

# A Complete Solution to LTE-U and Wi-Fi Hidden Terminal Problem

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**Abstract**—With the exponential growth in mobile data traffic, mobile operators are facing the unfortunate limit on the availability of licensed spectrum which has however, led to the popularity of Long Term Evolution (LTE) in unlicensed spectrum (LTE-U). Undeniably, it is expected from LTE-U that it fairly shares the spectrum with Wi-Fi. Along with fair sharing, efficient utilization of the unlicensed spectrum is also equally important, which in some sense requires coordination between the two Radio Access Technologies (RATs) *viz.*, LTE-U and Wi-Fi. In particular, the hidden terminal scenario between LTE-U and Wi-Fi, resulting mainly due to lack of coordination, threatens the spectrum utilization of unlicensed spectrum. Focusing on this hidden terminal problem between LTE-U and Wi-Fi, we highlight the deficiency of existing technologies from the Wi-Fi perspective, both at the user level and at the network level. We then propose a novel coexistence technique (similar to RTS-CTS mechanism in Wi-Fi) that solves the hidden terminal problem between LTE-U and Wi-Fi, and subsequently addresses the spectrum underutilization problem caused by hidden terminal collisions. The proposed mechanism achieves this by using a modified CTS frame of Wi-Fi. We have validated our proposed mechanism using a mathematical framework demonstrating its credibility.

**Index Terms**—IEEE 802.11 (Wi-Fi), LTE in Unlicensed (LTE-U), Hidden terminal problem, Self-CTS.

## I. INTRODUCTION

As the mobile Internet traffic grows aggressively, with a compound annual growth rate of 46% between 2016 and 2021, and reaching 48.3 EB per month by 2021 [2], there is a need for the telecommunication industry to boost their mobile network capacity to meet the upsurging traffic demand. This has led to the mobile operators propose their LTE deployment in the unlicensed spectrum. However, with Wi-Fi being one of the traditional, wide-spread, and far reaching technology in the unlicensed band, it necessitates the LTE to ensure fair coexistence with Wi-Fi networks, before any such deployment.

One such scheme is proposed by LTE-U forum [3], where LTE follows a discontinuous transmission—*e.g.*, Carrier Sense Adaptive Transmission (CSAT) [4], [5]—allowing the LTE-U eNB to transmit for some duration in a given duty cycle period (termed as the ON period) and mute its transmission for the rest of the duty cycle period (termed as OFF period). In [3], [4] the effectiveness of this ON-OFF transmission scheme is justified to be fairly coexisting with Wi-Fi. However, in most of the studies, the Wi-Fi performance is analyzed from a network perspective, and not from the users viewpoint. In

this paper, we focus on analyzing one of the major concerns arising in Wi-Fi network, when it is in the influence of LTE-U transmission, namely the hidden terminal problem. This problem, although merely conceivable from a network perspective, is highly disturbing with regards to individual Wi-Fi user performance.

The Wi-Fi network follows carrier sense multiple access with collision avoidance protocol to access the shared unlicensed channel. Each Wi-Fi device performs clear channel assessment and senses the channel as busy if it detects any Wi-Fi transmission exceeding the Carrier Sense Threshold (CST) (*i.e.*, -82 dBm), or any non Wi-Fi transmission exceeding the Energy Detection Threshold (EDT) (*i.e.*, -62 dBm). We refer to such Wi-Fi operation as Standard Wi-Fi (SW) operation. Based on these two threshold values EDT and CST, the configurations of an LTE-U and Wi-Fi pair operating on the same unlicensed channel can be classified into three scenarios as shown in Fig. 1.

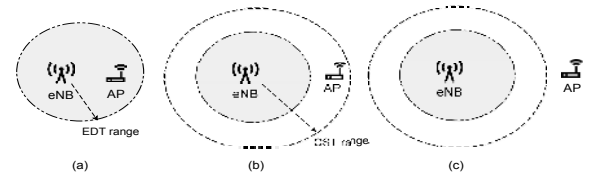


Fig. 1: An illustration of possible Wi-Fi AP positions with respect to LTE-U eNB, depicting the three scenarios: (a) Inside EDT range, (b) In-between EDT and CST range, (c) Outside CST range.

**a) Inside EDT:** In this scenario, the distance between Wi-Fi AP and LTE-U eNB is such that the Wi-Fi AP receives the LTE-U transmissions with a signal strength higher than EDT and these LTE-U transmissions will not allow the Wi-Fi AP to transmit simultaneously on the channel shared with LTE-U network.

**b) In-between EDT and CST:** In this scenario, the distance between Wi-Fi AP and LTE-U eNB is such that the Wi-Fi AP receives the LTE-U transmission with a signal strength lower than EDT but higher than CST. Here, as the LTE-U transmission is clearly a non-Wi-Fi signal, a signal strength lower than EDT implies that Wi-Fi AP can simultaneously transmit to some of its users even when LTE-U is ON. In this paper, the terms “in-between EDT and CST scenario” and “in-between scenario” are used interchangeably.

**c) Outside CST:** In this scenario, the distance between Wi-Fi AP and LTE-U eNB is such that LTE-U transmissions

received by the Wi-Fi AP are lower than CST. This inherently allows for simultaneous Wi-Fi transmissions. We refer to this scenario as “outside scenario”.

Note that in the latter two scenarios (*i.e.*, “in-between EDT and CST” and “outside CST”), the ability of Wi-Fi AP to transmit simultaneously causes non-uniformity among the Wi-Fi users. Although, AP can transmit to all of its users during the LTE-U ON period, those users who receive heavy LTE-U interference cannot decode Wi-Fi packets successfully. These users are referred to as victim users. On the other hand, remaining Wi-Fi users (or the non-victim users) do not face any such inter-RAT interference and hence achieve a higher throughput compared to the victim users, causing a non-uniformity in the achieved performance among users in the Wi-Fi network.

In all these scenarios, with two networks sharing the same unlicensed channel, coordination techniques are needed to assist in coexistence. In case of traditional homogeneous sharing among LTE networks and Wi-Fi networks, inter-cell interference coordination and RTS/CTS are used to help coordinate in hidden/expose terminal problems, respectively. In [6], the authors proposed employing point coordination function of Wi-Fi to achieve coordination, relying on the presence of a centralized controller. Since centralized approach hinders the scalability of the solution, a distributed mechanism for inter-RAT coexistence, in particular for LTE-U and Wi-Fi, needs to be explored.

In [7] and [8], the authors proposed to send a regular Self-CTS frame from LTE-U eNB (referred as LTE-U CTS or LCTS). If LTE-U eNB transmits a Self-CTS frame then AP would halt its transmission for the NAV (Network Allocation Vector) duration specified in the Self-CTS frame. This can achieve noteworthy performance gains for *inside EDT* and *in-between EDT and CST* scenarios as Wi-Fi can listen to Self-CTS frame and halts its transmissions for NAV duration. This solution does not completely solve the problem where Wi-Fi AP fails to receive the Self-CTS (*i.e.*, when Wi-Fi AP is outside the CST range). In this scenario, the performance of Wi-Fi network degrades as AP cannot receive Self-CTS frame transmitted by LTE-U eNB. Further, [8] proposed to transmit Self-CTS frame from an LTE UE instead of eNB (referred as UE-CTS). However, with the Self-CTS being the same as the one used by any other Wi-Fi node, the Wi-Fi AP cannot distinguish this from the CTS coming from another Wi-Fi node, thus limiting the efficient utilization of the unlicensed spectrum.

In this work, we propose a novel decentralized inter-RAT coexistence mechanism for LTE-U And Wi-Fi (LAW) providing exceptional throughput gains in unlicensed spectrum and solving the very fundamental issue of hidden terminal problem between LTE-U and Wi-Fi. Our approach is essentially build on the Self-CTS or CTS-to-Self mechanism of Wi-Fi with additional modification, but performed only using the available reserved fields in the CTS frame, thus facilitating easy implementation. The main contributions of this paper are summarized as follows:

- Proposed the categorization of Wi-Fi networks influenced by the LTE-U transmissions into three categories, and showed that the performance of the Wi-Fi network depends on the category to which the Wi-Fi AP belongs to.
- Highlighted the shortcomings of SW and LCTS in solving inter-RAT hidden terminal problem between LTE-U and Wi-Fi.
- Proposed a novel scheme, called LAW, that improves the spectral efficiency of Wi-Fi network by adopting a simple intelligence in scheduling, and by incorporating a feedback element that maintains fairness among Wi-Fi users with regards to the throughput achieved.
- Proposed a modified Self-CTS frame using the ‘reserved fields’ in existing CTS frame for achieving coordination between LTE-U and Wi-Fi networks in an entirely decentralized manner.

To the best of our knowledge this is the first work that addresses the hidden terminal problem between LTE-U and Wi-Fi networks in distributed manner comprehensively. In [1], a preliminary version of the proposed LAW scheme is presented. Compared to [1], in this paper, we have provided an analytical framework to analyze the throughput of LAW scheme. The analytical results are also validated with the simulation results. In addition, we have studied the proposed LAW scheme in detail like back-off results are collected to validate the throughput results. Finally, to quantify the percentage gain on an average over other baseline algorithms the proposed LAW scheme is studied in a generalized way with random placement of Wi-Fi users. The rest of the paper is organized as follows. Section II presents a motivational example describing the hidden terminal problem between LTE-U and Wi-Fi networks while Section III describes the related work. System model and problem formulation are discussed in Section IV. Section V and Section VI delineate on the proposed scheme and its analysis. Finally, Section VII and Section VIII contain experimental results and conclusions, respectively.

## II. INTER-RAT HIDDEN TERMINAL PROBLEM

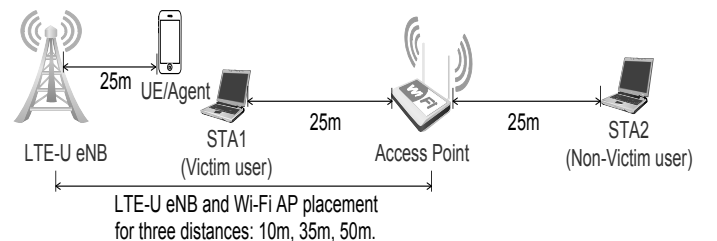


Fig. 2: Hidden terminal scenarios emulated by varying the distance between LTE-U eNB and Wi-Fi AP to 10 m, 35 m, and 50 m produce inside EDT, in-between EDT and CST, and outside CST scenarios, respectively. While the LTE-U users are placed at an arbitrary distance, Wi-Fi stations (STA1, STA2) are placed at 25 m from the Wi-Fi AP, on the either sides. Further, LTE-U is configured to follow a 50% duty cycle with a duty cycle period of 10 ms.

