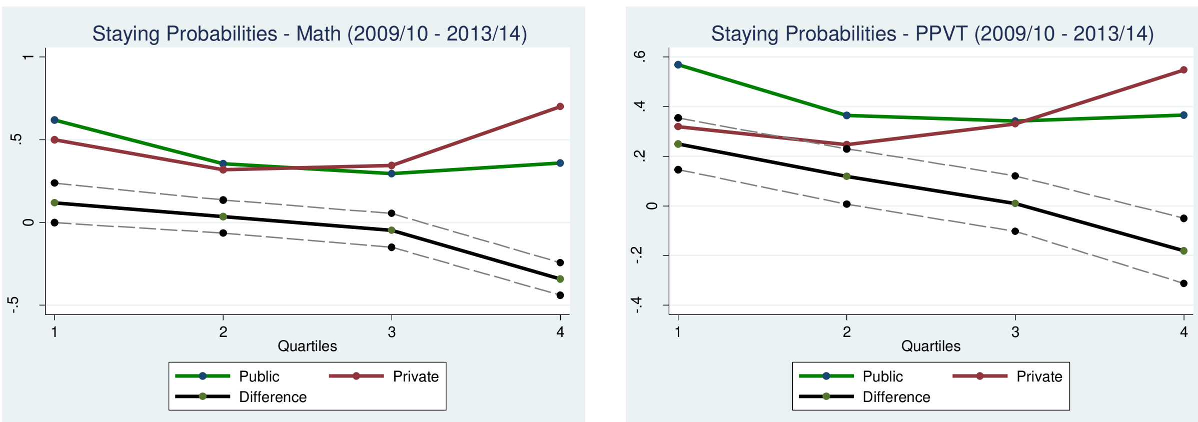
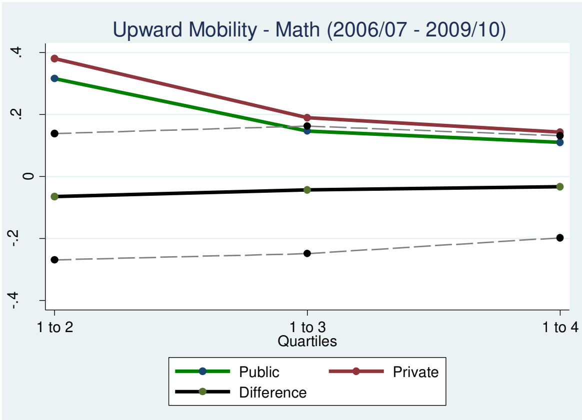
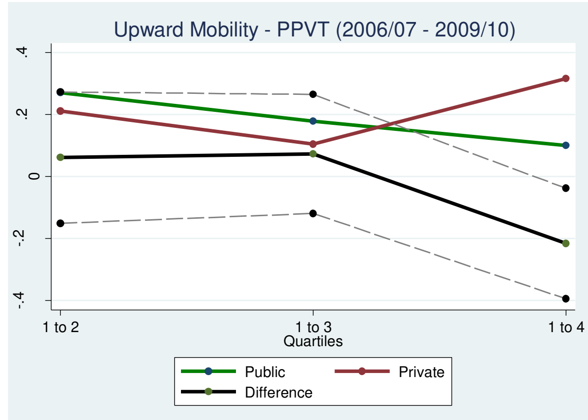


Figure 1. Staying Probabilities

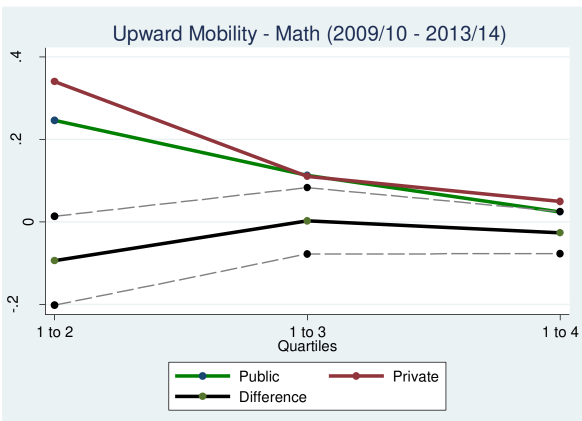
Notes: Y-axis shows estimated staying probabilities. The dashed lines represent the 95% confidence interval for the estimated difference between public and private staying probabilities.

Panel A: 2006/07 - 2009/10





Panel B: 2009/10 - 2013/14



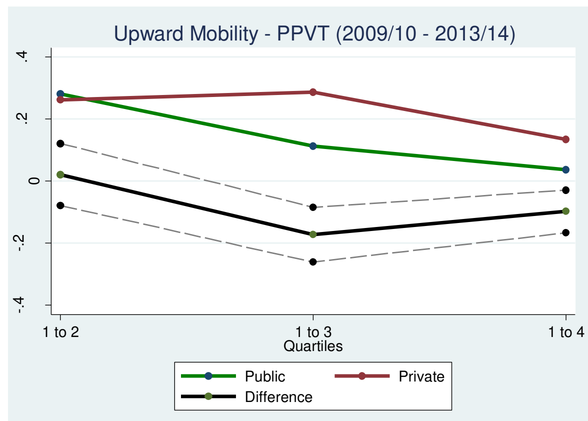
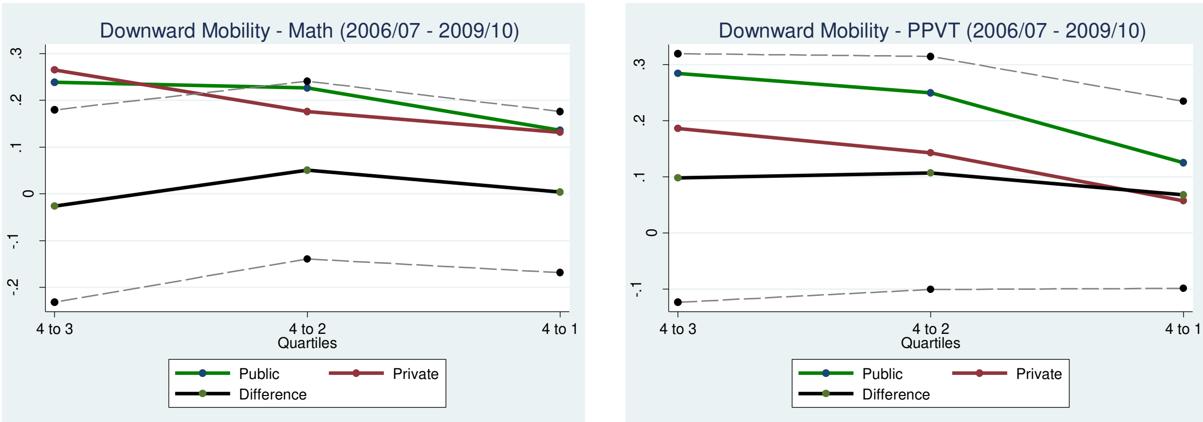


Figure 2. Upward Transition Probabilities

Notes: Y-axis shows estimated transition probabilities and X-axis shows movement from the first quartile in the initial period to some higher quartile in the final period. The dashed lines represent the 95% confidence interval for the estimated difference between public and private transition probabilities.

Panel A: 2006/07 - 2009/10



Panel B: 2009/10 - 2013/14

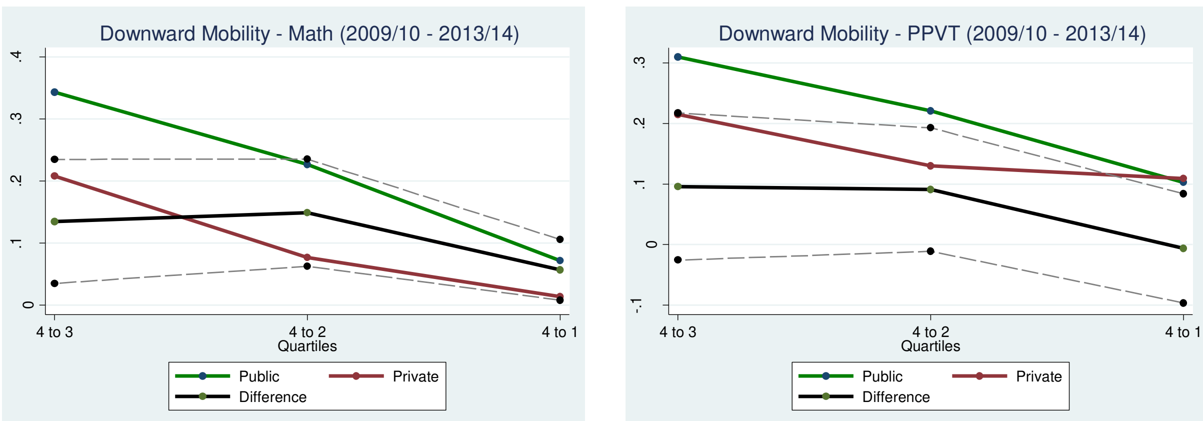
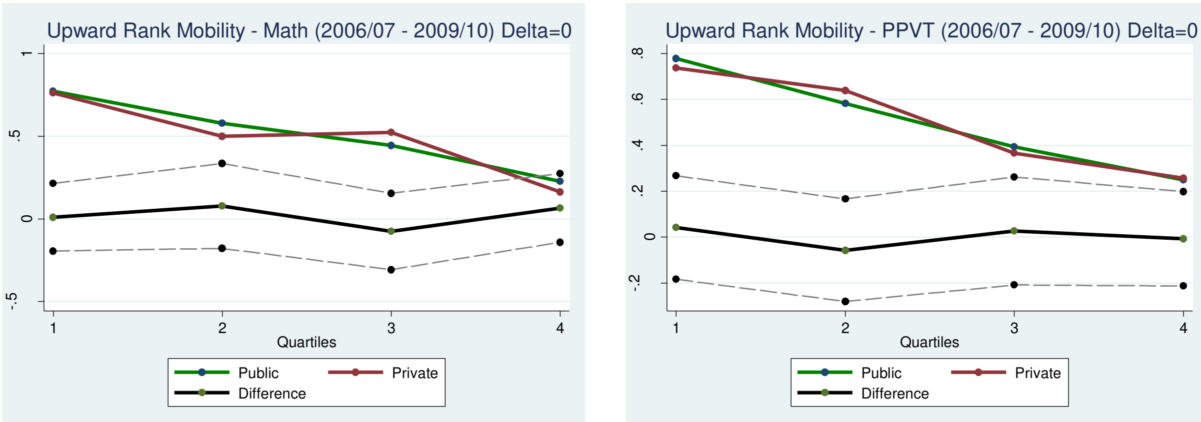