In the literature, a lot of importance is given to address hidden terminal problem in Wi-Fi networks. Similarly, we believe inter-RAT (LTE-U and Wi-Fi) hidden terminal problem

is equally important as we embark into 5G era with ultra-dense deployment of small cells in indoor environments like the way we currently see Wi-Fi deployment. To highlight the impact of inter-RAT hidden terminal problem on Wi-Fi network in the presence of LTE-U transmissions, in this section, we study the performance of a Wi-Fi network in each of the three scenarios described in the previous section. The placement shown in Fig 2, renders STA1 as a victim user and STA2 as a non-victim user similar to [6]. A full buffer traffic scenario is considered for the Wi-Fi network, implying that for (a) *Downlink (DL) only traffic scenario*, the Wi-Fi AP always has data to transmit; and for (b) *Uplink (UL) and DL traffic scenario*, the Wi-Fi users always have data to transmit. Similarly, a full buffer scenario is considered for the LTE-U network, where unlicensed spectrum is used only for transmitting DL traffic. As for the scheduling in the two networks, a Round-robin scheduler is employed for the LTE-U while the DCF mechanism is used in the Wi-Fi network. The simulation parameters used are as shown in Table I. In the following, we present the effect of inter-RAT hidden terminal problem on Wi-Fi network in both DL only and UL+ DL traffic cases.

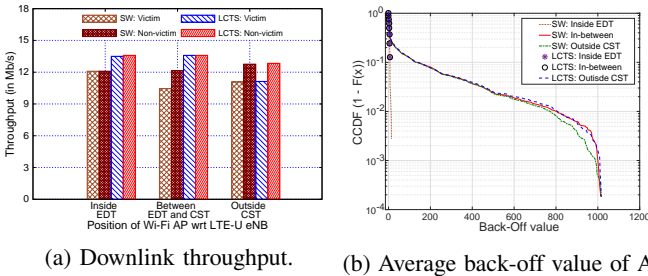


Fig. 3: DL throughput and back-off variation of Wi-Fi AP in DL only scenario.

1) *DL only traffic case*: Fig. 3a shows the DL throughput of victim and non-victim users in all the three scenarios. In the *inside EDT scenario*, Wi-Fi AP is able to detect the LTE-U transmissions in both the schemes (SW and LCTS), and consequently halts its transmissions during the LTE-U ON period. This eliminates any unfairness among the Wi-Fi users as the principle reason for the unfairness is the transmission of packets to some users during the LTE-U ON period. Nonetheless, the performance of LCTS scheme is better than that of SW. In SW case, the Wi-Fi frame transmitted just before the beginning of every LTE-U ON period is very likely to incur a collision. However, the inside EDT scenario is not of much interest in this paper as there is no hidden terminal issue.

In the *in-between EDT and CST scenario*, we see that an additional problem arises in SW; namely, the hidden terminal problem. The Wi-Fi AP being outside the EDT zone of LTE-U becomes completely unaware of two things: the LTE-U transmissions and the presence of a victim user. This makes Wi-Fi AP to involuntarily transmit to the victim user even during the LTE-U ON period. All of these transmissions simply result in packet losses which are followed by futile re-transmissions until either the re-transmission limit is reached or the LTE-U ON period expires. Further, since each loss manifests itself as an increase in average wait-time for the next packet (due to increase in back-off duration), the SW also suffers from a

performance degradation for both of its users. However, in the LCTS scheme with the use of Self-CTS frame, the Wi-Fi AP can refrain from transmitting during the LTE-U ON period. This helps in avoiding losses and consequently maintaining fairness among the users, but at the cost of completely halting during the LTE-U ON period.

In the *outside CST scenario*, both unfairness and a degradation in performance are observed for both the schemes (SW and LCTS), similar to *in-between EDT and CST scenario* described above. Once the Wi-Fi AP is outside the CST range of LTE-U eNB, it will fail to decode Self-CTS frames transmitted by the LTE-U eNB. This engenders the affects discussed above in the SW to appear in the LCTS scheme as well and thereby producing unfairness among the users and an overall performance degradation.

To further justify the above observations, the Complementary Cumulative Distribution Function (CCDF) of all the back-off values used by the Wi-Fi AP during the simulation period is plotted in Fig. 3b, for both the schemes (SW and LCTS). We can notice that there is a huge increase in back-off values for *in-between* and *outside CST scenarios* for SW and *outside CST scenario* for LCTS.

2) *UL+DL traffic case*: In this section, we study the performance of the Wi-Fi network with mixed UL and DL traffic. With the introduction of UL traffic in the Wi-Fi network, a new problem appears—imbalance between the UL and DL throughputs—as illustrated in Fig. 4a. To understand this imbalance, two major effects need to be considered.

- Firstly, the fact that AP unintelligently try serving even the victim users during the LTE-U ON period. Clearly, these transmissions cause an exponential growth of its contention window, and hence, the channel access opportunity of the AP in the LTE-U ON period reduces immensely.
- Secondly, as a consequence of the first effect, *the Wi-Fi AP is left in no competition with the non-victim user in regards to accessing the channel*. This is because once the channel access of Wi-Fi AP reduces, the non-victim user accesses the channel more frequently making the channel busier. But the AP, to decrease its high back-off values, needs the medium to be free. This further reduces the access opportunity of the AP multi-folds, and is particularly seen in the presence of UL traffic.

First, let us look at the two schemes (SW and LCTS) in the *in-between EDT and CST scenario*. As for the SW, the Wi-Fi AP being unaware of the LTE-U transmissions and the non-victim user having a full buffer UL traffic, it causes the Wi-Fi network to succumb to both the effects discussed above. As a result, its DL throughput is compromised during the LTE-U ON period, while its UL throughput is not hindered in any of the LTE-U ON and OFF periods. However, one might claim that the AP being like any other station in the network, it should contribute to one third of the total throughput (with three users in the network), which is what seems to happen. This point is made clear from Fig. 4b. As can be seen, most

Wi-Fi parameters		Common Parameters	
Parameter	Value	Parameter	Value
$CW_{min}, CW_{max}$	16, 1024	Transmission Power	20 dBm
PHY, MAC Header	128, 272 bits	Operating Frequency	5.3 GHz
ACK, RTS	240, 288 bits	Noise	-101 dBm
Payload, MPDU	8148 bits, 4	Bandwidth	20 MHz
Slottime, CTSimeout	9, 50 $\mu$ s	Antenna Ht.	10 meter
DIFS, SIFS	34, 16 $\mu$ s	User Antenna Ht.	1 meter
Beacon Interval, $\alpha$	100 ms, 0.5	Simulation Time	10 sec
Parameter	Value		
Wi-Fi PHY Rates (Mbps)	13, 26, 39, 52, 78, 104, 117, 130		
Required SNR (in dB)	5, 7, 9, 13, 17, 20, 22, 23		
Traffic	Full buffer via saturated UDP flows		
Channel	No shadow/Rayleigh fading		
Path Loss Model [9]	36.7log <sub>10</sub> (d[m])+22.7+26log <sub>10</sub> (freq[GHz])		

TABLE I: Wi-Fi &amp; LTE-U parameters

of the UL throughput comes from the non-victim user. The AP being no different from the non-victim user, it should have achieved similar throughput. If the victim user was to get similar throughput, the DL throughput would have been minimal compared to the total. This shows imbalance between the DL and UL throughputs.

And as for the LCTS scheme, the Wi-Fi AP being aware of the LTE-U transmissions because of Self-CTS frame, able to avoid above to reasons. but it is conservative in nature *i.e.*, not allowing simultaneous transmission of LTE-U eNB and Wi-Fi AP whenever possible causes the imbalance. The Wi-Fi AP completely avoids any transmissions in the LTE-U ON duration while it could serve the non-victim users and hence gets reduced throughput.

In the *outside CST scenario*, these effects are heavily intensified and both the schemes, SW and LCTS, perform almost the same. Both the effects highlighted above are clearly visible in terms of difference between the UL and DL contribution. Moreover, the share of the non-victim is much higher than that of the victim. Figs. 4a and 4b confirm the claim. Note that in this scenario, the UE-CTS scheme that was discussed earlier would perform the same way LCTS performed in in-between EDT and CST scenario. The AP would transmit very conservatively, avoiding all the simultaneous transmissions (while it is not required when being at such a far distance), and thus incurs imbalance between the UL and DL throughputs. Fig. 4c further confirms the above claims with CCDF of the back-off values used by the Wi-Fi AP for all the three scenarios.

### III. RELATED WORK

LTE in unlicensed spectrum has opened many research problems especially relating to the efficient utilization of the unlicensed spectrum and fair coexistence between LTE-U and Wi-Fi networks. With the Wi-Fi being one of the wide-spread consumer of the unlicensed bands; [10], [11], [12], [13] made it clear that modifications needs to be made in the always ON approach of LTE, to prevent the degradation in throughput performance of a neighboring Wi-Fi. For this reason, several modifications have been proposed in literature.

One of the approach proposed in [3],[4], is to follow a discontinuous transmission pattern termed as LTE-U, which

is claimed to fairly coexist with the Wi-Fi network. In [14], a muting technique was introduced within LTE, and in [15] authors suggested a modified almost blank subframe approach in LTE for fair coexistence with Wi-Fi. [16] has proposed a centralized optimization technique establishing inter-network coordination and dynamic spectrum management; [17], [18] have developed resource sharing and off-loading techniques for better coexistence. However, the definition of fairness, especially for hidden terminal problems, from the perspective of the users has received very less attention.

Nonetheless, in [19], the authors studied the above considered hidden terminal scenario using real equipments and showed the effect of LTE-U on Wi-Fi in terms of throughput performance and the proportion of beacon loss. To solve this, in [6], the authors proposed a centralized approach with the help of an inter-RAT controller, employing Point Coordination Function (PCF) of Wi-Fi to intelligently serve victim and non-victim users, and hence achieves fairness in the Wi-Fi network. With the use of PCF being optional in IEEE 802.11 standards and the centralized mechanism necessitating synchronization between LTE-U and Wi-Fi, it could be difficult to deploy [6] in real systems. Hence a simple technique, both in its functionality and the ease of adaptability (like RTS/CTS mechanism of Wi-Fi) needs to be introduced to solve this hidden terminal problem between LTE-U and Wi-Fi networks.

### IV. SYSTEM MODEL AND PROBLEM DESCRIPTION

We analyze the scenarios where LTE-U eNB and Wi-Fi AP, either from the same or different operators, coexist on the same unlicensed channel with the same amount of bandwidth available to both the networks. LTE-U follows a duty cycle based scheme to fairly coexistence with Wi-Fi, with the fraction of time LTE-U uses for its transmissions ( $\eta$ ) in a duty cycle period either predetermined or adaptively adjusted [20], [3]. Further, we make an assumption that the Wi-Fi interface of LTE-U users remains ON, especially when they are in coverage range of any Wi-Fi AP. Fig. 2 is a good demonstration of the considered scenario.

Two major concerns constitute our problem description, namely, the throughput unfairness among the Wi-Fi users and the overall throughput degradation in the Wi-Fi network. Note that, the usage of Self-CTS in LTE-U eNB (LCTS) [7], [8] helps to solve the problem to some extent by reducing

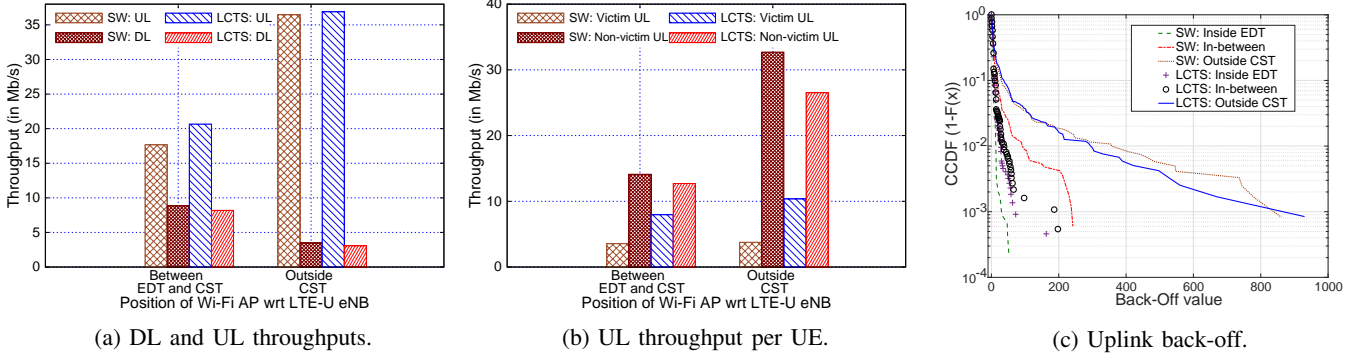


Fig. 4: Throughput results and back-off value in UL + DL scenario.

collisions, but fails to guarantee this in all scenarios (Fig. 3a), especially when the AP is in the *outside scenario* (as AP cannot receive the Self-CTS frame in the outside CST scenario). Hence, we propose a decentralized inter-RAT coordination mechanism, LAW, requiring minimal changes to LTE-U and Wi-Fi and simultaneously solving the above issues, especially in the *in-between and outside scenarios*. In principle, the proposed LAW mechanism solves the hidden terminal problem in a heterogeneous setting of LTE-U and Wi-Fi. Moreover, while achieving its main objective, it also equips the Wi-Fi AP with an elementary but powerful functionality of being able to schedule its users, to some extent.

## V. PROPOSED LAW MECHANISM

A careful insight yields that the essence of the inter-RAT hidden terminal problem is the lack of communication/coordination among the two RATs (LTE-U and Wi-Fi), leading to collisions and poor utilization of the channel. This would encourage one to propose mechanisms employing coordination between LTE-U and Wi-Fi using an inter-RAT controller or a logical interface. However, this solution would work only when both LTE-U and Wi-Fi networks belong to the same network operator which will hinder the scalability of the solution. Hence, the proposed solution is designed by using the already available Self-CTS frame which makes it scalable and operator independent.

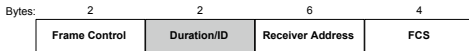


Fig. 5: Self-CTS frame format.

The Self-CTS frame format is as shown in Fig. 5. One of the important fields within the Self-CTS is the duration/ID field. This field is used to set the Network Allocation Vector (NAV) value, which informs the duration for which an upcoming Wi-Fi transmission will last. The LAW mechanism uses two values from the set of reserved values within the duration/ID field—32769 and 32770—to indicate the start of an LTE-U ON and OFF period, respectively, as shown in Table II. This is the only modification proposed within the existing Self-CTS frame and since it uses the reserved fields, it can be differentiated from the Self-CTS transmitted by the Wi-Fi network.

Bits 0-13	Bit 14	Bit 15	Usage
0-32767		0	Duration value ( $\mu$ s) for all frames except PS-poll frame
0	0	1	Fixed value in PCF during Contention Free Period
<b>1</b>	<b>0</b>	<b>1</b>	<b>When LTE-U is ON</b>
<b>2</b>	<b>0</b>	<b>1</b>	<b>When LTE-U is OFF</b>
3-16383	0	1	Reserved
0	1	1	Reserved
1-2007	1	1	Association ID in PS-Poll frames
2008-16383	1	1	Reserved

TABLE II: Duration/ID field encoding [21] with **suggested modification** for Self-CTS frame. The field is of 16 bits but not all the entries are used, or in other words it has many reserved values.

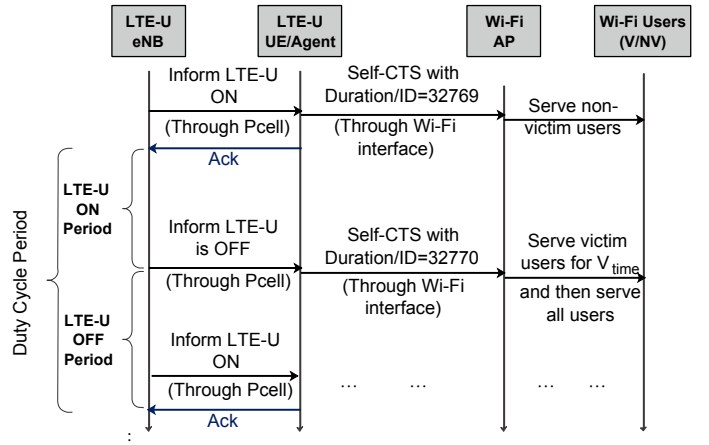


Fig. 6: Flow of events in the proposed LAW mechanism.

**Operation of the proposed LAW Mechanism:** The flow of events is shown in Fig. 6. The first objective is to inform the Wi-Fi of the upcoming ON or OFF period of LTE-U. To achieve this, LTE-U eNB selects one of its users as an LTE-U agent and instructs that user over licensed channel to transmit a Self-CTS frame using its Wi-Fi interface. The LTE-U eNB also informs the LTE-U agent of the value it needs to embed in the reserved fields, as discussed earlier. On receiving this command, the LTE-U agent uses its Wi-Fi interface to transmit this self-CTS frame. Further, in order to minimize the signaling overhead, instead of sending a Self-CTS frame every duty cycle period, the LTE-U eNB can inform the agent to transmit a Self-CTS frame only when there is a change in the LTE-U

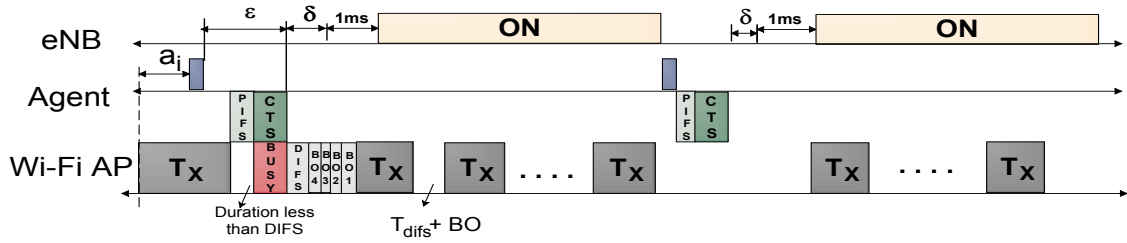


Fig. 7: The complete illustration of LAW scheme showing the coordination between LTE-U and Wi-Fi through agent (*i.e.*, LTE-U UE).

duty cycle ( $\eta$ ). In addition, to prioritize the transmission of this Self-CTS into the Wi-Fi network, we make the LTE-U agent wait for only the PCF Interframe Space (PIFS) interval before transmitting the Self-CTS frame as shown in Fig. 7 (more details are presented in Section VI). It is followed by an acknowledgment is sent by the LTE-U agent to the LTE-U eNB through the licensed channel.

When Wi-Fi AP receives this Self-CTS frame, based on the value embedded within the duration/ID field, it operates as follows: The value of 32769 in duration/ID field implies that the LTE-U is ON and so the Wi-Fi AP will serve only the non-victim users until it receives another Self-CTS frame with the duration/ID field as 32770. Now, the Wi-Fi AP first serves the disadvantaged victim users up to a predetermined time. After this, if the LTE-U OFF period still remains, it follows the regular Wi-Fi behavior and serves all the users with equal priority.

**1) Time to serve victim users ( $V_{time}$ ):** If the entire LTE-U OFF period is used to serve the victim users, then the victim users can gain undue advantage. To avoid this, we are finding  $V_{time}$  within the LTE-U OFF duration, that is required for ensuring throughput fairness among the Wi-Fi users. Variation in  $V_{time}$  can give control on how much perquisite the victim users have over the non-victim. When  $V_{time}$  is made to zero, the performance of LAW mechanism degenerates to legacy DCF mechanism of Wi-Fi. Any value of  $V_{time}$  between 0 and maximum *i.e.*, till the end of LTE-U OFF period will give better throughput for the victim users (if present) as well for the entire Wi-Fi network. One approach to realizing optimum  $V_{time}$  duration is to adjust  $V_{time}$  by observing the throughputs of Wi-Fi users in the previous duty cycle periods. The updated duration ( $V_{time}$ ) in terms of previous ( $V_{time}^{old}$ ) durations is expressed using weighted moving average as follows

$$V_{time} = \min\left(\frac{R_{nv}^{new}}{R_v^{new}} \cdot V_{time}^{old}, \text{LTE-U\_OFF\_Period}\right) \quad (1)$$

$$R_v^{new} = (1 - \alpha)R_v^{curr} + \alpha R_v^{old} \quad \text{and} \quad (2)$$

$$R_{nv}^{new} = (1 - \alpha)R_{nv}^{curr} + \alpha R_{nv}^{old}$$

Where  $R_v^{curr}$  and  $R_{nv}^{curr}$  are average instantaneous throughputs,  $R_v^{old}$  and  $R_{nv}^{old}$  are the past throughputs of victim and non-victim users, respectively and  $\alpha$  is a smoothing parameter. A complete illustration of adaptive learning of LTE-U ON-OFF durations in CSAT and accordingly learning of  $V_{time}$  in our proposed LAW scheme is shown in Fig. 8. An LTE-U eNB senses the unlicensed channel and sets its LTE-U ON duration in such a way that it will be fair with Wi-Fi operating on

the same unlicensed channel. The change in LTE-U ON-OFF durations affects throughputs of victim and non-victim users which in-turn changes  $V_{time}$ . Thus, based on the throughputs of victim and non-victim users, our proposed LAW mechanism is determining  $V_{time}$  (*i.e.*, *time to serve victim users*) in such a way that throughput fairness among Wi-Fi users is achieved.

**Why  $R_{nv}/R_v * V_{time}$  can be efficient?** We provide the following intuitive explanation in lieu of Eqn .1. If the average throughput of victim users is lower than that of non-victim users, then the value of  $V_{time}$  should proportionally be increased and vice-versa. In essence, if the ratio of average throughput of non-victim users to average throughput of victim users is more than one, we seek to increase  $V_{time}$ , and similarly if the ratio is less than one, we intend to decrease  $V_{time}$ . To achieve this, we multiply the above ratio (*i.e.*,  $\frac{R_{nv}}{R_v}$ ) by the previous  $V_{time}$ . This ensures that we get the desired variation while incorporating a feedback mechanism. In addition, its simplicity can be easily incorporated into practical systems. As for the smoothing parameter  $\alpha$ , the main intention is to employ the history to ensure throughput fairness in long term.

**2) Which LTE-U UE to choose to send Self-CTS frame?** The rationale behind this selection is that the chosen agent *i.e.*, LTE-U UE must be the closest to the effected Wi-Fi AP. This can be achieved as follows. Once an LTE-U UE comes within the coverage area of an Wi-Fi AP, it starts to receive beacons (with SSID information) transmitted by the Wi-Fi AP through its Wi-Fi interface. This UE can extract the SSID and transmit this information to the eNB, making the eNB aware of all the neighboring Wi-Fi networks. The eNB then chooses the UE with highest Received Signal Strength Indicator (RSSI) of the received beacon for each Wi-Fi network, as an LTE-U agent.