Figure 3. Downward Transition Probabilities

Notes: Y-axis shows estimated transition probabilities and X-axis shows movement from fourth quartile in the initial period to some lower quartile in the final period. The dashed lines represent the 95% confidence interval for the estimated difference between public and private transition probabilities.

Panel A: 2006/07 - 2009/10



Panel B: 2009/10 - 2013/14

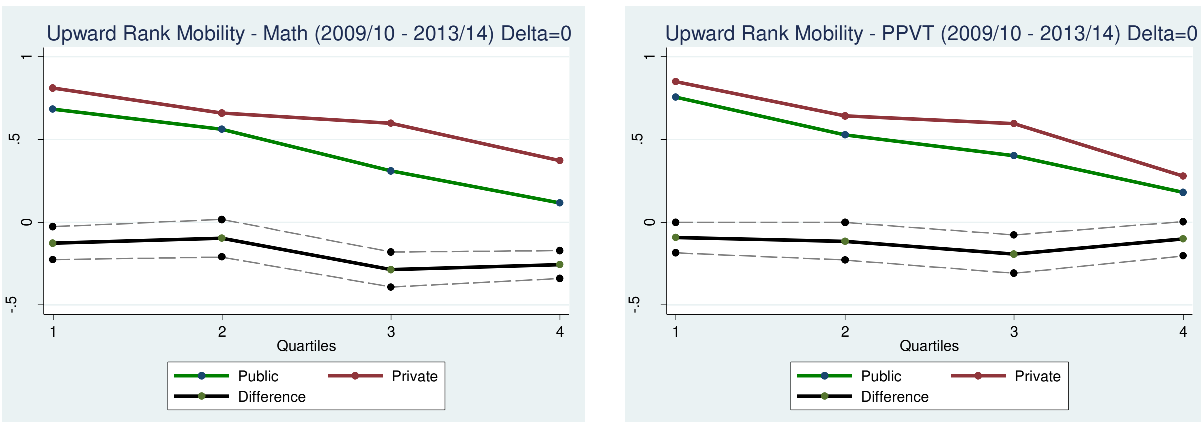
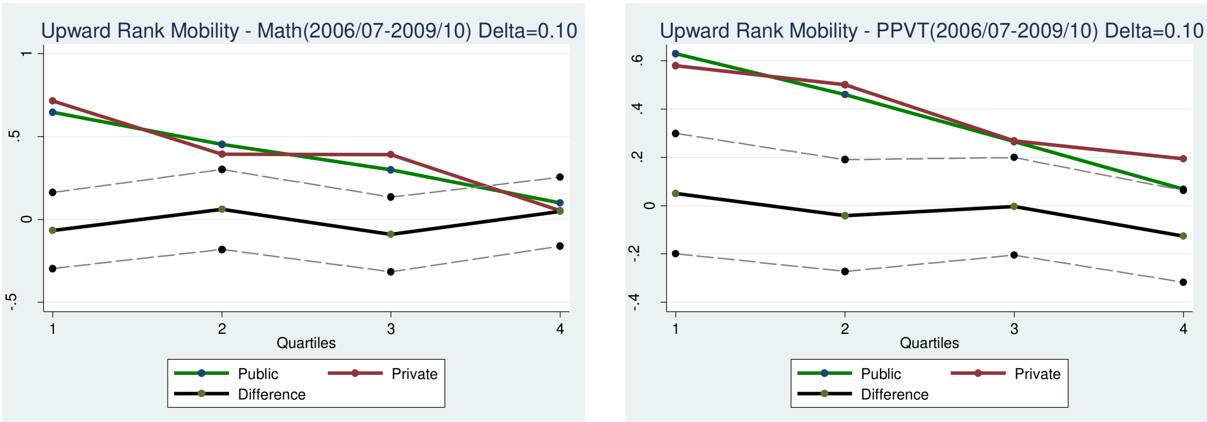


Figure 4. Upward Rank Mobility ($\delta = 0$)

Notes: Y-axis shows estimated upward rank mobilities. The dashed lines represent the 95% confidence interval for the estimated difference between public and private rank mobilities.

Panel A: 2006/07 - 2009/10



Panel B: 2009/10 - 2013/14

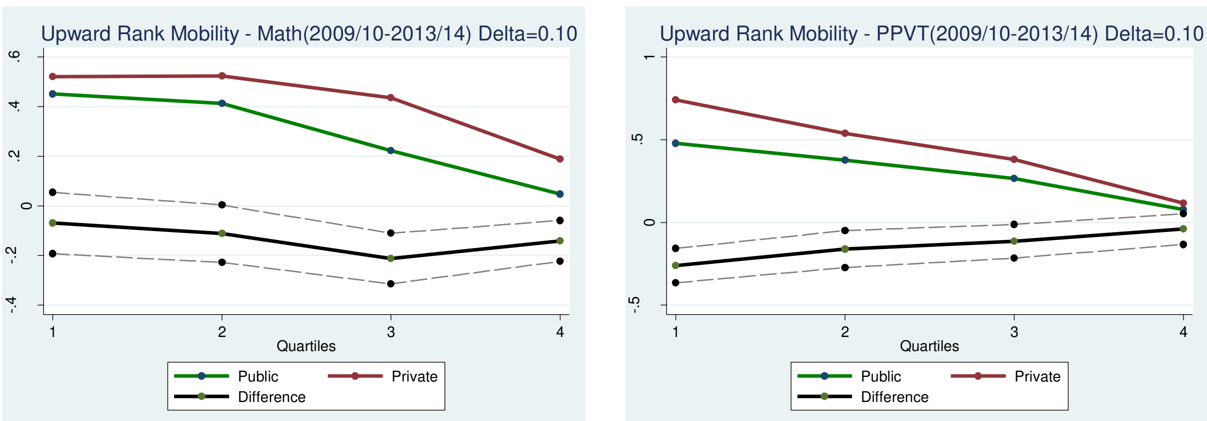
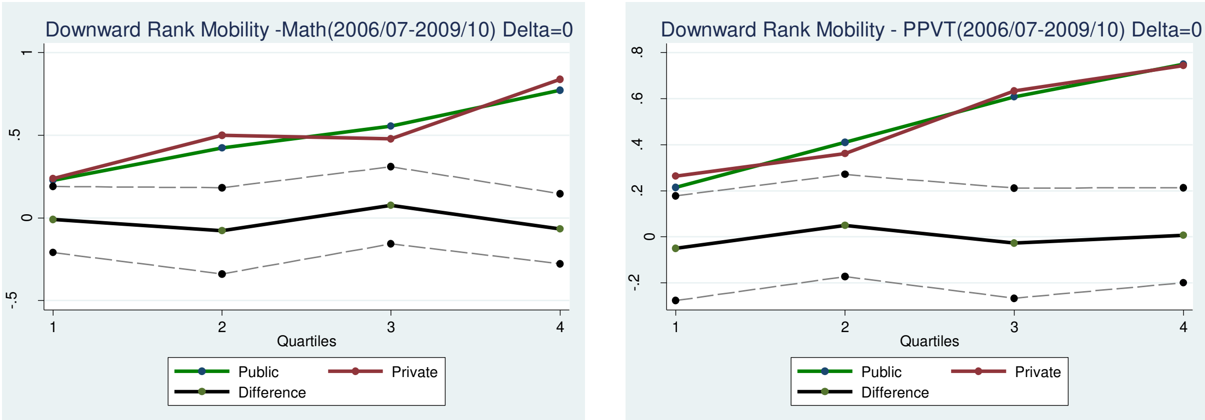


Figure 5. Upward Rank Mobility ($\delta = 0.10$)

Notes: Y-axis shows estimated upward rank mobilities. The dashed lines represent the 95% confidence interval for the estimated difference between public and private rank mobilities.