**3) How Wi-Fi AP knows which are victim and non-victim users?** When an LTE-U operates in a scenario similar to Fig. 2, then a victim user can receive Wi-Fi frames only during the LTE-U OFF period while a non-victim user can receive packets uninterrupted (both during the LTE-U OFF and ON periods). Once the Wi-Fi AP detects the presence of an LTE-U network, it starts to closely monitor the throughput variation of its users. The users achieving positive throughput continuously are classified as non-victim users while on the other hand those users which receive positive throughput for a specific duration and zero or near zero throughput for a specific duration; and then repeats this behavior, are classified as victim users.

**4) What happens if the process of transmitting Self-CTS fails?** A Self-CTS frame is sent right after sensing the

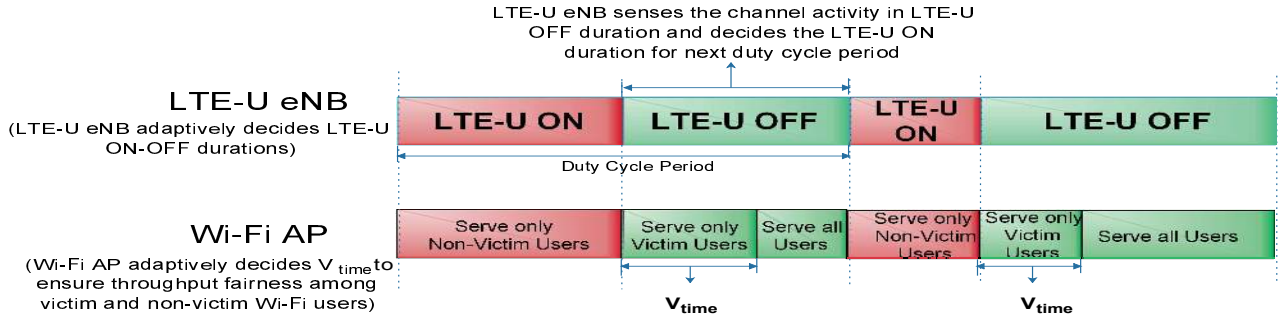


Fig. 8: Adaptive learning of LTE-U ON and LTE-U OFF durations in CSAT scheme and correspondingly learning of  $V_{time}$  in the proposed LAW mechanism.

channel idle for PIFS duration which ensures that it occupies the channel earlier than any other Wi-Fi transmission. This reduces the chances of losing this frame. In any case, if the process still fails the proposed scheme behaves as follows;

If *Self-CTS with LTE-U ON message fails*, the Wi-Fi network would still consider LTE-U to be OFF though LTE-U eNB is now in LTE-U ON period and would serve all of its users. This would simply result in the performance of LAW to be the same as the DCF scheme during this LTE-U ON period.

If *Self-CTS with LTE-U OFF message fails*, the Wi-Fi network assumes LTE-U to be still ON and serves only the non-victim users, while the LTE-U is OFF. This would result in an unnecessary increase in the throughput of non-victim users which the LAW mechanism takes into account by increasing  $V_{time}$  in subsequent duty cycles, eventually matching the average throughput of its victim and non-victim users. Therefore, performance of the LAW mechanism still outperforms the standard Wi-Fi even when Self-CTS message fails.

**5) How about the inter-RAT interference caused by the Wi-Fi AP on the LTE-U users?** The inter-RAT interference caused by the Wi-Fi AP to the LTE-U users is ignored because of the following two reasons. Firstly, LTE-U having access to both the licensed and unlicensed spectrum can serve its victim users using the licensed spectrum and can provide more resources to its non-victim users in the unlicensed spectrum. This clearly is not possible for the Wi-Fi network as it lacks any licensed anchor. Secondly, LTE-U has the ability to control its duty cycle period which it can manipulate (like increase/decrease the LTE-U ON fraction) as long as it is fair with Wi-Fi. Moreover, since the LAW mechanism improves the performance of the Wi-Fi network, LTE-U can find some room to increase its duty-cycle, in-turn leading to a win-win situation, for both LTE-U and Wi-Fi. Therefore, we have focused only on classifying the Wi-Fi users and addressing their problem in coexistence scenarios in this work.

## VI. THROUGHPUT ANALYSIS OF LAW MECHANISM

To analyze the throughput of the proposed LAW mechanism, we use the approach similar to [22]. We assume Wi-Fi network behavior to be a process comprising busy and idle states, with busy state marked by an ongoing transmission and idle state involving the DIFS and random back-off (BO). Further, we define the following random variables.

$B_k$  - Duration of  $k^{th}$  busy period

$I_k$  - Random duration of  $k^{th}$  idle period

$Q_k$  - Random number of back-offs during the period  $I_k$

The aim of this analysis is to establish a mathematical support for all the previous discussion on the proposed LAW mechanism. Since Wi-Fi involves excessive randomness, we provide here the analysis of the DL only scenario, demonstrating the supremacy of the proposed LAW mechanism without being lost into the very subtle details of the Wi-Fi. Therefore, as a DL only scenario involves transmission of packets only from the Wi-Fi AP, no collisions occur in the network, and hence the contention window remains at its minimum size. This transmission duration, assuming that all the packets have same payload size, will be the sum of time for transmitting payload and ack ( $t_{payload} + t_{ack}$ ), and also the SIFS duration in between ( $t_{sifs}$ )  $\implies P(B_K = t_{payload} + t_{sifs} + t_{ack}) = 1$ . As the back-off value follows a uniform distribution,  $P(Q_K = q_k) = 1/W$ , for  $0 \leq q_k < W$ . Since the Self-CTS commands from LTE-U agent to Wi-Fi AP can arrive in either of the two periods (busy or idle), to incorporate the randomness caused due to their random arrivals, we define a few more variables below:

$S_i$  - Random variable describing the arrival of Self-CTS –  $S_i \in \{B, I\}$

$N_i$  - Random variable denoting the renewal period

$A_i$  - Random offset inside  $i^{th}$  busy period  $C_i$  - Random offset inside  $i^{th}$  idle period

$r_i^{on}$  and  $r_i^{off}$  - remaining time from the end of busy period until the arrival of next Self-CTS command to the LTE-U agent encompassing the ON and OFF periods of LTE-U, respectively.

Since the ON duration of LTE-U ( $T_{on}$ ) and the OFF duration of LTE-U ( $T_{off}$ ) are in the multiples of subframe interval (i.e., 1ms), it can be safely assumed that at least one Wi-Fi transmission will occur in either of the ON and OFF periods. This implies that the maximum number of Wi-Fi transmissions that can take place is  $K_{max} = \lceil \frac{r_i}{t_x + t_{difs}} \rceil$ .

Let  $k \in \{1, 2, \dots, K_{max}\}$  denote the number of Wi-Fi transmissions that occur between two consecutive Self-CTS frame arriving to the LTE-U agents, then the average number of transmissions within this interval can be written as

$$E_x[k] = \sum_{s_i \in \{I, B\}} E_x[k | (S_i = s_i)] \cdot Pr(S_i = s_i) \cdot u(\lambda_{EDT} - P_r) \quad (3)$$

where  $x \in \{ON, OFF\}$ ,  $u(\cdot)$  denotes the standard unit step function,  $\lambda_{EDT}$  is the energy detection threshold, and  $P_r$  denotes the received power of the LTE-U signals at the Wi-Fi AP.

In order to evaluate Eqn. (3), we proceed as following,

$$E_x[k|(S_i = s_i)] = \sum_{s_{i+1} \in \{I, B\}} E_x[k|S_{i+1} = s_{i+1}, S_i = s_i] \cdot P[S_{i+1} = s_{i+1}|S_i = s_i] \quad (4)$$

The second term inside the summation represents the probability for the next state to be either idle or busy, given the current state is either idle or busy. We call these probability expressions as the conditional probabilities of states, and to evaluate them we need to compute the probabilities of future states (*i.e.*,  $S_{i+1}$ ) conditioned on  $S_i$  being Idle (I) or Busy (B). The Self-CTS command can arrive to agent either when Wi-Fi network is busy or idle. These two instances are given below:

1) **Instance I**:  $S_i = B$ : This means that the Self-CTS command has arrived to the agent when the Wi-Fi network was busy. If we assume  $k \in \{1, 2, \dots, K_{max}\}$  transmissions occur in the succeeding interval, from the arrival of current Self-CTS command to the arrival of the next Self-CTS, then we can characterize  $Pr(A_{i+1} = a_{i+1}|A_i = a_i)$ . Using this, the two conditional probabilities, (*i.e.*,  $Pr(S_{i+1} = B|S_i = B)$  and  $Pr(S_{i+1} = I|S_i = B)$ ) can be easily computed. We know that if  $S_i = B$ ,

$$\begin{aligned} r_{b,i}^x &= T_x + 2 - \{t_x - a_i + T_{pifs} + T_{cts}\} \\ \implies Pr\left(\sum_{j=1}^k I_{N_i+j} = r_{b,i}^x - a_{i+1} - t_x(k-1)\right) \\ &= Pr\left(\sum_{j=1}^k Q_{N_i+j} = q_b^x\right) \end{aligned} \quad (5)$$

where  $q_b^x$  is given by

$$q_b^x = \frac{r_{b,i}^x - a_{i+1} - (k-1) \cdot t_x - k \cdot t_{difs}}{t_{bo}}$$

Since we analyze here for the DL only scenario,  $Q_k$  would be a discrete uniform random variable, and its sum distribution can be computed as described in [23]. This implies,

$$\begin{aligned} Pr\left(\sum_{j=1}^k Q_{N_i+j} = q_b^x\right) \\ = \frac{k}{(W+1)^k} \cdot \sum_{u=0}^{\lfloor q_b^x/W \rfloor} \frac{\Gamma(k + q_b^x - u \cdot W)(-1)^u}{\Gamma(u+1)\Gamma(k-u+1)\Gamma(q_b^x - u \cdot W + 1)} \end{aligned} \quad (6)$$

$$\implies Pr(A_{i+1} = a_{i+1}|A_i = a_i) = \sum_{k=1}^{K_{max}} Pr\left(\sum_{j=1}^k Q_{N_i+j} = q_b^x\right) \quad (7)$$

where  $\Gamma$  is a Gamma function for integer arguments. For  $S_{i+1} = B$ ,  $A_{i+1}$  can take values from  $[1, t_x]$ , where  $t_x$  denotes the duration for which the channel remains busy after

a transmission starts, with  $x \in \{ON, OFF\}$ .

$$\begin{aligned} \implies Pr(S_{i+1} = B|S_i = B) &= \sum_{a_{i+1}=1}^{t_x} Pr(A_{i+1} = a_{i+1}|S_i = B) \\ &= \frac{1}{t_{\hat{x}}} \cdot \sum_{a_i=1}^{t_{\hat{x}}} \sum_{a_{i+1}=1}^{t_x} Pr(A_{i+1} = a_{i+1}|A_i = a_i) \end{aligned} \quad (8)$$

where  $\hat{x} = \begin{cases} ON & \text{if } x = OFF \\ OFF & \text{if } x = ON \end{cases}$

Similar, for  $S_{i+1} = I$ ,  $Pr(S_{i+1} = I|S_i = B) = 1 - Pr(S_{i+1} = B|S_i = B)$

2) **Instance II**:  $S_i = I$ : Now again, we follow a similar procedure. First, to evaluate  $Pr(S_{i+1} = B|S_i = I)$ , we start by finding  $r_i^x$ .

When  $S_i = I$ ,

$$\begin{aligned} r_i^x &= T_x + 2 - \{T_{pifs} + T_{cts}\} \quad \text{and} \\ q_i^x &= \frac{r_i^x - a_{i+1} - (k-1) \cdot t_x - k \cdot t_{difs}}{t_{bo}} \end{aligned} \quad (9)$$

$$\implies Pr(S_{i+1} = B|S_i = I) = \sum_{a_{i+1}=1}^{t_x} Pr(A_{i+1} = a_{i+1}|S_i = I) \quad (10)$$

with  $r_i^x$  being independent on  $i$ ,

$$Pr(A_{i+1} = a_{i+1}|S_i = I) = \sum_{k=1}^{K_{max,i}} Pr\left(\sum_{j=1}^k Q_{N_i+j} = q_i^x\right) \quad (11)$$

$$\implies Pr(S_{i+1} = B|S_i = I) = \sum_{a_{i+1}=1}^{t_x} \sum_{k=1}^{K_{max,i}} Pr\left(\sum_{j=1}^k Q_{N_i+j} = q_i^x\right) \quad (12)$$

where  $K_{max,i}$  and  $q_i^x$  uses  $r_i^x$  given by Eqn. (9). Substituting the above equation in Eqn. (10), we get a similar discussion as for Eqn. (6) leads us to a closed form expression for  $Pr\left(\sum_{j=1}^k Q_{N_i+j} = q_i^x\right)$  as

$$\begin{aligned} Pr\left(\sum_{j=1}^k Q_{N_i+j} = q_i^x\right) \\ = \frac{k}{(W+1)^k} \sum_{u=0}^{\lfloor q_i^x/W \rfloor} \frac{\Gamma(k + q_i^x - u \cdot W)(-1)^u}{\Gamma(u+1)\Gamma(k-u+1)\Gamma(q_i^x - u \cdot W + 1)} \end{aligned} \quad (13)$$

This provides us with all the equations to evaluate  $Pr(S_{i+1} = B|S_i = I)$  and  $Pr(S_{i+1} = I|S_i = I)$ .

After the probability expressions, we need to compute the expected number of transmissions in LTE-U ON/OFF periods conditioned on the current and next states. This expectation is given as

$$\begin{aligned} E[k|S_{i+1} = s_{i+1}, S_i = s_i] \\ = \sum_{s_i, s_{i+1} \in \{I, B\}} k \cdot Pr(k|S_{i+1} = s_{i+1}, S_i = s_i) \end{aligned} \quad (14)$$

But again, Eqn. (14) requires evaluating four different double conditioned probability expressions, *i.e.*,  $Pr(k|S_{i+1} = s_{i+1}, S_i = s_i)$  with different combinations of  $(s_i, s_{i+1}) \in \{I, B\}$  which are presented below.



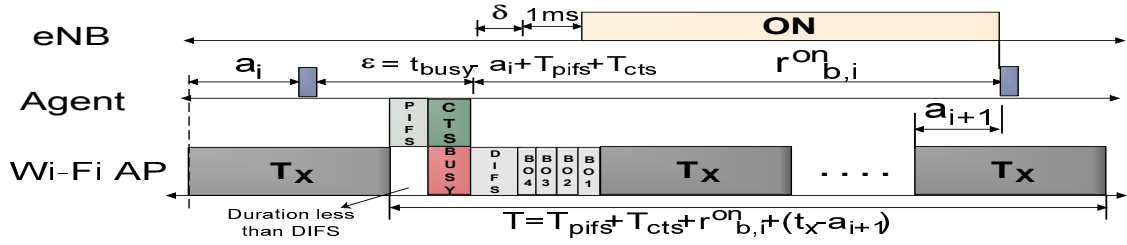


Fig. 9: Scenario depicting the arrival of the current Self-CTS in Busy period and the next Self-CTS in Busy period, i.e.,  $S_{i+1} = B, S_i = B$ .

1) **Case I** :  $S_{i+1} = B, S_i = B$ : Fig. 9 shows a pictorial representation of the case when  $s_{i+1} = B$  and  $s_i = B$ . This means that the LTE-U agent will receive the current and as well the next Self-CTS commands from LTE-U eNB when the Wi-Fi network is busy with an ongoing transmission.

Now,  $Pr(k|S_{i+1} = B, S_i = B)$  can be expressed using  $Pr(k|A_{i+1} = a_{i+1}, A_i = a_i)$  as

$$\begin{aligned} Pr(k|S_{i+1} = B, S_i = B) &= \frac{Pr(k \cap S_{i+1} = B|S_i = B)}{Pr(S_{i+1} = B|S_i = B)} \\ &= \frac{\frac{1}{t_{\hat{x}}} \sum_{a_i=1}^{t_{\hat{x}}} \sum_{a_{i+1}=1}^{t_x} Pr(k, A_{i+1} = a_{i+1}|A_i = a_i)}{Pr(S_{i+1} = B|S_i = B)} \end{aligned} \quad (15)$$

Note that

$$\begin{aligned} Pr(k, A_{i+1} = a_{i+1}|A_i = a_i) &= Pr\left(\sum_{j=1}^k Q_{N_i+j} = q_b^x\right) \quad (16) \\ \implies Pr(k|S_{i+1} = B, S_i = B) &= \frac{\frac{1}{t_{\hat{x}}} \sum_{a_i=1}^{t_{\hat{x}}} \sum_{a_{i+1}=1}^{t_x} Pr\left(\sum_{j=1}^k Q_{N_i+j} = q_b^x\right)}{Pr(S_{i+1} = B|S_i = B)} \end{aligned} \quad (17)$$

By knowing  $Pr(\sum_{j=1}^k Q_{N_i+j} = q_b^x)$  from Eqn. (6),  $Pr(k|S_{i+1} = B, S_i = B)$  can be computed easily.

2) **Case II** :  $S_{i+1} = B, S_i = I$ :

$$\begin{aligned} \implies Pr(k|S_{i+1} = B, S_i = I) &= \frac{Pr(k \cap S_{i+1} = B|S_i = I)}{Pr(S_{i+1} = B|S_i = I)} \\ &= \frac{\sum_{a_{i+1}=1}^{t_x} Pr(k, A_{i+1} = a_{i+1}|S_i = I)}{Pr(S_{i+1} = B|S_i = I)} \end{aligned} \quad (18)$$

With  $s_i = I$ ,  $r_i^x$  is given by  $r_i^x = T_x + 2 - \{T_{pifs} + T_{cts}\}$  (can be derived from Fig. 10). Since  $r_i^x$  is independent of  $i$ ,  $Pr(k, A_{i+1} = a_{i+1}|S_i = I)$  can be derived from Eqn. (11) as

$$\begin{aligned} Pr(k, A_{i+1} = a_{i+1}|S_i = I) &= Pr\left(\sum_{j=1}^k Q_{N_i+j} = q_i^x\right) \\ \implies Pr(k|S_{i+1} = B, S_i = I) &= \frac{\frac{1}{t_{\hat{x}}} \sum_{a_i=1}^{t_{\hat{x}}} \sum_{a_{i+1}=1}^{t_x} Pr\left(\sum_{j=1}^k Q_{N_i+j} = q_i^x\right)}{Pr(S_{i+1} = B|S_i = I)} \end{aligned} \quad (19)$$

3) **Case III** :  $S_{i+1} = I$  and  $S_i = B$ : Fig. 11 shows a pictorial representation of the case  $s_i = B$  and  $s_{i+1} = I$ . This allows  $Pr(k|S_{i+1} = I, S_i = B)$  to be written as

$$Pr(k|S_{i+1} = I, S_i = B) = \frac{Pr(k \cap S_{i+1} = I|S_i = B)}{Pr(S_{i+1} = I|S_i = B)} \quad (20)$$

With  $S_{i+1} = I$  and  $S_i = B$ , the expression  $Pr(k \cap S_{i+1} = I|S_i = B)$  can be expanded as

$$\begin{aligned} Pr(k \cap S_{i+1} = I|S_i = B) &= \frac{1}{t_{\hat{x}}} \sum_{a_i=1}^{t_{\hat{x}}} \sum_{c_{i+1}=1}^{t_{difs} + (W-1)t_{slot}} Pr(k, C_{i+1} = c_{i+1}|A_i = a_i) \end{aligned} \quad (21)$$

Furthermore,  $Pr(k|C_{i+1} = c_{i+1}, A_i = a_i)$  can be re-written using  $Pr(\sum_{j=1}^k Q_{N_i+j} = q_b^x)$  as in Eqn. (16), but with an additional term to accommodate the appearance of  $C_{i+1}$  as compared to the earlier  $A_{i+1}$ .

$$\begin{aligned} \implies Pr(k, C_{i+1} = c_{i+1}|A_i = a_i) &= Pr\left(\sum_{j=1}^k Q_{N_i+j} = q_b^x\right) \cdot \left(1 - \frac{u\left(\left\lceil \frac{C_{i+1} - T_{difs}}{T_{slot}} \right\rceil\right)}{W}\right) \\ \implies Pr(k|S_{i+1} = I, S_i = B) &= \frac{\frac{1}{t_{\hat{x}}} \sum_{a_i=1}^{t_{\hat{x}}} \sum_{c_{i+1}=1}^{t_{difs} + (W-1)t_{slot}} Pr\left(\sum_{j=1}^k Q_{N_i+j} = q_b^x\right)}{Pr(S_{i+1} = I|S_i = B)} \\ &\quad \left(1 - \frac{u\left(\left\lceil \frac{C_{i+1} - T_{difs}}{T_{slot}} \right\rceil\right)}{W}\right) \end{aligned} \quad (22)$$

where  $Pr(\sum_{j=1}^k Q_{N_i+j} = q_b^x)$  can be computed using Eqn. (6).

4) **Case IV** :  $S_{i+1} = I$  and  $S_i = I$ : When  $s_i = I$  and  $s_{i+1} = I$  (Fig. 12),  $Pr(k|S_{i+1} = I, S_i = I)$  can be expanded in a similar manner as

$$\begin{aligned} Pr(k|S_{i+1} = I, S_i = I) &= \frac{Pr(k \cap S_{i+1} = I|S_i = I)}{Pr(S_{i+1} = I|S_i = I)} \\ &= \frac{\sum_{c_{i+1}=1}^{t_{difs} + (W-1)t_{slot}} Pr(k, C_{i+1} = c_{i+1}|S_i = I)}{Pr(S_{i+1} = I|S_i = I)} \end{aligned} \quad (23)$$

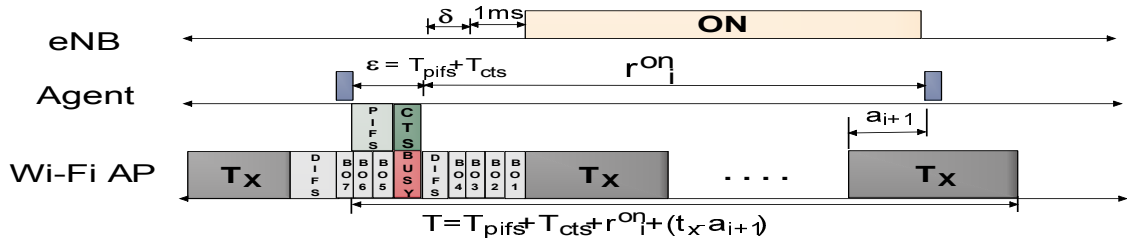


Fig. 10: Scenario depicting the arrival of the current Self-CTS in Idle period and the next Self-CTS in Busy period, i.e.,  $S_{i+1} = B, S_i = I$ .