Panel A: 2006/07 - 2009/10



Panel B: 2009/10 - 2013/14

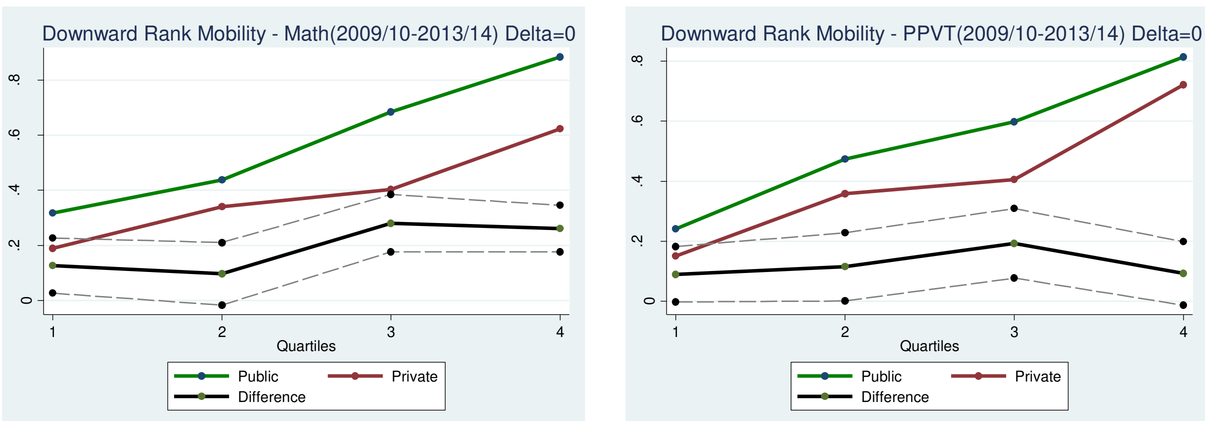
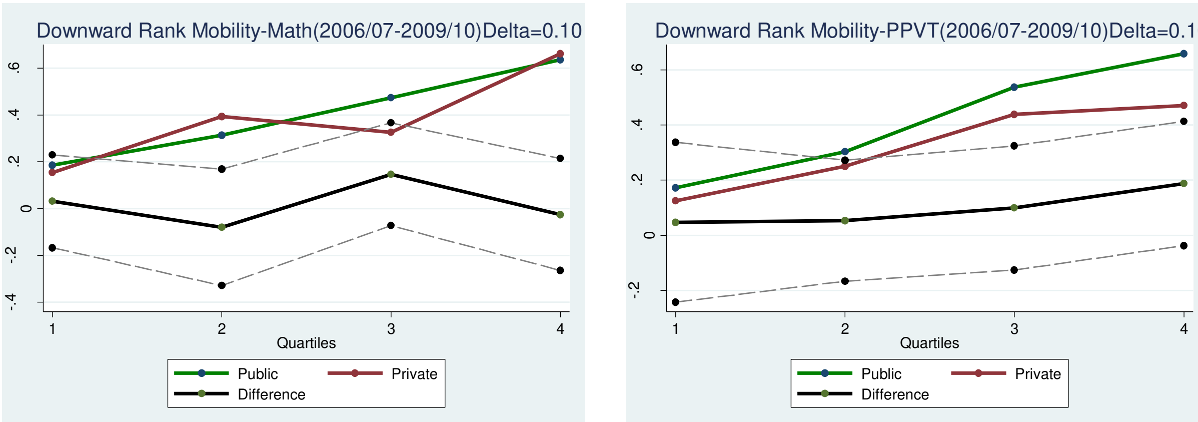


Figure 6. Downward Rank Mobility ($\delta = 0$)

Notes: Y-axis shows estimated downward rank mobilities. The dashed lines represent the 95% confidence interval for the estimated difference between public and private rank mobilities.

Panel A: 2006/07 - 2009/10

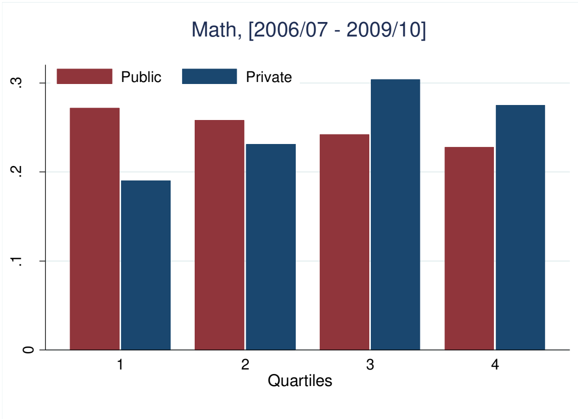


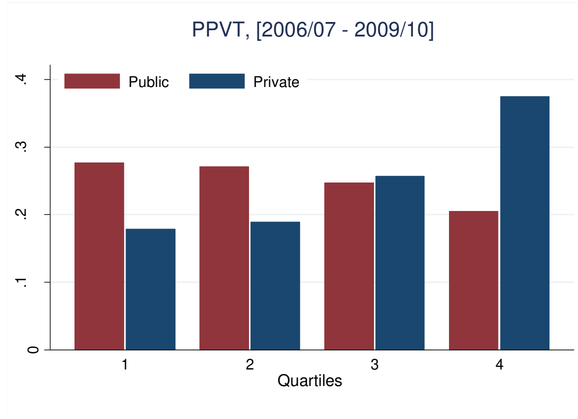
Panel B: 2009/10 - 2013/14

Figure 7. Downward Rank Mobility (δ = 0.10)

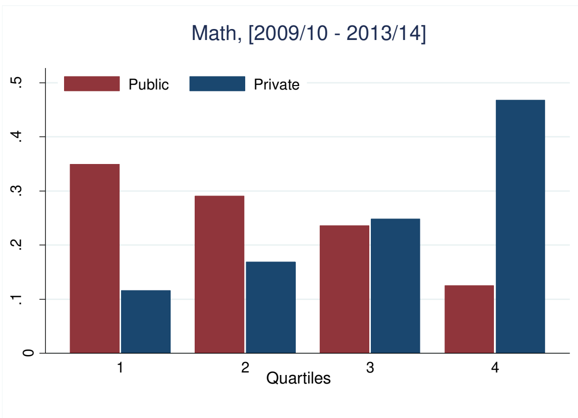
Notes: Y-axis shows estimated downward rank mobilities. The dashed lines represent the 95% confidence interval for the estimated difference between public and private rank mobilities.