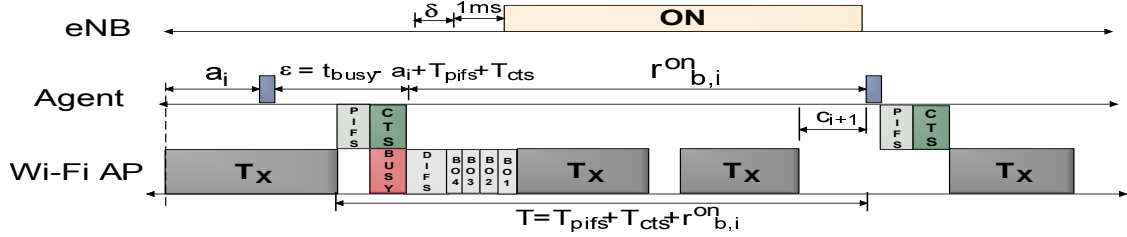


Fig. 11: Scenario depicting the arrival of the current Self-CTS in Busy period and the next Self-CTS in Idle period, i.e.,  $S_{i+1} = I, S_i = B$ .

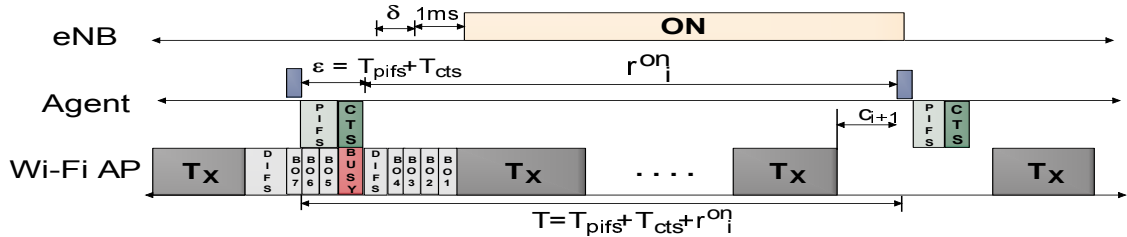


Fig. 12: Scenario depicting the arrival of the current Self-CTS in Idle period and as well the next Self-CTS in Idle period, i.e.,  $S_{i+1} = I, S_i = I$ .

Similarly, using  $Pr(k|C_{i+1} = c_{i+1}, S_i = I)$  as

$$\begin{aligned}
 & Pr(k, C_{i+1} = c_{i+1} | A_i = a_i) \\
 &= Pr\left(\sum_{j=1}^k Q_{N_i+j} = q_i^x\right) \cdot \left(1 - \frac{u\left(\left[\frac{c_{i+1} - T_{difs}}{T_{slot}}\right]\right)}{W}\right) \\
 \implies & Pr(k | S_{i+1} = I, S_i = I) = \\
 & \frac{\sum_{c_{i+1}=1}^{t_{difs}+(W-1)t_{slot}} Pr\left(\sum_{j=1}^k Q_{N_i+j} = q_i^x\right) \cdot \left(1 - \frac{u\left(\left[\frac{c_{i+1} - T_{difs}}{T_{slot}}\right]\right)}{W}\right)}{Pr(S_{i+1} = I | S_i = I)} \quad (24)
 \end{aligned}$$

Eqns. (17), (19), (22), and (24) give us the closed form expressions for all the four doubly conditioned probabilities. Now, the only expressions to be evaluated are the probabilities of individual states. One helpful observation at this point would be that the process of these states follows a Markov Process with states as Idle and Busy and the transition matrix ( $P$ ) given by

$$P = \begin{bmatrix} Pr(S_{i+1} = B | S_i = B) & Pr(S_{i+1} = I | S_i = B) \\ Pr(S_{i+1} = B | S_i = I) & Pr(S_{i+1} = I | S_i = I) \end{bmatrix} \quad (25)$$

With all the elements in the above transition matrix being non zero, the irreducibility and aperiodicity of the Markov chain can be established. In addition, using the fact that the number of states is finite, the existence of a stationary distribution can

be proved. This gives

$$\begin{aligned}
 Pr(S_i = B) &= \\
 & \frac{Pr(S_{i+1} = B | S_i = I)}{Pr(S_{i+1} = I | S_i = B) + Pr(S_{i+1} = B | S_i = I)} \quad (26)
 \end{aligned}$$

$$\begin{aligned}
 Pr(S_i = I) &= \\
 & \frac{Pr(S_{i+1} = I | S_i = B)}{Pr(S_{i+1} = I | S_i = B) + Pr(S_{i+1} = B | S_i = I)} \quad (27)
 \end{aligned}$$

Therefore, using Eqns. (17), (19), (22), and (24) the Eqn. (14) can be computed. This will finally give us the value of *expected number of transmissions* in both the ON and OFF periods ( $E_{on}[k]$  and  $E_{off}[k]$ ).

In order to evaluate throughput of the network ( $\Gamma^{PS}$ ), we evaluate the average rates during the LTE-U ON ( $R_{on}$ ) and LTE-U OFF ( $R_{off}$ ) periods as below,

$$R_{on} = \frac{E_{on}[k] * PS_{on}}{E_{on}[T]} \quad \text{and} \quad R_{off} = \frac{E_{off}[k] * PS_{off}}{E_{off}[T]}$$

where  $PS_{on}$  and  $PS_{off}$  denote the average packet sizes in the ON and OFF periods, respectively,  $E_{on}[T]$  and  $E_{off}[T]$  denote the ON and OFF periods in terms of  $T_{on}$  and  $T_{off}$ , respectively.

The average packet sizes could be computed by making a realistic assumption that the duration of transmission of a packet by the Wi-Fi AP in a particular interval  $x$  (where  $x \in \{ON, OFF\}$ ) is constant and is equal to  $B_{on}$  and  $B_{off}$  for ON and OFF periods, respectively. This assumption is very practical in scenarios where the Wi-Fi nodes transmit for a duration called 'transmission opportunity'; defined as

the maximum duration for which the channel can be used by a Wi-Fi node once the node gains access to the channel. Assume that  $j^{th}$  user of Wi-Fi uses rate  $R_j^x$  during the period  $x \in \{ON, OFF\}$  and has a probability  $p_j^x$  for a given ongoing transmission to be meant for  $j^{th}$  user (this is same as the probability that whenever Wi-Fi AP gets access to the channel, it transmits the packet belonging to  $j^{th}$  user). Now, the average packet size is given by

$$PS_x = t_x \cdot \sum_j R_j^x \cdot p_j^x$$

$$\implies R_x = \frac{E_x[k] \cdot t_x \cdot \sum_j R_j^x \cdot p_j^x}{E_x[T]} \quad (28)$$

To evaluate  $E_{on}[T]$  and  $E_{off}[T]$ , we use the same approach used above for calculating the *expected number of transmissions*.

To evaluate the duration of any period  $x$ , where  $x \in \{ON, OFF\}$ , the above expectation equations can be modified as

$$E_x[T] = \sum_{s_i \in \{I, B\}} E_x[T|(S_i = s_i)] \cdot Pr(S_i = s_i) \cdot u(EDT - P_r) \quad (29)$$

where  $E_x[T|(S_i = s_i)] = \sum_{s_{i+1} \in \{I, B\}} E_x[T|S_{i+1} = s_{i+1}, S_i = s_i] \cdot P[S_{i+1} = s_{i+1}|S_i = s_i]$  and  $E_x[T|S_{i+1} = s_{i+1}, S_i = s_i]$  for all four possibilities can be given as follows:

$$E_x[T|S_{i+1} = B, S_i = B]$$

$$= \frac{\frac{1}{t_{\hat{x}}} \sum_{a_i=1}^{t_{\hat{x}}} \sum_{a_{i+1}=1}^{t_{\hat{x}}} (T_{pifs} + T_{cts} + r_{b,i}^x + (t_x - a_{i+1}))}{Pr(S_{i+1} = B|S_i = B)} \left( Pr(A_{i+1} = a_{i+1}|A_i = a_i) \right) \quad (30)$$

$$E_x[T|S_{i+1} = I, S_i = B]$$

$$= \frac{\frac{1}{t_{\hat{x}}} \sum_{a_i=1}^{t_{\hat{x}}} \sum_{c_{i+1}=1}^{t_{difs} + (W-1)t_{slot}} (T_{pifs} + T_{cts} + r_{b,i}^x)}{Pr(S_{i+1} = I|S_i = B)} \left( Pr(C_{i+1} = c_{i+1}|A_i = a_i) \right) \quad (31)$$

where  $Pr(C_{i+1} = c_{i+1}|A_i = a_i)$

$$= \sum_{k=1}^{K_{max}} Pr(k, C_{i+1} = c_{i+1}|A_i = a_i)$$

$$E_x[T|S_{i+1} = B, S_i = I]$$

$$= \frac{\sum_{a_{i+1}=1}^{t_x} (T_{pifs} + T_{cts} + r_i^x + (t_x - a_{i+1}))}{Pr(S_{i+1} = B|S_i = I)} \left( Pr(A_{i+1} = a_{i+1}|S_i = I) \right) \quad (32)$$

$$E_x[T|S_{i+1} = I, S_i = I]$$

$$= \frac{t_{difs} + (W-1)t_{slot} \sum_{c_{i+1}=1} (T_{pifs} + T_{cts} + r_i^x)}{Pr(C_{i+1} = c_{i+1}|S_i = I)} \quad (33)$$

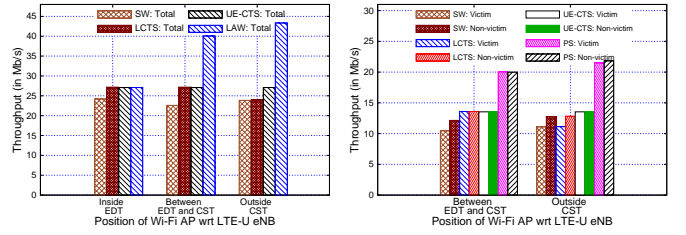
Further, the above equations (obtained through a similar mechanism used for deriving  $E_x[k|S_{i+1} = s_{i+1}, S_i = s_i]$ ) can be used to compute the above expressions. Now, the rate  $R$  averaged over the ON and OFF periods can be computed as

$$R = \frac{R_{on} \cdot E_{on}[T] + R_{off} \cdot E_{off}[T]}{E_{on}[T] + E_{off}[T]} \quad (34)$$

However, when Wi-Fi AP is inside EDT scenario and can receive the LTE-U transmissions during the LTE-U ON period, with signal strength above the energy detection threshold, Wi-Fi AP will not transmit. This causes the throughput to be zero during the LTE-U ON period and hence  $E_{on}[k] = 0$  and  $E_{off}[k] = E_{off}[k]$ .

## VII. PERFORMANCE EVALUATION

We begin by first simulating the inter-RAT hidden terminal scenario presented in Fig. 2 using a MATLAB simulator employing SW, LCTS and the proposed mechanisms, with the system model described in Section IV and the parameters tabulated in Table I. Further, we investigate the performance of LAW against SW and LCTS schemes using the same MATLAB simulator for various experiments. These experiments are differentiated by the direction of traffic flow, amount of traffic, number of users in the network, and the placement of users.