Panel A: 2006/07 - 2009/10





Panel B: 2009/10 - 2013/14



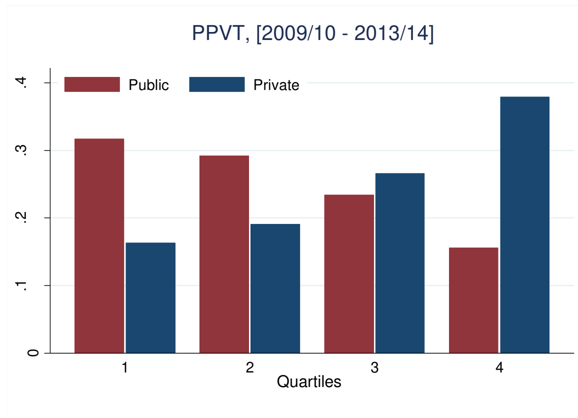


Figure 8. Markov Chain Steady State Distributions

Table 1. Summary Statistics

		Full Sample		Public School		Private School	
		Mean	SD	Mean	SD	Mean	SD
(A) 2006/07	Math test Score	0.100	0.985	-0.018	0.954	0.473	0.993
	PPVT test score	0.138	0.908	0.010	0.851	0.544	0.966
	Wealth Index	0.426	0.186	0.371	0.153	0.600	0.173
	Age of Child	5.005	0.090	5.004	0.084	5.011	0.106
	Grade Completion by the child	0.301	0.501	0.277	0.504	0.376	0.486
	Male	0.521	0.500	0.506	0.500	0.567	0.497
	Perceived Child's Ability	12.460	2.054	12.219	2.100	13.230	1.687
	Ethnicity (Schedule Caste)	0.203	0.402	0.229	0.421	0.118	0.323
	Ethnicity (Schedule Tribe)	0.185	0.389	0.210	0.408	0.107	0.310
	Ethnicity (Backward Caste)	0.431	0.496	0.448	0.498	0.376	0.486
	Ethnicity (Other Caste)	0.181	0.385	0.113	0.317	0.399	0.491
	Religion (Hindu)	0.887	0.317	0.896	0.306	0.860	0.348
	Household Size	5.515	2.341	5.552	2.348	5.399	2.320
	Residence (Rural)	0.862	0.345	0.959	0.197	0.551	0.499
	N	745		567		178	
		Mean	SD	Mean	SD	Mean	SD
(B) 2009/10	Math test Score	0.058	0.925	-0.110	1.015	0.274	0.741
	PPVT test score	0.042	0.931	-0.133	0.974	0.265	0.820
	Wealth Index	0.516	0.179	0.427	0.148	0.631	0.147
	Age of Child	7.836	0.383	7.854	0.370	7.813	0.397
	Grade Completion by the child	1.684	0.966	1.902	0.941	1.405	0.925
	Male	0.522	0.500	0.466	0.499	0.594	0.491
	Perceived Child's Ability	12.478	2.087	11.916	2.256	13.196	1.581
	Ethnicity(Schedule Caste)	0.181	0.385	0.232	0.422	0.116	0.321
	Ethnicity(Schedule Tribe)	0.148	0.355	0.203	0.402	0.077	0.267
	Ethnicity(Backward Caste)	0.467	0.499	0.469	0.499	0.466	0.499
	Ethnicity(Other Caste)	0.204	0.403	0.097	0.296	0.341	0.474
	Religion(Hindu)	0.874	0.332	0.902	0.297	0.837	0.369
	Household Size	5.476	2.320	5.541	2.319	5.392	2.320
	Residence (Rural)	0.745	0.436	0.922	0.268	0.519	0.500
	N	1767		992		775	
		Mean	SD	Mean	SD	Mean	SD
(C) 2013/14	Math test Score	0.076	0.898	-0.130	0.933	0.384	0.745
	PPVT test score	0.047	0.930	-0.114	0.847	0.290	0.996
	Wealth Index	0.587	0.165	0.516	0.147	0.695	0.131
	Age of Child	11.840	0.379	11.860	0.366	11.808	0.398
	Grade Completion by the child	5.524	1.224	5.684	1.158	5.282	1.280
	Male	0.533	0.499	0.479	0.500	0.614	0.487
	Perceived Child's Ability	12.495	2.065	12.016	2.160	13.215	1.673
	Ethnicity(Schedule Caste)	0.180	0.384	0.238	0.426	0.093	0.291
	Ethnicity(Schedule Tribe)	0.145	0.352	0.197	0.398	0.067	0.251
	Ethnicity(Backward Caste)	0.468	0.499	0.467	0.499	0.471	0.499
	Ethnicity(Other Caste)	0.207	0.405	0.099	0.299	0.369	0.483
	Religion(Hindu)	0.878	0.327	0.894	0.309	0.856	0.352
	Household Size	5.465	2.323	5.529	2.319	5.369	2.327
	Residence (Rural)	0.724	0.447	0.896	0.305	0.464	0.499
	N	1751		1052		699	

Notes: Math test score: IRT Math score; PPVT test score: IRT PPVT score; N = Number of observations. SD = Standard deviation. N in each panel correspond to the number of students enrolled in a type of school in each wave with no missing value for the covariates. Perceived Child's Ability is measured by the caregiver's perception about the level of education likely to be completed by the child.

Table 2. Conditional Mobility Gaps, Math

	2006/07-2009-10								2009/10-2013/14							
	Upward				Downward				Upward				Downward			
	δ=0.00	δ=0.00	δ=0.10	δ=0.10	δ=0.00	δ=0.00	δ=0.10	δ=0.10	δ=0.00	δ=0.00	δ=0.10	δ=0.10	δ=0.00	δ=0.00	δ=0.10	δ=0.10
Public School	0.130***	0.196***	0.107**	0.190***	-0.138***	-0.215***	-0.085*	-0.182***	-0.130***	-0.167***	-0.077***	-0.145***	0.130***	0.168***	0.111***	0.123***
	(0.046)	(0.063)	(0.044)	(0.058)	(0.046)	(0.063)	(0.046)	(0.062)	(0.026)	(0.036)	(0.025)	(0.035)	(0.026)	(0.036)	(0.024)	(0.033)
Household size	-	0.005	-	-0.005	-	-0.005	-	0.002	-	0.004	-	0.004	-	-0.004	-	0.001
	-	(0.007)	-	(0.008)	-	(0.007)	-	(0.007)	-	(0.005)	-	(0.005)	-	(0.005)	-	(0.005)
Ethnicity (Schedule Caste)	-	0.278***	-	0.271***	-	-0.280***	-	-0.310***	-	-0.145***	-	-0.111**	-	0.146***	-	0.156***
	-	(0.062)	-	(0.059)	-	(0.062)	-	(0.061)	-	(0.045)	-	(0.044)	-	(0.045)	-	(0.041)
Ethnicity (Backward Caste)	-	0.228***	-	0.220***	-	-0.231***	-	-0.223***	-	-0.134***	-	-0.097**	-	0.133***	-	0.131***
	-	(0.055)	-	(0.049)	-	(0.054)	-	(0.056)	-	(0.039)	-	(0.039)	-	(0.039)	-	(0.034)
Ethnicity (Other Caste)	-	0.310***	-	0.277***	-	-0.304***	-	-0.292***	-	-0.160***	-	-0.150***	-	0.161***	-	0.145***
	-	(0.070)	-	(0.068)	-	(0.070)	-	(0.070)	-	(0.048)	-	(0.047)	-	(0.048)	-	(0.043)
Religion (Hindu)	-	0.131**	-	0.057	-	-0.117*	-	-0.116*	-	0.0157	-	-0.023	-	-0.017	-	0.025
	-	(0.066)	-	(0.063)	-	(0.066)	-	(0.064)	-	(0.042)	-	(0.040)	-	(0.042)	-	(0.039)
Wealth Index	-	0.083	-	0.291**	-	-0.100	-	-0.229*	-	-0.025	-	-0.154	-	0.028	-	-0.179**
	-	(0.144)	-	(0.138)	-	(0.143)	-	(0.138)	-	(0.100)	-	(0.096)	-	(0.100)	-	(0.090)
Rural	-	0.050	-	0.090	-	-0.044	-	-0.077	-	0.024	-	0.023	-	-0.021	-	-0.064*
	-	(0.072)	-	(0.070)	-	(0.072)	-	(0.071)	-	(0.038)	-	(0.036)	-	(0.038)	-	(0.034)
Age	-	0.139	-	0.026	-	-0.156	-	-0.032	-	-0.110***	-	-0.044	-	0.109***	-	0.067**
	-	(0.173)	-	(0.195)	-	(0.173)	-	(0.185)	-	(0.034)	-	(0.033)	-	(0.034)	-	(0.031)
Perceived Child's Ability	-	0.011	-	0.004	-	-0.015	-	-0.017*	-	0.002	-	0.007	-	-0.002	-	0.005
	-	(0.010)	-	(0.010)	-	(0.010)	-	(0.010)	-	(0.007)	-	(0.006)	-	(0.007)	-	(0.007)
Male	-	0.005	-	-0.006	-	-0.010	-	-0.046	-	-0.004	-	-0.029	-	0.005	-	0.013
	-	(0.041)	-	(0.039)	-	(0.041)	-	(0.039)	-	(0.027)	-	(0.025)	-	(0.027)	-	(0.025)
Constant	0.410***	-0.896	0.301***	-0.353	0.609***	2.047**	0.449***	1.373	0.578***	1.498***	0.383***	0.834***	0.422***	-0.495*	0.243***	-0.294
	(0.039)	(0.907)	(0.037)	(1.015)	(0.039)	(0.903)	(0.040)	(0.959)	(0.020)	(0.292)	(0.019)	(0.283)	(0.020)	(0.292)	(0.017)	(0.270)
Observations	604	604	604	604	604	604	604	604	1,491	1,491	1,491	1,491	1,491	1,491	1,491	1,491
R-squared	0.013	0.071	0.009	0.071	0.015	0.074	0.006	0.083	0.016	0.035	0.006	0.023	0.016	0.036	0.014	0.031

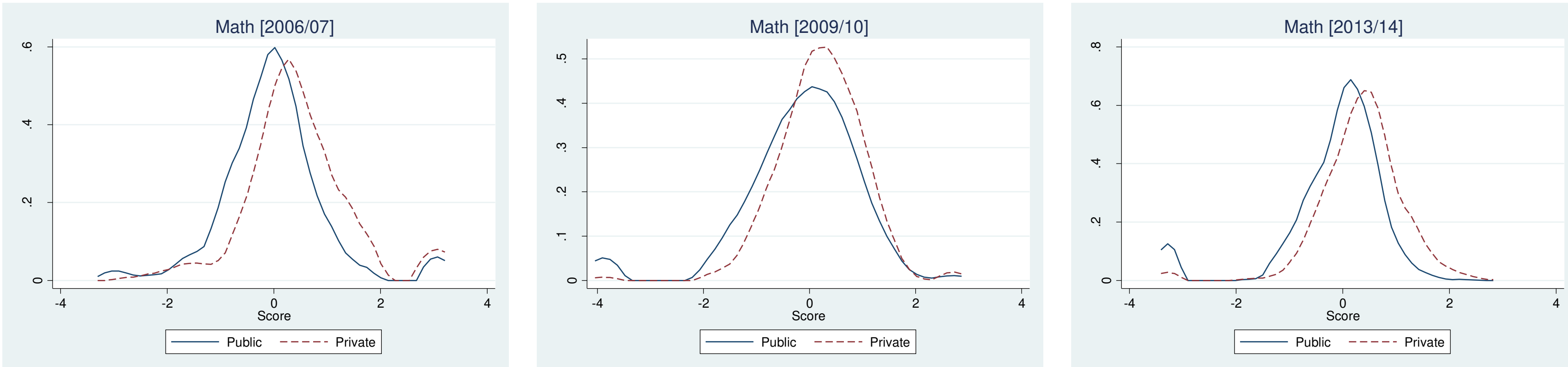
Notes: We include the coefficient estimates on the public school indicator without any controls to provide a sense of how these estimates change as additional covariates are added to the model; robust standard errors in parentheses.
*** p<0.01, ** p<0.05, * p<0.1