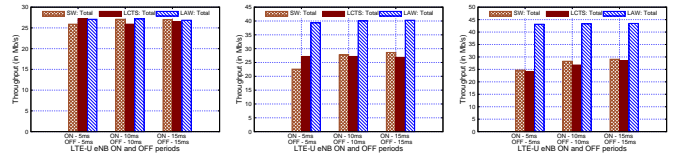


(a) Total DL throughput.

(b) Per UE DL throughput.

Fig. 13: Throughput results in Downlink only traffic for the three scenarios (i.e., inside EDT, between EDT and CST, and outside CST). LTE-U follows a 50% duty cycle with a period of 10ms, implying 5ms ON and OFF periods.

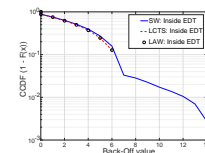
### A. Results of DL only traffic case



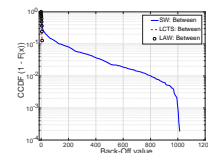
(a) Throughput inside scenario.

(b) Throughput in-between scenario.

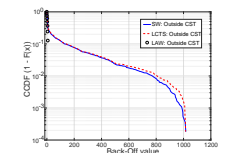
(c) Throughput outside scenario.



(d) back-off inside scenario.



(e) back-off in-between scenario.



(f) back-off outside scenario.

Fig. 14: Throughput and back-off of all three schemes.

Fig. 13a shows the improvement in total throughput by employing LAW mechanism, for the scenario presented in

Fig. 2 (discussed in Section II), in comparison with the SW and LCTS schemes. To show the improvement in all the three scenarios, we deal with them individually. In [8], the authors proposed transmitting Self-CTS from LTE-U user (UE-CTS). However, maximum gains cannot be extracted using UE-CTS scheme. This is because the AP cannot distinguish if the Self-CTS was sent from an LTE-U agent signaling an ON period or from the Wi-Fi node trying to reserve the channel by sending legacy Self-CTS frame. Irrespective of the source of Self-CTS, the Wi-Fi AP, after listening to the CTS, it will not access the channel for the NAV duration and as a result, will not transmit to any of its users. But, the proposed LAW mechanism leverages its ability to differentiate; and uses this distinction to perform simultaneous transmissions to its non-victim users. The throughput gains which the proposed scheme offers will be discussed next for each of the scenarios.

1) *Inside EDT scenario*: When the Wi-Fi AP is inside EDT of LTE-U eNB, for all the three schemes (*i.e.*, SW, LCTS, and LAW schemes), the Wi-Fi AP can detect LTE-U transmissions with signal strength higher than EDT. This will cause them to transmit only during the LTE-U OFF period. Hence, this scenario incurs no retransmission losses, nor does it allow any successful packet transmission during the LTE-U ON period (which we call as simultaneous transmission of LTE-U and Wi-Fi), thereby hindering any scope of improvement. Therefore the proposed LAW mechanism, unable to exploit any advantage, performs same as other two schemes and it is not substandard in any regard.

However, the SW appears to have a slightly declined throughput compared to the other two schemes. This decline is due to the packet losses occurring at the ON-OFF transition as it was explained earlier in the motivational results. This further can be seen from Fig. 14 where the duty cycle period of LTE-U eNB is varied as 10ms, 20ms, and 30ms with a 50% duty cycle. The foremost observation is that the throughput declination in SW scheme in Fig. 14a is only when the duty cycle period is small. This confirms transition losses. When the duty cycle period is low, *i.e.*, the ON-OFF switching frequency is high, it causes a higher number of packets to be lost at the transitions. One of the advantages of LCTS scheme (and also of the UE-CTS scheme) is that it avoids these collisions and therefore its the throughput is slightly higher compared to SW scheme for lower duty cycle periods. A similar mechanism is employed in LAW scheme thus preventing these collisions.

Furthermore, CCDF of back-off values of AP in Fig. 14d confirms the above observations. The CCDF curve of SW extends upto 14, indicating an increased contention window due to collisions, whereas, the CCDF curves of LCTS and LAW has values less than or equal to 6, thus proving the fact that packet losses at the interface are successfully prevented.

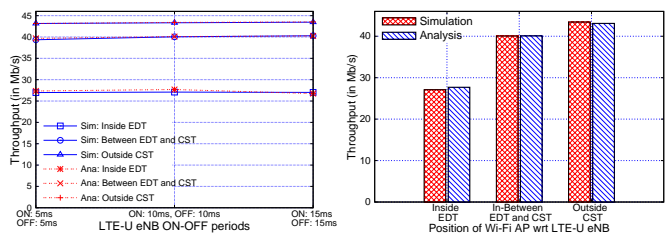
2) *In-between EDT and CST scenario*: As was explained in Section II, the poor performance of SW was due to its retransmission losses and that of LCTS was because of its conservative nature. LAW successfully avoids these by transmitting only to the non-victim users during the LTE-U ON period and later to all of its users during the LTE-

U OFF period, thereby avoiding retransmission losses and also gaining the advantage of simultaneous transmissions. The aggregated result on throughput can be observed from Fig 14b. A noteworthy observation between SW and LCTS from Fig 14b is that the throughput of SW slightly surpasses that of LCTS at higher periods. This is because, as the duty cycle period increases, the chances that the Wi-Fi AP after reaching the retransmission limit, selects a non-victim user is also increases. This results in a successful transmission within the LTE-U ON period, thus increasing the throughput of SW scheme. The Fig 14e shows the effect of such behavior on the back-off values selected by Ap in the three schemes. As SW incurs many packet losses during the LTE-U ON period, its back-off values extend all the way up to 1024. On the contrary, LCTS and LAW schemes appear to select very low back-off values, lower than the minimum contention window size, due to their successful collision avoidance. Interestingly, this benefit of lower back-off values, in the proposed LAW scheme, manifest itself into increased throughput, but remains dormant for the LCTS scheme.

3) *Outside CST scenario*: In this scenario, SW as well as LCTS cannot detect the presence of LTE-U transmissions, thus transmit to the victim users during the LTE-U ON period. As a result, abundant packet losses occur, causing immense channel wastage. On the other hand, LAW can detect the LTE-U presence using the modified Self-CTS which helps to achieve the gains. Fig. 14c shows these gains in terms of throughput for *outside CST scenario*. Furthermore, Fig. 14f shows the exponential growth of the contention window in SW and LCTS schemes, while it remains at the minimum value in the proposed LAW scheme.

Apart from all the throughput gains, LAW also ensures fairness among the Wi-Fi users for all the three scenarios. Fig. 13b shows per user throughput for all three schemes. Performance of both victim and non-victim users have drastically improved and most importantly the so called victim user is not victim anymore, having achieved performance comparable to that of non-victim user.

## B. Validation



(a) Varying duty cycle period. (b) LAW in all three scenarios.

Fig. 15: Analytical and simulation results for proposed LAW schemes in all three scenarios.

Figs. 15a and 15b show a comparison graph between the analytical and simulated throughputs obtained for the three scenarios, *i.e.*, *inside EDT*, *between EDT and CST*, and *outside EDT*. The very close match between the analytical and

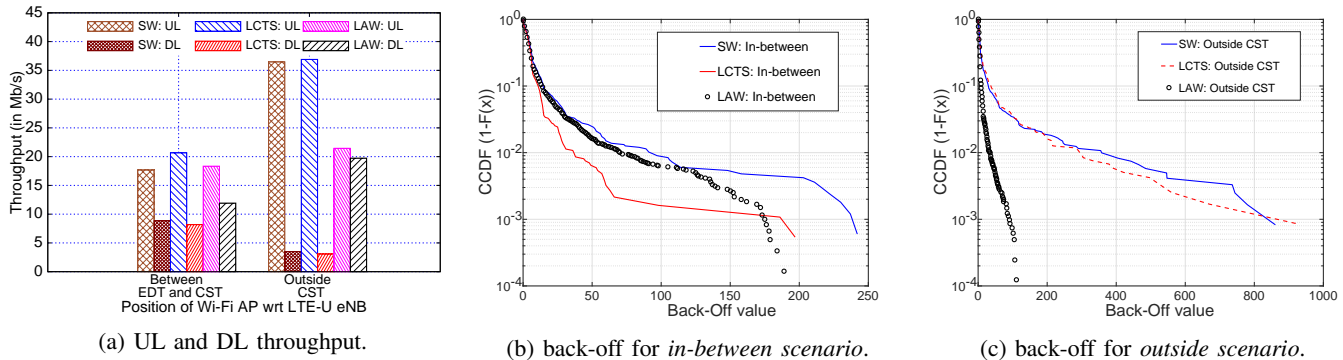


Fig. 16: Variation in throughput and back-off for SW, LCTS, and LAW schemes in UL+DL traffic case.

simulated results justifies correctness of the above propounded analysis for the proposed LAW scheme.

### C. Results for UL + DL traffic case

The issues concerning the imbalance or unfairness among the UL and DL traffic were discussed in Section II-2. The proposed LAW scheme helps overcome these issues by the following. Firstly, the Wi-Fi AP is allowed to intelligently schedule its users (by deferring from transmitting to the victim users in the LTE-U ON period), and thus prevents the packet losses due to inter-RAT interference. This avoids the exponential growth of the contention window and thereby helps in maintaining quick channel accessibility. Secondly, by maintaining a fair channel accessibility, the non-victim user would not get a chance to completely occupy the channel, which avoids the second issue discussed in Section II-2. Apparently, as shown in Fig. 16a, UL throughput still remains higher than DL throughput because of the fact that more number of devices contribute to the UL throughput. In addition, Figs. 16b and 16c also confirm this behavior by demonstrating the CCDF of back-off values chosen by the Wi-Fi AP.

### D. Results of DL with varying UL traffic

Although, the above results conveyed a rosy side the proposed LAW scheme in various scenarios, these scenarios however do not cover all the practical deployment scenarios. A more general case is when DL traffic is saturated while UL is unsaturated, which finds its applications in web-browsing, live streaming, etc. Furthermore, we have presented the averaged behavior of the schemes by randomizing the user placement. We varied the UL traffic from an extreme low load (of 10 packets/sec) to a significantly high load (of 1280 packets/sec), essentially spanning the whole range from almost no UL traffic to a saturated UL traffic. User placement follows a random distribution with 10 users being placed randomly and uniformly in a circular region around the AP of radius 50 m. The simulation is performed for 100 different seeds. An important statistic observed here was the average percentage of victim users, with the numbers being 45% and 22% for the *in-between* and *outside* CST, respectively. The UL and DL throughputs are captured in Fig. 17.