Table 3. Conditional Mobility Gaps, PPVT

	2006/07-2009-10								2009/10-2013/14							
	Upward				Downward				Upward				Downward			
	δ=0.00	δ=0.00	δ=0.10	δ=0.10	δ=0.00	δ=0.00	δ=0.10	δ=0.10	δ=0.00	δ=0.00	δ=0.10	δ=0.10	δ=0.00	δ=0.00	δ=0.10	δ=0.10
Public School	0.121***	0.110*	0.106**	0.126**	-0.123***	-0.110*	-0.010	-0.011	-0.038	-0.066*	-0.038	-0.092***	0.039	0.065*	0.004	0.024
	(0.046)	(0.062)	(0.043)	(0.061)	(0.046)	(0.062)	(0.045)	(0.060)	(0.026)	(0.036)	(0.025)	(0.035)	(0.026)	(0.036)	(0.025)	(0.034)
Household size	-	-0.003	-	-0.003	-	0.003	-	0.008	-	0.001	-	0.009	-	-0.001	-	-0.000
	-	(0.008)	-	(0.007)	-	(0.008)	-	(0.008)	-	(0.006)	-	(0.005)	-	(0.006)	-	(0.005)
Ethnicity (Schedule Caste)	-	0.320***	-	0.253***	-	-0.312***	-	-0.263***	-	0.054	-	0.022	-	-0.053	-	-0.040
	-	(0.060)	-	(0.056)	-	(0.060)	-	(0.062)	-	(0.046	-	(0.044)	-	(0.046)	-	(0.044)
Ethnicity (Backward Caste)	-	0.330***	-	0.246***	-	-0.322***	-	-0.266***	-	0.022	-	-0.030	-	-0.023	-	-0.017
	-	(0.052)	-	(0.048)	-	(0.052)	-	(0.055)	-	(0.040)	-	(0.038)	-	(0.040)	-	(0.038)
Ethnicity (Other Caste)	-	0.469***	-	0.364***	-	-0.461***	-	-0.362***	-	0.024	-	-0.042	-	-0.023	-	-0.042
	-	(0.067)	-	(0.065)	-	(0.067)	-	(0.069)	-	(0.049)	-	(0.047)	-	(0.049)	-	(0.047)
Religion (Hindu)	-	0.107*	-	0.058	-	-0.108*	-	-0.078	-	-0.020	-	-0.053	-	0.027	-	-0.002
	-	(0.061)	-	(0.059)	-	(0.061)	-	(0.060)	-	(0.042)	-	(0.041)	-	(0.042)	-	(0.040)
Wealth Index	-	-0.146	-	-0.052	-	0.144	-	-0.103	-	0.137	-	0.008	-	-0.140	-	-0.099
	-	(0.145)	-	(0.133)	-	(0.145)	-	(0.139)	-	(0.101)	-	(0.095)	-	(0.101)	-	(0.096)
Rural	-	0.139**	-	0.090	-	-0.140**	-	-0.185***	-	0.141***	-	0.086**	-	-0.143***	-	-0.091**
	-	(0.070)	-	(0.068)	-	(0.070)	-	(0.068)	-	(0.038)	-	(0.036)	-	(0.038)	-	(0.036)
Age	-	0.261	-	0.395**	-	-0.260	-	-0.117	-	-0.014	-	-0.002	-	0.013	-	-0.003
	-	(0.165)	-	(0.169)	-	(0.165)	-	(0.182)	-	(0.034)	-	(0.032)	-	(0.034)	-	(0.032)
Perceived Child's Ability	-	-0.006	-	-0.004	-	0.007	-	0.016*	-	0.010	-	0.004	-	-0.009	-	0.0001
	-	(0.010)	-	(0.010)	-	(0.010)	-	(0.009)	-	(0.007)	-	(0.007)	-	(0.007)	-	(0.006)
Male	-	0.028	-	0.030	-	-0.026	-	-0.055	-	-0.053**	-	-0.051**	-	0.055**	-	0.043*
	-	(0.040)	-	(0.039)	-	(0.040)	-	(0.040)	-	(0.027)	-	(0.025)	-	(0.027)	-	(0.025)
Constant	0.410***	-1.242	0.276***	-1.991**	0.590***	2.220**	0.385***	1.254	0.535***	0.243	0.363***	0.243	0.463***	0.757**	0.329***	0.548**
	(0.039)	(0.866)	(0.036)	(0.881)	(0.039)	(0.865)	(0.039)	(0.943)	(0.020)	(0.297)	(0.019)	(0.282)	(0.020)	(0.297)	(0.019)	(0.276)
Observations	604	604	604	604	604	604	604	604	1,491	1,491	1,491	1,491	1,491	1,491	1,491	1,491
R-squared	0.011	0.103	0.009	0.073	0.012	0.101	0.000	0.076	0.001	0.015	0.002	0.013	0.001	0.016	0.000	0.008

Notes: We include the coefficient estimates on the public school indicator without any controls to provide a sense of how these estimates change as additional covariates are added to the model; robust standard errors in parentheses.
*** p<0.01, ** p<0.05, * p<0.1

Panel A: Math Test Scores



Panel B: PPVT Test Scores

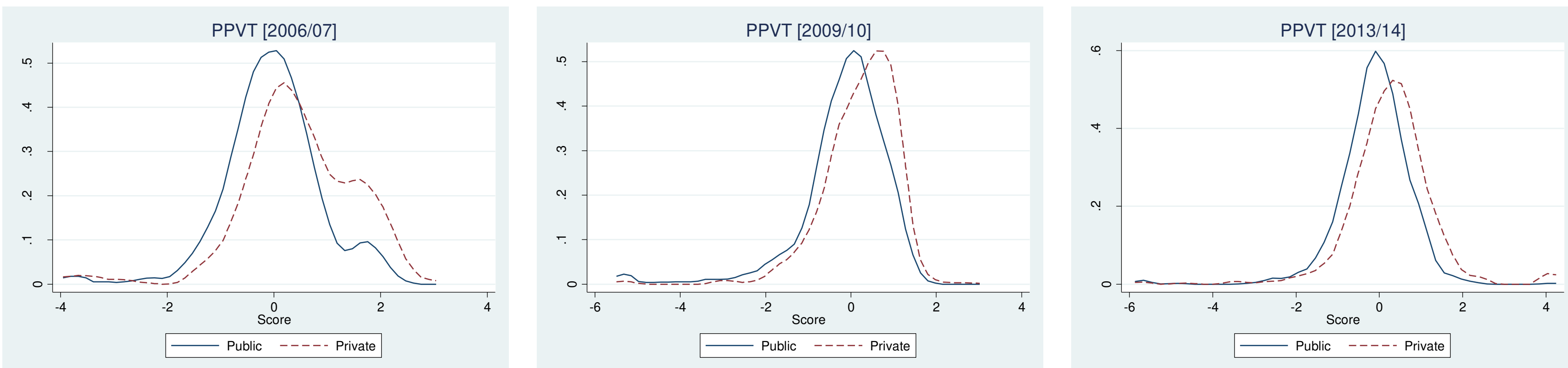


Figure A1. Kernel Density Plots of Test Scores in Math and PPVT by School Type

Panel A: 2006/07 - 2009/10

Panel B: 2009/10 - 2013/14

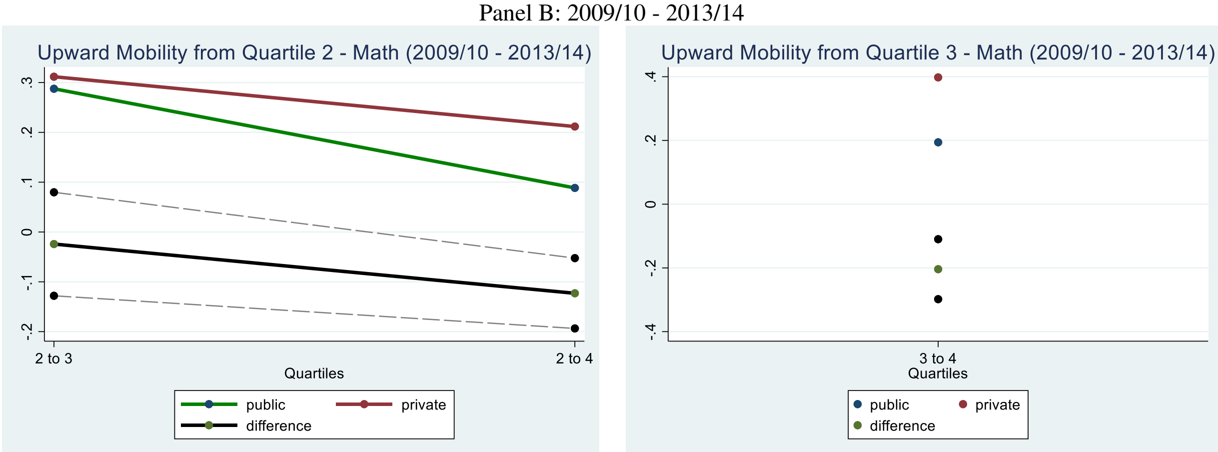


Figure A2. Upward Transition Probabilities – Math

Notes: Y-Axis shows estimated transition probabilities and X-axis shows movement from the second and third quartiles to some higher quartile in the final period. The dashed lines represent the 95% confidence interval for the estimated difference between public and private transition probabilities.

Panel A: 2006/07 - 2009/10

Panel B: 2009/10 - 2013/14

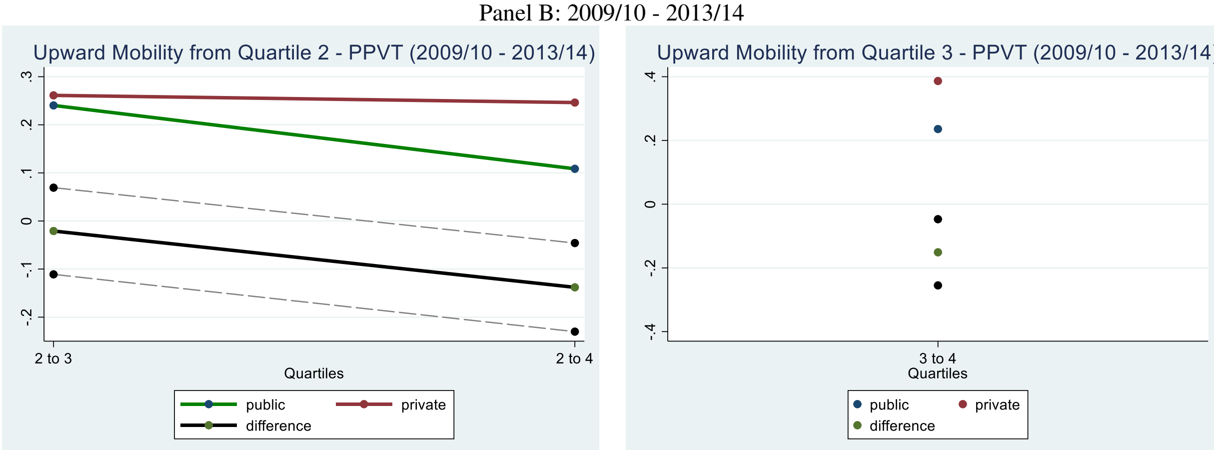
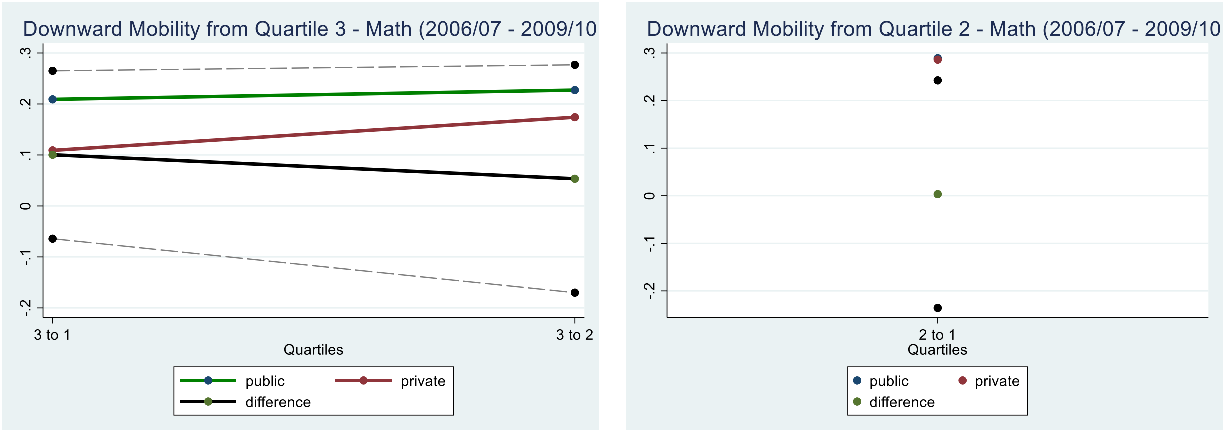


Figure A3. Upward Transition Probabilities – PPVT

Notes: Y-Axis shows estimated transition probabilities and X-axis shows movement from the second and third quartiles to some higher quartile in the final period. The dashed lines represent the 95% confidence interval for the estimated difference between public and private transition probabilities.

Panel A: 2006/07 - 2009/10

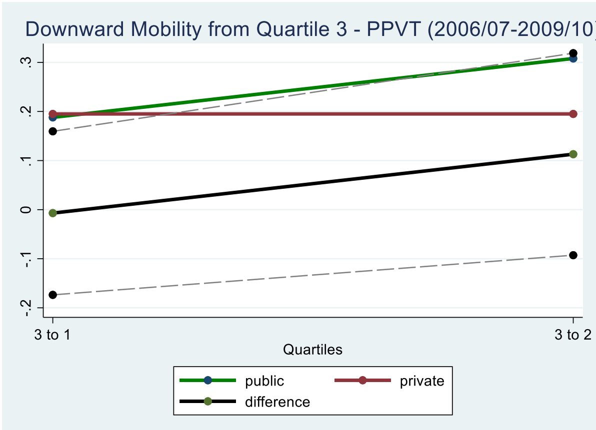


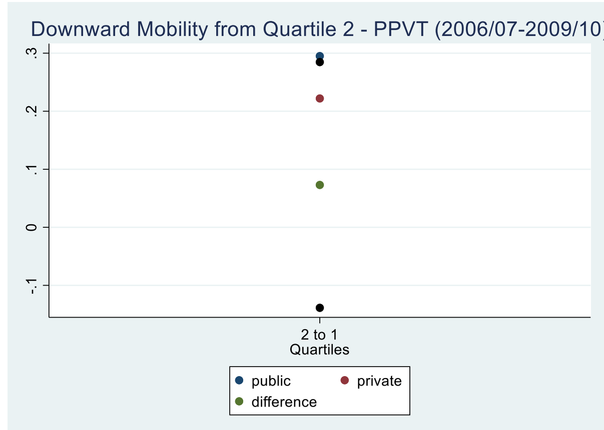
Panel B: 2009/10 - 2013/14

Figure A4. Downward Transition Probabilities – Math

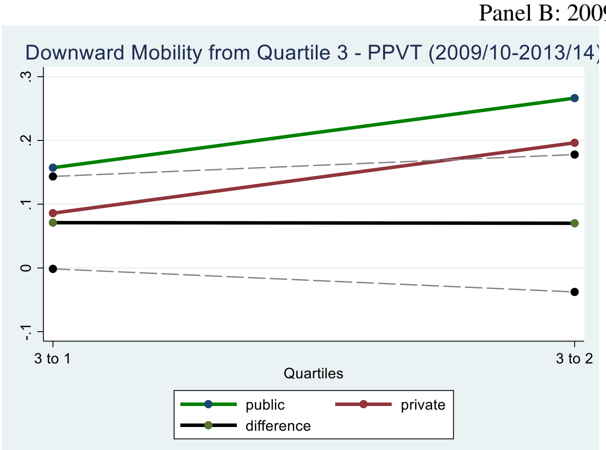
Notes: Y-Axis shows estimated transition probabilities and X-axis shows movement from the third and second quartiles to some lower quartile in the final period. The dashed lines represent the 95% confidence interval for the estimated difference between public and private transition probabilities.

Panel A: 2006/07 - 2009/10





Panel B: 2009/10 - 2013/14



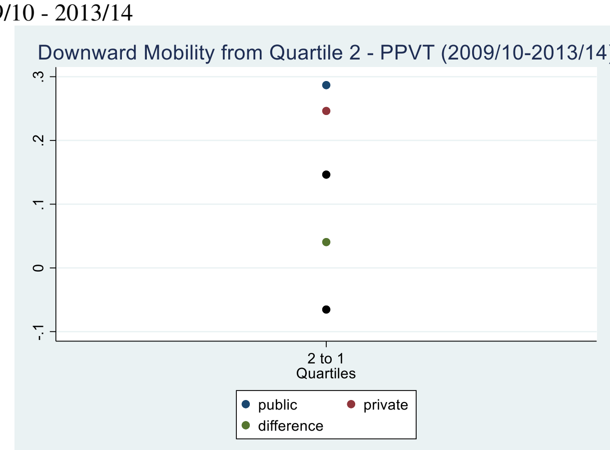


Figure A5. Downward Transition Probabilities – PPVT

Notes: Y-Axis shows estimated transition probabilities and X-axis shows movement from the third and second quartiles to some lower quartile in the final period. The dashed lines represent the 95% confidence interval for the estimated difference between public and private transition probabilities.