The first observation (from Fig. 17a) is that the proposed LAW scheme leads the DL performance which is followed by LCTS scheme while the lowest is SW. The explanation for this follows from Section VII-C where the two reasons

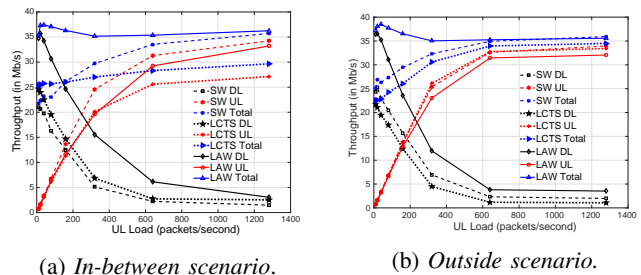


Fig. 17: Variation in Wi-Fi network throughput for all three schemes for in-between, and outside CST scenarios.

highlighted hinder SW the most. While LCTS saves itself, its conservative nature does not allow its throughput to be maximized. However, LAW uses the intelligence imbibed and hence achieves the maximum in terms of DL throughput. The second observation is in the behavior of the UL throughput. Here, UL throughput of SW scheme is highest because, DL being lowest confirms that the Wi-Fi AP's access is affected and hence non-victim users now dominate the network. On the contrary, the UL throughput is lowest in LCTS scheme as in the process of preserving the access of Wi-Fi AP, it has also limited other non-victim users accessing the channel. LAW is in the middle—not allowing the UL to dominate and as well not limiting it too much, ensuring a good UL-DL balance. Finally, as for the total throughput is concerned, LAW wins over both LCTS and SW by utilizing the channel most efficiently. Similarly, moving on to Fig. 17b, all the explanations given for the *in-between* scenario hold here as well. By resolving the issues faced in SW and LCTS (discussed in Section VII-C), the proposed LAW scheme shows an advantage over these schemes in the DL throughput as well as the total throughput.

### E. Results of varying number of victim and non-victim users

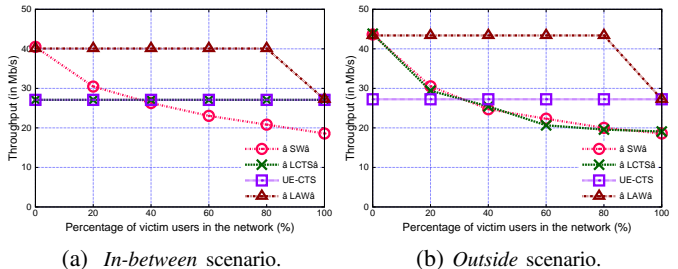


Fig. 18: Wi-Fi network throughput with varying percentage of victim users in the network—achieved by altering the number of victim and non-victim users from 0 to 10, while maintaining the total user count at 10.

One of the important aspects to study how the proposed LAW mechanism behaves when the ratio of the victim and non-victim users gets unbalanced—including when there are no victim users or no non-victim users in the network. In Fig. 18a, as for SW, when there are no victim users, there will not be any packet loss in the LTE-U ON period, and as a result it achieves a throughput comparable to that of LAW. But as the percentage of victim users increases, the losses increase and the throughput of SW degrades substantially. On the other hand, LCTS being conservative, its throughput remains at a constant level. Finally, LAW by intelligently scheduling its users, not only remains at a constant level but also achieves improved throughput compared to the former two schemes. However, it experiences a dip when there are no non-victim users, simply because it has to squander the entire LTE-U ON period, therefore performs similar to the LCTS scheme. Similarly, in Fig. 18b, both SW and LCTS experience a considerable throughput degradation with increase in percentage of victim users, while LAW remains robust to such an increase and performs substantially better than the other two schemes.

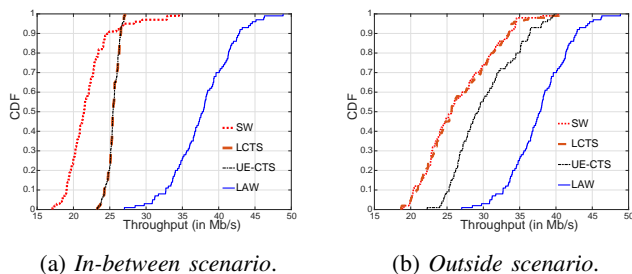


Fig. 19: CDF of Wi-Fi network throughput for SW, LCTS, UE-CTS, LAW schemes for in-between and outside CST scenarios.

#### F. Results by varying placement of users

An important aspect which justifies the employment of LAW is the improvement in the average throughput of the network. For this, 10 users are placed *uniformly at random* in a circle of radius 50 m from the Wi-Fi AP. The CDF of user throughput is generated by repeating the experiment 100 times, each time with different user placement. Figs. 19a and 19b demonstrate the advantage of the proposed scheme, even in an average sense. It provides an improvement of 83.63%, 54.69%, and 54.7% compared to SW, LCTS, and UE-CTS schemes, at the median for the *in-between* scenario and 47.27%, 47.3%, and 31% for the *outside* scenario, respectively.

### VIII. CONCLUSIONS

This paper focused on achieving efficient coordination to improve coexistence between LTE-U and Wi-Fi networks. We observed the degradation in the performance of Wi-Fi network in different hidden terminal scenarios. Further, to resolve this issue, we have proposed a decentralized approach, LAW mechanism, by reusing Self-CTS frame of Wi-Fi to aid in inter-RAT (*i.e.*, LTE-U and Wi-Fi) coexistence. The proposal of utilizing the reserved fields in Self-CTS frame has the potential for designing coexistence schemes for RATs willing to co-exist with Wi-Fi. In addition, the proposed scheme is

also modeled, providing a new direction to analyze problems involving arrivals of deterministic signals (like the Self-CTS in LAW mechanism) into the Wi-Fi network. Finally, the efficacy of the proposed LAW mechanism is shown in various scenarios by conducting extensive number of experiments.

### REFERENCES

- [1] A. M. Baswade, T. A. Atif, B. R. Tamma, and A. A. Franklin, "LAW: A Novel Mechanism for Addressing Hidden Terminal Problem in LTE-U and Wi-Fi Networks," *IEEE Communications Letters*, vol. 22, no. 4, pp. 1280 – 1283, 2018.
- [2] Cisco, "Cisco Visual Networking Index: Global Mobile Data Traffic Forecast Update, 2016-2021." Cisco White Paper, Feb 2017.
- [3] LTE-U Forum, "LTE-U SDL Coexistence Specifications." [online] <http://www.lteuforum.org/documents.html>, 2015.
- [4] Qualcomm, "LTE in Unlicensed Spectrum: Harmonious Coexistence with Wi-Fi." Qualcomm White Paper, June 2014.
- [5] LTE-U Forum, "LTE-U SDL CSAT Technical Specifications V1.0." [online] <http://www.lteuforum.org/documents.html>, 2015.
- [6] A. M. Baswade, T. A. Atif, B. R. Tamma, and A. A. Franklin, "A Novel Coexistence Scheme for IEEE 802.11 for User Fairness and Efficient Spectrum Utilization in the Presence of LTE-U," *Computer Networks*, vol. 139, pp. 1–18, 2018.
- [7] E. Chai, K. Sundaresan, M. A. Khojastepour, and S. Rangarajan, "LTE in Unlicensed Spectrum: Are We There Yet?," in *Proc. of the 22nd Annual International Conference on Mobile Computing and Networking*, pp. 135 – 148, Mobicom ACM, October 2016.
- [8] J. Jeon, H. Niu, Q. C. Li, A. Papatthassiou, and G. Wu, "LTE in the Unlicensed Spectrum: Evaluating Coexistence Mechanisms," in *Proc. of Globecom Workshops (GC Wkshps)*, 2014, pp. 740–745, IEEE, 2014.
- [9] 3GPP, "3GPP-TSG-RAN-WG1: Evolved Universal Terrestrial Radio Access," Tech. Rep. TR 36.814 V9.0.0, 2010.
- [10] F. M. Abinader, E. P. Almeida, F. S. Chaves, A. M. Cavalcante, R. D. Vieira, R. C. Paiva, A. M. Sobrinho, S. Choudhury, E. Tuomaala, K. Doppler, *et al.*, "Enabling the Coexistence of LTE and Wi-Fi in Unlicensed Bands," *IEEE Communications Magazine*, vol. 52, no. 11, pp. 54–61, 2014.
- [11] A. Babaei, J. Andreoli-Fang, Y. Pang, and B. Hamzeh, "On the Impact of LTE-U on Wi-Fi Performance," *International Journal of Wireless Information Networks*, vol. 22, no. 4, pp. 336–344, 2015.
- [12] A. M. Cavalcante, E. Almeida, *et al.*, "Performance Evaluation of LTE and Wi-Fi Coexistence in Unlicensed Bands," in *Proc. of IEEE 77th Vehicular Technology Conference (VTC Spring)*, 2013.
- [13] Nihar Jindal and Donald Breslin, "LTE and Wi-Fi in Unlicensed Spectrum: A Coexistence Study." Google white paper, 2016.
- [14] T. Nihtilä, V. Tykhomyrov, O. Alanan, *et al.*, "System Performance of LTE and IEEE 802.11 Coexisting on a Shared Frequency Band," in *Proc. of IEEE Wireless Communications and Networking Conference (WCNC)*, 2013.
- [15] E. Almeida, A. M. Cavalcante, *et al.*, "Enabling LTE/WiFi Coexistence by LTE Blank Subframe Allocation," in *Proc. of IEEE International Conference on Communications (ICC)*, 2013.
- [16] S. Sagari, S. Baysting, D. Saha, I. Seskar, W. Trappe, and D. Raychaudhuri, "Coordinated Dynamic Spectrum Management of LTE-U and Wi-Fi Networks," in *Proc. of IEEE International Symposium on Dynamic Spectrum Access Networks (DySPAN)*, pp. 209–220, September 2015.
- [17] Q. Chen, G. Yu, A. Maaref, G. Li, and A. Huang, "Rethinking Mobile Data Offloading for LTE in Unlicensed Spectrum," *IEEE Transactions on Wireless Communications*, vol. 15, no. 7, pp. 4987 – 5000, July 2016.
- [18] Q. Chen, G. Yu, H. Shan, A. Maaref, G. Y. Li, and A. Huang, "Cellular Meets WiFi: Traffic offloading or resource sharing?," *IEEE Transactions on Wireless Communications*, vol. 15, no. 5, pp. 3354–3367, 2016.
- [19] A. M. Baswade, T. A. Atif, B. R. Tamma, and A. A. Franklin, "LTE-U and Wi-Fi Hidden Terminal Problem: How Serious is it for Deployment Consideration?," in *Proc. of 10th International Conference on Communication Systems & Networks (COMSNETS)*, pp. 33–40, IEEE, 2018.
- [20] 3GPP, "TSGRAN; Study on LAA to Unlicensed Spectrum," Tech. Rep. TR 36.889 V13.0.0, June 2015.
- [21] I. S. Association *et al.*, "802.11-2012-IEEE Standard for Information technology–Telecommunications and information exchange between systems Local and metropolitan area networks–Specific requirements Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications," 2012.

- [22] H.-A. Safavi-naeini, S. Roy, and S. Ashrafi, "Spectrum Sharing of Radar and Wi-Fi Networks: The Sensing/Throughput Tradeoff," *IEEE Transactions on Cognitive Communications and Networking*, vol. 1, pp. 372 – 382, December 2015.
- [23] C. C. Caiado and P. N. Rathie., "Polynomial Coefficients and Distribution of the Sum of Discrete Uniform Variables." [online] <http://community.dur.ac.uk/c.c.d.s.caiado/multinomial.pdf>, Accessed on Apr 2017.
- [24] ITU, "IMT traffic estimates for the years 2020 to 2030." Report ITU-R M.2370-0, [https://www.itu.int/dms\\_pub/itu-r/opb/rep/R-REP-M.2370-2015-PDF-E.pdf](https://www.itu.int/dms_pub/itu-r/opb/rep/R-REP-M.2370-2015-PDF-E.pdf), July 2015.