Table A1. Transition probability estimates by school type, Math

Panel A: 2006/07 - 2009/10

	Public				Private				Public - Private			
	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
Q1	0.426	0.316	0.147	0.110	0.286	0.381	0.190	0.143	0.141	-0.065	-0.043	-0.033
[N_{PU} = 136, N_{PR} = 21]	(0.057)	(0.059)	(0.050)	(0.042)	(0.090)	(0.087)	(0.089)	(0.073)	(0.114)	(0.104)	(0.105)	(0.084)
Q2	0.289	0.250	0.266	0.195	0.286	0.250	0.357	0.107	0.003	0.000	-0.092	0.088
[N_{PU} = 128, N_{PR} = 28]	(0.063)	(0.064)	(0.058)	(0.051)	(0.087)	(0.077)	(0.089)	(0.056)	(0.122)	(0.107)	(0.115)	(0.080)
Q3	0.209	0.227	0.327	0.236	0.109	0.174	0.370	0.348	0.100	0.053	-0.042	-0.111
[N_{PU} = 110, N_{PR} = 46]	(0.064)	(0.078)	(0.076)	(0.074)	(0.048)	(0.071)	(0.071)	(0.069)	(0.084)	(0.114)	(0.104)	(0.110)
Q4	0.136	0.227	0.239	0.398	0.132	0.176	0.265	0.426	0.004	0.051	-0.026	-0.029
[N_{PU} = 88, N_{PR} = 68]	(0.072)	(0.078)	(0.084)	(0.094)	(0.044)	(0.057)	(0.056)	(0.068)	(0.088)	(0.097)	(0.105)	(0.129)

Panel B: 2009/10 - 2013/14

	Public				Private				Public - Private			
	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
Q1	0.618	0.246	0.113	0.024	0.500	0.340	0.110	0.050	0.118*	-0.094*	0.003	-0.026
[N_{PU} = 293, N_{PR} = 100]	(0.028)	(0.026)	(0.021)	(0.009)	(0.049)	(0.044)	(0.032)	(0.024)	(0.061)	(0.055)	(0.041)	(0.026)
Q2	0.270	0.354	0.288	0.089	0.159	0.318	0.312	0.212	0.111**	0.036	-0.024	-0.123***
[N_{PU} = 226, N_{PR} = 170]	(0.034)	(0.035)	(0.035)	(0.022)	(0.031)	(0.034)	(0.034)	(0.029)	(0.049)	(0.051)	(0.053)	(0.036)
Q3	0.194	0.316	0.296	0.194	0.097	0.161	0.344	0.398	0.097**	0.154***	-0.048	-0.204***
[N_{PU} = 206, N_{PR} = 186]	(0.033)	(0.036)	(0.038)	(0.031)	(0.019)	(0.026)	(0.032)	(0.030)	(0.040)	(0.046)	(0.052)	(0.048)
Q4	0.072	0.227	0.343	0.359	0.014	0.077	0.208	0.700	0.057**	0.149***	0.135***	-0.341***
[N_{PU} = 181, N_{PR} = 207]	(0.022)	(0.037)	(0.041)	(0.037)	(0.010)	(0.021)	(0.027)	(0.029)	(0.025)	(0.044)	(0.051)	(0.050)

Notes: See text for description of the estimator. Balanced panels were constructed. Bootstrap standard errors based on 250 repetitions are in parentheses.

Table A2. Transition probability estimates by school type, PPVT

Panel A: 2006/07 - 2009/10

	Public				Private				Public - Private			
	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
Q1	0.450	0.271	0.179	0.100	0.368	0.211	0.105	0.316	0.082	0.061	0.073	-0.216**
[N_{PU} = 136, N_{PR} = 21]	(0.057)	(0.059)	(0.046)	(0.039)	(0.096)	(0.081)	(0.085)	(0.080)	(0.130)	(0.108)	(0.098)	(0.091)
Q2	0.295	0.254	0.295	0.156	0.222	0.250	0.361	0.167	0.073	0.004	-0.066	-0.011
[N_{PU} = 122, N_{PR} = 36]	(0.068)	(0.068)	(0.065)	(0.055)	(0.070)	(0.088)	(0.087)	(0.054)	(0.108)	(0.125)	(0.117)	(0.081)
Q3	0.188	0.308	0.239	0.265	0.195	0.195	0.390	0.220	-0.007	0.113	-0.151	0.045
[N_{PU} = 117, N_{PR} = 41]	(0.055)	(0.068)	(0.071)	(0.066)	(0.062)	(0.072)	(0.074)	(0.069)	(0.085)	(0.105)	(0.114)	(0.102)
Q4	0.125	0.250	0.284	0.341	0.057	0.143	0.186	0.614	0.068	0.107	0.098	-0.273**
[N_{PU} = 88, N_{PR} = 68]	(0.074)	(0.095)	(0.093)	(0.095)	(0.035)	(0.043)	(0.057)	(0.055)	(0.085)	(0.106)	(0.113)	(0.115)

Panel B: 2009/10 - 2013/14

	Public				Private				Public - Private			
	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
Q1	0.569	0.281	0.113	0.037	0.319	0.261	0.286	0.134	0.250***	0.021	-0.173***	-0.098***
[N_{PU} = 274, N_{PR} = 119]	(0.030)	(0.027)	(0.022)	(0.013)	(0.041)	(0.042)	(0.039)	(0.031)	(0.053)	(0.051)	(0.045)	(0.035)
Q2	0.287	0.364	0.240	0.109	0.246	0.246	0.261	0.246	0.041	0.118**	-0.021	-0.138***
[N_{PU} = 258, N_{PR} = 134]	(0.030)	(0.032)	(0.029)	(0.023)	(0.038)	(0.038)	(0.034)	(0.037)	(0.054)	(0.057)	(0.046)	(0.047)
Q3	0.157	0.266	0.341	0.236	0.086	0.196	0.331	0.387	0.071*	0.070	0.009	-0.151***
[N_{PU} = 229, N_{PR} = 163]	(0.029)	(0.038)	(0.037)	(0.031)	(0.021)	(0.031)	(0.034)	(0.033)	(0.037)	(0.055)	(0.057)	(0.053)
Q4	0.103	0.221	0.310	0.366	0.109	0.130	0.215	0.547	-0.006	0.091*	0.096	-0.181***
[N_{PU} = 145, N_{PR} = 247]	(0.033)	(0.043)	(0.051)	(0.050)	(0.023)	(0.023)	(0.026)	(0.031)	(0.046)	(0.052)	(0.062)	(0.067)

Notes: See text for description of the estimator. Balanced panels were constructed. Bootstrap standard errors based on 250 repetitions are in parentheses.

Table A3. Upward rank mobility estimates by school type, Math

Panel A: 2006/07 - 2009/10

	$\delta = 0.00$			$\delta = 0.05$			$\delta = 0.10$		
	Public	Private	Public - Private	Public	Private	Public - Private	Public	Private	Public - Private
Q1	0.772	0.762	0.010	0.728	0.714	0.014	0.647	0.714	-0.067
[$N_{PU} = 136$, $N_{PR} = 21$]	(0.053)	(0.057)	(0.059)	(0.057)	(0.085)	(0.113)	(0.060)	(0.087)	(0.117)
Q2	0.578	0.500	0.078	0.531	0.500	0.031	0.453	0.393	0.060
[$N_{PU} = 128$, $N_{PR} = 28$]	(0.069)	(0.098)	(0.131)	(0.067)	(0.096)	(0.128)	(0.070)	(0.091)	(0.123)
Q3	0.445	0.522	-0.076	0.373	0.478	-0.106	0.300	0.391	-0.091
[$N_{PU} = 110$, $N_{PR} = 46$]	(0.084)	(0.072)	(0.118)	(0.084)	(0.069)	(0.118)	(0.082)	(0.071)	(0.115)
Q4	0.227	0.162	0.066	0.216	0.103	0.113	0.100	0.053	0.047
[$N_{PU} = 88$, $N_{PR} = 68$]	(0.088)	(0.054)	(0.106)	(0.098)	(0.043)	(0.111)	(0.092)	(0.051)	(0.106)

Panel B: 2009/10 - 2013/14

	$\delta = 0.00$			$\delta = 0.05$			$\delta = 0.10$		
	Public	Private	Public - Private	Public	Private	Public - Private	Public	Private	Public - Private
Q1	0.683	0.810	-0.127**	0.509	0.600	-0.091	0.451	0.520	-0.069
[$N_{PU} = 293$, $N_{PR} = 100$]	(0.030)	(0.041)	(0.051)	(0.031)	(0.051)	(0.063)	(0.031)	(0.051)	(0.063)
Q2	0.562	0.659	-0.097*	0.478	0.612	-0.134**	0.412	0.524	-0.112*
[$N_{PU} = 226$, $N_{PR} = 170$]	(0.039)	(0.036)	(0.058)	(0.039)	(0.037)	(0.056)	(0.035)	(0.041)	(0.059)
Q3	0.311	0.597	-0.286***	0.262	0.516	-0.254***	0.223	0.435	-0.212***
[$N_{PU} = 206$, $N_{PR} = 186$]	(0.036)	(0.034)	(0.054)	(0.031)	(0.036)	(0.056)	(0.033)	(0.034)	(0.052)
Q4	0.116	0.372	-0.256***	0.071	0.243	-0.172***	0.048	0.189	-0.141***
[$N_{PU} = 181$, $N_{PR} = 207$]	(0.026)	(0.032)	(0.043)	(0.025)	(0.031)	(0.042)	(0.022)	(0.032)	(0.042)

Notes: See text for description of the estimator. Balanced panels were constructed. Bootstrap standard errors based on 250 repetitions are in parentheses.

Table A4. Upward rank mobility estimates by school type, PPVT

Panel A: 2006/07 - 2009/10

	δ = 0.00			δ = 0.05			δ = 0.10		
	Public	Private	Public - Private	Public	Private	Public - Private	Public	Private	Public - Private
Q1	0.779	0.737	0.042	0.679	0.684	-0.006	0.629	0.579	0.050
[N_{PU} = 136, N_{PR} = 21]	(0.055)	(0.057)	(0.059)	(0.057)	(0.095)	(0.121)	(0.061)	(0.098)	(0.127)
Q2	0.582	0.639	-0.057	0.508	0.583	-0.075	0.459	0.500	-0.041
[N_{PU} = 122, N_{PR} = 36]	(0.071)	(0.079)	(0.114)	(0.070)	(0.082)	(0.115)	(0.067)	(0.085)	(0.118)
Q3	0.393	0.366	0.027	0.325	0.293	0.032	0.265	0.268	-0.003
[N_{PU} = 117, N_{PR} = 41]	(0.075)	(0.087)	(0.120)	(0.070)	(0.077)	(0.106)	(0.069)	(0.070)	(0.103)
Q4	0.250	0.257	-0.007	0.135	0.264	-0.129	0.068	0.194	-0.127
[N_{PU} = 88, N_{PR} = 68]	(0.087)	(0.050)	(0.105)	(0.085)	(0.053)	(0.103)	(0.075)	(0.060)	(0.097)

Panel B: 2009/10 - 2013/14

	δ = 0.00			δ = 0.05			δ = 0.10		
	Public	Private	Public - Private	Public	Private	Public - Private	Public	Private	Public - Private
Q1	0.755	0.849	-0.093**	0.569	0.782	-0.212***	0.478	0.740	-0.261***
[N_{PU} = 274, N_{PR} = 119]	(0.029)	(0.033)	(0.047)	(0.030)	(0.038)	(0.051)	(0.030)	(0.041)	(0.053)
Q2	0.527	0.642	-0.115**	0.453	0.590	-0.136**	0.376	0.537	-0.161***
[N_{PU} = 258, N_{PR} = 134]	(0.036)	(0.041)	(0.058)	(0.035)	(0.041)	(0.056)	(0.034)	(0.041)	(0.057)
Q3	0.402	0.595	-0.193***	0.323	0.454	-0.131**	0.266	0.380	-0.114**
[N_{PU} = 229, N_{PR} = 163]	(0.040)	(0.037)	(0.059)	(0.031)	(0.041)	(0.058)	(0.034)	(0.034)	(0.052)
Q4	0.179	0.279	-0.100*	0.153	0.209	-0.057	0.077	0.117	-0.040
[N_{PU} = 145, N_{PR} = 247]	(0.041)	(0.026)	(0.053)	(0.038)	(0.029)	(0.051)	(0.037)	(0.031)	(0.047)

Notes: See text for description of the estimator. Balanced panels were constructed. Bootstrap standard errors based on 250 repetitions are in parentheses.

Table A5. Downward rank mobility estimates by school type, Math

Panel A: 2006/07 - 2009/10	δ = 0.00			δ = 0.05			δ = 0.10		
	Public	Private	Public - Private	Public	Private	Public - Private	Public	Private	Public - Private
Q1	0.228	0.238	-0.010	0.202	0.188	0.014	0.185	0.154	0.031
[N_{PU} = 136, N_{PR} = 21]	(0.053)	(0.057)	(0.059)	(0.057)	(0.094)	(0.119)	(0.064)	(0.077)	(0.101)
Q2	0.422	0.500	-0.078	0.375	0.500	-0.125	0.313	0.393	-0.080
[N_{PU} = 128, N_{PR} = 28]	(0.069)	(0.100)	(0.133)	(0.068)	(0.094)	(0.130)	(0.064)	(0.093)	(0.127)
Q3	0.555	0.478	0.076	0.500	0.413	0.087	0.473	0.326	0.147
[N_{PU} = 110, N_{PR} = 46]	(0.084)	(0.073)	(0.119)	(0.085)	(0.070)	(0.119)	(0.080)	(0.071)	(0.112)
Q4	0.773	0.838	-0.066	0.682	0.735	-0.053	0.636	0.662	-0.025
[N_{PU} = 88, N_{PR} = 68]	(0.089)	(0.054)	(0.108)	(0.095)	(0.059)	(0.119)	(0.097)	(0.061)	(0.122)

Panel B: 2009/10 - 2013/14	δ = 0.00			δ = 0.05			δ = 0.10		
	Public	Private	Public - Private	Public	Private	Public - Private	Public	Private	Public - Private
Q1	0.317	0.190	0.127**	0.214	0.090	0.124**	0.116	0.042	0.074*
[N_{PU} = 293, N_{PR} = 100]	(0.029)	(0.040)	(0.051)	(0.032)	(0.039)	(0.050)	(0.031)	(0.026)	(0.042)
Q2	0.438	0.341	0.097*	0.385	0.276	0.108*	0.314	0.206	0.108**
[N_{PU} = 226, N_{PR} = 170]	(0.039)	(0.036)	(0.058)	(0.039)	(0.034)	(0.056)	(0.039)	(0.031)	(0.053)
Q3	0.684	0.403	0.281***	0.592	0.376	0.216***	0.510	0.285	0.225***
[N_{PU} = 206, N_{PR} = 186]	(0.036)	(0.034)	(0.053)	(0.031)	(0.033)	(0.055)	(0.041)	(0.030)	(0.054)
Q4	0.884	0.623	0.261***	0.779	0.473	0.306***	0.696	0.348	0.348***
[N_{PU} = 181, N_{PR} = 207]	(0.026)	(0.032)	(0.043)	(0.033)	(0.031)	(0.047)	(0.036)	(0.031)	(0.050)

Notes: See text for description of the estimator. Balanced panels were constructed. Bootstrap standard errors based on 250 repetitions are in parentheses.

Table A6. Downward rank mobility estimates by school type, PPVT

Panel A: 2006/07 - 2009/10

	δ = 0.00			δ = 0.05			δ = 0.10		
	Public	Private	Public - Private	Public	Private	Public - Private	Public	Private	Public - Private
Q1	0.214	0.263	-0.049	0.196	0.067	0.130	0.172	0.125	0.047
[N_{PU} = 136, N_{PR} = 21]	(0.053)	(0.057)	(0.059)	(0.052)	(0.092)	(0.119)	(0.060)	(0.114)	(0.148)
Q2	0.410	0.361	0.049	0.369	0.306	0.063	0.303	0.250	0.053
[N_{PU} = 122, N_{PR} = 36]	(0.070)	(0.078)	(0.113)	(0.073)	(0.073)	(0.112)	(0.070)	(0.075)	(0.112)
Q3	0.607	0.634	-0.027	0.564	0.488	0.076	0.538	0.439	0.099
[N_{PU} = 117, N_{PR} = 41]	0.075	(0.088)	(0.122)	(0.076)	(0.082)	(0.117)	(0.075)	(0.078)	(0.115)
Q4	0.750	0.743	0.007	0.739	0.614	0.124	0.659	0.471	0.188
[N_{PU} = 88, N_{PR} = 68]	(0.087)	(0.050)	(0.105)	(0.089)	(0.060)	(0.113)	(0.093)	(0.059)	(0.115)

Panel B: 2009/10 - 2013/14

	δ = 0.00			δ = 0.05			δ = 0.10		
	Public	Private	Public - Private	Public	Private	Public - Private	Public	Private	Public - Private
Q1	0.241	0.151	0.090*	0.175	0.107	0.069	0.098	0.096	0.002
[N_{PU} = 274, N_{PR} = 119]	(0.029)	(0.033)	(0.047)	(0.029)	(0.032)	(0.046)	(0.027)	(0.037)	(0.047)
Q2	0.473	0.358	0.115**	0.395	0.291	0.104*	0.333	0.231	0.102**
[N_{PU} = 258, N_{PR} = 134]	(0.036)	(0.041)	(0.058)	(0.033)	(0.040)	(0.055)	(0.032)	(0.039)	(0.050)
Q3	0.598	0.405	0.193***	0.550	0.362	0.188***	0.472	0.331	0.140**
[N_{PU} = 229, N_{PR} = 163]	(0.040)	(0.037)	(0.059)	(0.031)	(0.036)	(0.060)	(0.041)	(0.035)	(0.058)
Q4	0.814	0.721	0.093*	0.752	0.611	0.140**	0.614	0.518	0.096
[N_{PU} = 145, N_{PR} = 247]	(0.041)	(0.027)	(0.054)	(0.046)	(0.031)	(0.061)	(0.050)	(0.031)	(0.066)

Notes: See text for description of the estimator. Balanced panels were constructed. Bootstrap standard errors based on 250 repetitions are in parentheses.